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Static Universe and Nebular Red Shift

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Hubble's recent nebular counts have brought the question of a static universe into the forefront. The red shift of the nebulae in a static universe can be explained if instead of postulating a growing world radius we proceed on the assumption of a shrinkage of the universal lengths of atomic physics, this being equivalent to a decrease with time of the quantum of action. A new quantity, $\dot{h}$, is thereby defined and, if introduced into the Dirac equation of the electron, leads to gravitation. The consequences of this connection and other features of the theory are discussed and compared with experience.

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HUBBLE'S systematic nebular counts and his subsequent discussion of the consequences which follow for the various cosmological theories[1] give rise to grave doubts regarding the validity of the conception of an expanding universe. The uniform spatial distribution of nebulae up to a distance of about $1.6 \cdot 10^8$ parsecs suggests the idea of a static universe and reopens the question as to the physical causes of the red shift.

Before going into this question let us again regard the models of a closed world with an increasing radius R and try to determine the meaning of the assertion "the radius of the world increases with time." According to Laue[2] all bodies participate in the expansion, every material area being proportional to R^2. That would mean that the question of the dependence of the world radius on time could in principle be answered only by physical observation of processes in other parts of space (which for practical purposes must be at a very large distance from the observer), e.g., nebular distribution, Doppler effects and by considering certain statements capable of being proved by experiment with regard to the propagation of light in an expanding universe. On the other hand, according to this view, no answer could be obtained regarding the question of expansion by measuring physical processes at different times at one and the same point of space, seeing that together with every element of space its physical substance also undergoes a similarity transformation.

As against this the question must be asked whether the assertion "the radius of the world increases with time" does not in principle involve the existence of a constant, reproducible standard of length, compared with which R, or the distance between two nebulae, increases as time goes on. Such a standard of length cannot be looked for within the framework of the theory of relativity, which does not contain any assumption regarding the physical nature of matter, or, to be more exact, its atomistic structure. On the other hand, the universal lengths of atomic physics and the quantum theory are to be regarded as standard units to which the increase of R stands in relation. In other words, the assertion that R increases can have a meaning only if R is measured in units of the radius of the electron e^2/mc^2 or the Compton wave-length h/mc or the radius of the hydrogen atom in its ground state $h^2/(4\pi^2me^2)$. If, however, one accepts this relation to a standard length, the above mentioned assertion can also be formulated as follows: "Expressed as fractions of the constant world radius R, the radius of the electron and the other universal lengths shrink with time." This assertion is equivalent to the above one; it has, however, the advantage of postulating a static cosmos which is in accordance with Hubble's nebular counts. The dynamics of expansion are transferred into the dimensions of atomistic phenomena where expansion appears as its reverse-shrinkage.

Now it is well known that the atomistic quantities of the dimension of a length are re-

[1] Nature **138**, 1001 (1936).
[2] Laue, Berl. Ber. 1931, 123; Zeits. f. Astrophys. 208 (1936).

ductible to the two universal lengths e^2/mc^2 and $\hbar/mc$, the quotient of which represents the fine structure constant $\alpha = 2\pi e^2/hc$. It can be said, therefore, that all these lengths are proportional to Planck's h, the main significance of which, as has often been pointed out,[3] lies in the introduction of a new standard of length into physics. We shall now try to interpret the shrinkage of the atomic standards of length as a decrease with time of the quantum of action. Assuming the velocity of light to remain constant and taking into consideration the fact that the dimensionless fine structure constant also must remain constant with time for reasons of quantum mechanical stability, it follows that the elementary quantum of electricity also undergoes a shrinkage in such a way that e^2 shrinks at the same rate as h.

We therefore postulate that a static universe with a quantum of action decreasing with time is equivalent to an expanding universe with a constant quantum of action. This equivalence does not, of course, signify an isomorphy of the two models; that is evident from the above mentioned fact that the first model correctly describes the observed constant nebular density, while the second finds itself in contradiction to this phenomenon. The new model must, however, describe those features of the expanding universe which relate to the propagation of light, particularly the nebular red shifts which increase with the distance. That it does do so is evident under the assumption that the energy $\epsilon = h\nu$ connected with every definite quantum process remains constant in time. The more remote the observed nebula is, the more remote in time therefore the quantum process which produced the light which we observe, the greater must have been Planck's h, the smaller therefore the emitted frequency. The propagation of light in empty space and its measurement at the point of observation, which in practice is carried out by dispersion or diffraction, can be described by means of the wave theory. And since the proper time of the light wave is zero, the emitted frequency ν is transmitted to us without undergoing any change; this frequency, as compared with that which is produced today by the respective quantum process, therefore displays a red shift.

The decrease with time of h can be calculated from Hubble's distance red shift relation of spiral nebulae. By differentiating $\epsilon = h\nu$ with respect of time we get

$$0 = \dot{h}\nu + h\dot{\nu}$$

or

$$\dot{h} = -h\frac{\dot{\nu}}{\nu} = -h_0\frac{\dot{\nu}}{\nu_0} = -\frac{h_0}{\nu_0}\frac{\nu_0 - \nu}{t_0 - t}.$$

The indices refer to the present time. Furthermore we have

$$\nu = \nu_0(1 - u/c),$$

where u is the radial velocity of the nebula and c the velocity of light. We, therefore, get

$$\dot{h} = -h_0\frac{u/c}{t_0 - t}. \qquad (1)$$

Because of the uncertainty in determining the distances of the nebulae, the velocity of expansion, as is well known, is known to us only with a low degree of accuracy. According to Ten Bruggencate[4] the error can be as great as 50 percent. Of late the value has been estimated as 560 km/(sec. 10^6 parsec). For reasons which will soon be clear to us we shall assume the true value to be about 14 percent lower, e.g. 485 km/(sec. 10^6 parsec). Then there follows from (1)

$$\dot{h} = -1.03 \cdot 10^{-43} \text{ erg}. \qquad (2)$$

For obvious reasons a linear decrease of h makes no sense except as a first approximation, say for a period of 10^8 years. The true form of the function $h(t)$ can only be such that h vanishes asymptotically for $t = \infty$. We are not contradicted by experience if we assume an exponential decrease of h, i.e. $h = h_0 \cdot e^{-\kappa t}$. The quotient $\kappa = \dot{h}/h$ is then a constant, i.e., the well-known Hubble factor.

2

The hypothesis of a static universe with shrinking quantum of action which replaces a constant by a decreasing physical quantity, however slow the decrease, can only be justified either by the discovery of new physical relations,

[3] B. Podolsky, Phys. Rev. **46**, 734 (1934); compare e.g., the quotation from L. L. Whyte mentioned there.

[4] Ten Bruggencate, Naturwiss. **24**, 609 (1936).

e.g., between quantum and cosmic phenomena or if it permits a simpler description of certain empirical facts.

The shrinkage of h could be proved, in principle, by measuring the wave-length of a spectral line, which is produced at the same point at two different times, since the decrease of h provides us with a new way for distinguishing the future from the past. The frequency of every spectral line is in the course of time shifted towards the violet. It can easily be shown that the wave-length of the red cadmium line would decrease within 350 years by one unit of the third decimal; in other words, we cannot, for the time being, count on this experimental proof.

It is possible to establish a relation between the new quantity $\dot{h}$ and gravitation, since the gravitational constant G can be shown to be proportional to $\dot{h}$. Since $\dot{h}$ has the dimension of an energy we must have

$$[G]=[l/m^2]\cdot\dot{h}.$$

The lengths as well as the masses must be atomic quantities. By assuming $\dot{h}$ to be equal (2) we in fact get

$$G=\frac{2\pi e^2\dot{h}}{M^2mc^2}, \tag{3}$$

where e is the charge of the electron, M and m the masses of the proton and the electron respectively and c the velocity of light. If one writes (3) in the form

$$\frac{GMm}{e^2}=\frac{2\pi\dot{h}}{Mc^2}, \tag{4}$$

it becomes evident that the relation of the gravitational energy of the hydrogen atom to its Coulomb energy which stands on the left side of the equation can be expressed by $\dot{h}$ and the rest energy of the atom.

The relation between G and $\dot{h}$ which is expressed in equations (3) and (4) can also be derived by introducing h as a function of time, and generally, by introducing the exponential shrinkage of all lengths into the quantum mechanical calculus and by correspondingly transforming the Dirac equation of the electron.[5] This operation yields an additional term for the energy of the system which can be interpreted, in accordance with (3) or (4), as the gravitational energy of the hydrogen atom. If we also take into account that Dirac's coefficients α_κ are now functions of the coordinates, and if we substitute for

$$\alpha_i\alpha_\kappa+\alpha_\kappa\alpha_i=2\delta_{i\kappa}$$

the more general relation $\alpha_i\alpha_\kappa+\alpha_\kappa\alpha_i=2g_{i\kappa}$, then the $g_{i\kappa}$ which results from the new α_κ yields the metric of the world which contains the shrinking hydrogen atom. The result is a hyperbolic world with the constant radius

$$R=hc/\dot{h}=1.9\cdot 10^{27}\text{ cm.} \tag{5}$$

The value of (5) is in good agreement with the other values of R which have been obtained in different ways. From Eqs. (5) and (3) it can also be deduced that, if h is constant, gravitation vanishes and the finite curved universe degenerates into an infinite Euclidean space.

The attempt has often been made to establish relations between G and various combinations of the fundamental units of atomic physics. Stewart[6] makes the assumption that the red shift can be expressed by $\nu=\nu_0\cdot e^{-x/H}$ and gets the result that the length H which is identical with the world radius can easiest be expressed, with a 20 percent deviation from the experimental data, by the combination $H=e^6/(hGm^3c^3)$. The fact that the introduction of $\dot{h}$ permits the so much simpler relation (3) or (5), is an argument in favor of the reality of $\dot{h}$.

3

The relation between $\dot{h}$ and gravitation leads to further consequences which are susceptible of experimental proof. According to (3) G is not a constant, but, since it is proportional to e^2 and $\dot{h}$, a quantity which decreases with h^2. If one considers that the effect of the gravitational forces upon the dynamics of the creation of stars and stellar systems is determined by the product GM, where M is the mass of the system,

[5] This calculation and the following ones in which I was greatly aided by Mr. M. Schiffer, with whom I also had many valuable discussions of the problem, will be given more fully in a separate paper.

[6] Stewart, Phys. Rev. **38**, 2070 (1931).

it becomes obvious that, on the basis of an average value for GM, the masses of those stars which originated earlier must have been smaller in the proportion in which G was greater. The most reliable data on stellar masses have been obtained by the observation of binary stars. Nernst[7] has recently investigated the statistical material relative to about 60 binary stars. In doing so he makes two assumptions which today are shared probably by most astrophysicists: 1. The Russell diagram is a purely evolutionary diagram; 2. The duration of life of a star is approximately 10^{10} years. Nernst arrives at the result that in the course of their development the stars lose considerably more of their mass than can be accounted for by radiation, on the basis of Einstein's law of equivalence. According to Table II[7] this decrease in mass is about 100 : 1 in 10^{10} years, compared with which the relativistic decrease is negligible.

On the basis of the above considerations, the "nonrelativistic" decrease in mass can now be explained in the simplest way. Taking a mean GM as a basis, the older binary stars were characterized by a larger G, and therefore must have had a smaller mass. Numerical calculations lead to a satisfactory agreement with the data given by Nernst from a qualitative point of view, and for a period of 10^8 years there is also quantitative agreement.

As regards the influence of the expansion of the universe on the luminosity of distant sources of light which is dealt with by Laue in the above mentioned papers, there is no difficulty in tracing its analog within the framework of our theory, if one assumes the sources of light (nebulae) to be black radiators. The constant of Wien's law of displacement $\lambda_{max}\cdot T=\text{const}$ is, as is well known, proportional to h and therefore decreases continuously. The wave-length λ_{max} of a nebula of a certain effective temperature therefore undergoes a shift towards the red in proportion to its distance from us and at the same time its luminosity is changed in such a way as though it belonged to an object the temperature of which was smaller by the factor $e^{\kappa t}$.

The energy density u of a nebula is proportional to T^4, where T is its effective absolute temperature. The factor of proportionality, according to Planck's law, is inversely proportional to h^3. We, therefore, get

$$u/\nu^3=\text{const.} \tag{6}$$

Against this increase in the energy density of radiation with time, there is a decrease in the gravitational energy of the cosmic masses (decrease of G), which probably has its origin in the reciprocal action of both energies in the nebulae; the increase of the one is compensated by the decrease of the other. The energy of radiation undergoes no change during its propagation, while in an expanding universe it decreases on its way through space in the measure in which it participates in the expansion.

It has already been pointed out that the assumption of a shrinking h entails a new physical criterion for distinguishing the future from the past. Of two states of the universe the one with the smaller h is to be regarded as the later. Without any doubt there is a connection between this criterion and the entropy law. At any rate, there is no contradiction between the two. The entropy constant of an ideal monoatomic gas, e.g., is proportional to $\log 1/h^3$. For this reason the entropy of a gas of this kind increases in a static universe with decreasing h, even if it is subject only to reversible changes of state.

The question of how far the hypothesis whose fundamentals have been outlined here is tenable and capable of being developed further, depends not only on its compatibility with experience but also to a very high degree on the progress of quantum mechanics and quantum electrodynamics, and in particular on the possibility of a more exact treatment of the many-electron problem within the framework of Dirac's theory.

[7] Nernst, Zeits. f. Physik **97**, 511 (1935).