The Efficiency of Counters and Counter Circuits

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(Received June 17, 1937)

Existing formulas for correcting counter data for "missing" are found to apply to conditions which often cannot be realized in practice. Let τ_c, τ_i, and τ_r be the recovery times of the counter, amplifier and recording unit. It is shown that excellent performance can be obtained by using an amplifier which controls the voltage recovery of the counter so that $\tau_c=\tau_i=$ constant. When the counter receives f particles/sec., capable of activating it, the efficiency of the entire apparatus is $(1+f\tau_i)^{-1}$ if $\tau_r<\tau_i$. It is $e^{-f(\tau_r-\tau_i)}/(1+f\tau_i)$ if $\tau_r>\tau_i$ and the recorder is not influenced by any impulse which reaches it while it is operating.

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THERE has been considerable misunderstanding as to the correction of counter data for particles missed because of the finite recovery times of the counter and of its recording circuits. Formulas for this purpose published by different authors are not identical and closer inspection shows that they really refer to different arrangements of apparatus. These formulas can be used to correct data obtained under the conditions assumed in their derivations, but often these conditions are not realized in practice. All formulas proposed up to the present contain only a single recovery time. Some authors have considered the recovery time of a counter fed with a random distribution of events, paying no attention to the recording circuits. Correction equations obtained in this way can be used only when the recovery times of the circuit-elements are negligible compared to that of the counter. Conversely, other authors have discussed the behavior of a hypothetical recording circuit characterized by a single recovery time, neglecting the counter recovery time and the fact that the impulses from a counter do not form a random distribution. In this paper we derive new formulas, taking into account all recovery times which affect the over-all counting efficiency, and show how apparatus can be arranged to make these formulas valid.

Suppose that on the average a counter receives f particles per unit of time, following the Bateman distribution law, and that the recovery time is τ_c. This quantity is variable and its highest value will be called $\tau_{c\,\max}$. For present purposes we can treat the amplifier as a single unit with recovery time τ_i, and the recorder as another unit with recovery time τ_r. The word "recorder" includes both the mechanical recorder and any circuit which precedes it to scale down the counts. In such a case τ_r is the recovery time of the first stage in the scaling-down circuit.

The number of counts recorded per unit of time is f', and the problem is to get f from observations of f' and of the three recovery times, $\tau_{c\,\max}$, τ_i, and τ_r.

Skinner[1] has given correction formulas and curves for the tube counter itself. Of necessity, his very complete analysis is based on certain reasonable assumptions about the mechanism of recovery, derived partly from Danforth's[2] experiments on the variation of counter voltage during a "kick." To avoid extreme complexity, Skinner neglects the variation of efficiency with voltage, and the fact that τ_c cannot be less than the duration τ_d of the discharge in the tube counter. Nevertheless, his treatment is the best we have, and until detailed experimental work indicates the necessity of modifications, it may be used to correct counter data when certain conditions are satisfied. First, the recovery times of the amplifier and recorder should be small compared with $\tau_{c\,\max}$. Second, the average counting rate should be small compared with the reciprocal of τ_d.

2. EFFICIENCIES OF RECORDERS FED WITH RANDOM EVENTS

We now consider cases in which the counter and amplifier are very fast compared with the

[1] Skinner, Phys. Rev. **48**, 438 (1935).
[2] Danforth, Phys. Rev. **46**, 1026 (1934).

recorder. More specifically, it is supposed that the recorder receives a Bateman distribution of impulses. Previous authors have not pointed out that there are two types of recorders, with different efficiencies.

Type I is exemplified by the Cenco recorder. Suppose that a current impulse excites the magnet of such a recorder, moving the ratchet wheel ahead one tooth, and that the ratchet pawl starts to fall back. If a second impulse arrives before the cycle of movement is complete, the pawl simply slides up over the tooth on which it is riding, and starts back again. The result is that the second impulse is not counted, and the recovery time after the second impulse is shorter than that required for the whole cycle of movement. Thus a recorder of this type has a variable recovery time when counting random impulses. The average recovery time depends on the counting rate and on the details of the ratchet motion, so no general statements can be made about it. Nevertheless the efficiency can be obtained, for an impulse coming at an arbitrary instant will be recorded only if the recorder has not received another in a period equal to the maximum recovery time τ_r. This simple argument was given by Volz[3] but he did not mention the fact that it applies only to ratchet-like recorders. His efficiency formula is

$$f'/f=e^{-f\tau_r}. \quad (1)$$

Type II. The second type of recorder is one which goes through its cycle after an impulse without paying any attention to the arrival of additional ones. A properly designed thyratron set approximates this behavior closely. (This is true even when a ratchet-type recorder is used after the final pair of tubes, provided the recovery time of this pair is larger than that of the mechanical recorder.) The efficiency formula is

$$f'/f=1/(1+f\tau_r). \quad (2)$$

Skinner mentioned this result, but did not give the simple proof, which is as follows. Consider the interval τ_r following the arrival of an impulse. The average number of impulses lying in this interval is $f\tau_r$, but there are f' such intervals per second, so $f'f\tau_r$ impulses are missed per second. Thus $f=f'+f'f\tau_r$, which gives Eq. (2).

As f increases, the counting rate of a Type I recorder passes through a maximum value, $1/e\tau_r$, and decreases again; the recorder is said to "jam." A Type II recorder simply goes up to the maximum possible counting rate, $1/\tau_r$, which is e times larger than the value for a Type I recorder. In this laboratory we have counting outfits which behave approximately in this way. For one of them, we found a recovery time of 3.5 milliseconds from oscillograph observations. The maximum counting rate was very close to 300/sec., leading to a recovery time of 3.3 milliseconds. Such high-speed tests can be avoided if one has two similar radioactive sources of known ratio, and of suitable strength. If the numbers of particles supplied to the counter by the sources are f_1 and f_2, we write $R=f_2/f_1$. Then Eq. (2) shows that

$$\tau_r=(Rf_1'-f_2')/[(R-1)f_1'f_2']. \quad (3)$$

3. Desirable Characteristics of an Apparatus for Fast Counting

We now consider how the entire counting apparatus can be designed to make it follow simple efficiency formulas to a high degree of approximation. There are two general types of amplifying equipment. The first is exemplified by any amplifier which has the conventional input and no arrangements by which the amplifier can control the course of voltage recovery in the counter. By "conventional input" we mean that the counter wire is separated from ground by a high resistance and is coupled to the grid of the first tube through a condenser, a grid resistance of the order of 1 megohm being provided. As an idealization, one may say that the grid has no influence on the recovery of the counter, and it will be assumed that the remainder of the amplifier follows the first stage perfectly. It is easy to arrange matters so that the recovery time of the input grid obeys the conditions $\tau_d<\tau_i<\tau_{c\ \mathrm{max}}$, or alternatively the condition $\tau_{c\ \mathrm{max}}<\tau_i$. No matter which choice we make, we can see from Skinner's discussion that the impulses handed on to the recorder will not form a Bateman distribution.

To avoid this complicated situation, one must

[3] Volz, Zeits. f. Physik **93**, 539 (1935); Schiff, Phys. Rev. **50**, 88 (1936).

use the Neher-Harper circuit[4] or any similar circuit which automatically lowers the counter voltage when a count occurs and raises it briskly to a level above threshold after the discharge ceases.[5] The important point is that the conditions $\tau_i = \tau_c =$ constant should be satisfied. Then the counter and amplifier will work together as a single unit with a definite recovery time. If the counter misses an event, the amplifier also does, and conversely the amplifier cannot miss any impulse from the counter. The amplifier gives $f/(1+f\tau_i)$ impulses per second and the value of the recorder efficiency depends on the relative sizes of the recovery times. If τ_r is less than τ_i the recorder follows the amplifier perfectly, no matter whether it is of Type I or Type II. Thus the efficiency of the entire counting apparatus is given by Eq. (2). If τ_r is greater than τ_i and the recorder is of a kind which cannot be reexcited while it is in action (Type II), the efficiency of the apparatus is

$$f'/f = e^{-f(\tau_r - \tau_i)}/(1+f\tau_i). \qquad (4)$$

To see this, we consider the interval τ_r following an amplifier pulse at time zero. There is certainly no additional pulse until time τ_i and the probability that none occurs in the interval τ_i to τ_r is $e^{-f(\tau_r - \tau_i)}$. This, then, is the probability that the recorder catches the next pulse, and the efficiency (4) of the whole apparatus is gotten by multiplying this factor by the efficiency of the counter and the amplifier. When $\tau_r > \tau_i$, the efficiency of a ratchet recorder (Type I) depends on the detailed nature of its cycle of motion. The only general statement which can be made is that the efficiency of the entire apparatus is not less than that given by Eq. (4).

[4] Neher and Harper, Phys. Rev. **49**, 940 (1936). We have employed a multivibrator circuit which is a modification of one described by Gingrich, Evans and Edgerton, R. S. I. **7**, 450 (1936); see their Fig. 6.

[5] If τ_i is made less than τ_d, reexcitation occurs. We have found it convenient to adjust τ_i to the optimum value by oscillographic observations. Starting with a low value, τ_i is increased until double amplifier pulses are eliminated.

When Eq. (4) holds true the counting rate rises to a maximum as f increases. This maximum occurs when

$$2\tau_i f = -1 + (1 + 4\tau_i/(\tau_r - \tau_i))^{\frac{1}{2}}. \qquad (5)$$

We are now in a position to make definite recommendations for choosing the essential constants of a counting apparatus which operates without jamming and with minimum loss up to the highest possible speed, and which has the simple correction formula (2) with τ_i substituted for τ_r. The amplifier should control the voltage recovery of the counter itself, and should feed the recorder unit with pulses whose duration is less than τ_r. The recovery time τ_r should be less than τ_i, and τ_i should be made as short as possible, always keeping in mind the requirement that deionization of the counter must be complete before the amplifier returns to its initial state. Circuits which fulfill these requirements will be described in another communication.

A word of caution as to the discharge time of tube counters is desirable. Different authors[6] have found wide variations in this time, depending on the size of the counter and the nature of the gas. Values from 10^{-2} to 2×10^{-5} seconds have been reported for counters operating at a few cm of mercury. It is not possible to say whether the higher values reported are due to the use of voltages too far above the threshold. However, we have observed a value of about 0.003 second for a counter 3.25 cm in diameter and 15 cm long, filled with dry tank hydrogen at atmospheric pressure. Larger counters filled in the same way are still slower.

The experimental work which led to the writing of this paper was facilitated by a grant from the National Research Council to one of us (A.E.R.). This aid is gratefully acknowledged.

[6] Medicus, Zeits. f. Physik **74**, 350 (1932); Hummel, Physik. Zeits. **35**, 997 (1934); Trost, Physik. Zeits. **36**, 801 (1935).