values of E_A from experiments on capture of slow neutrons. The γ-ray breadth is according to Bethe and Placzek[13] about 10^{-7} MV for an excitation energy of ~ 8 MV. If it is permissible to assume this value valid also for higher excitation, the capture of high energy neutrons ($E > 3$ MV) will be extremely improbable. Even a transition of the compound nucleus to a less energetic unstable state by emission of a γ-ray will be an improbable process since according to the above table the probability of neutron emission is in general several orders of magnitude greater than that for radiation.

I would like to express my thanks to Professor N. Bohr and to Dr. F. Kalckar for the fruitful collaboration which led to the production of this paper. I would also like to thank Drs. G. Placzek, L. Landau, and H. Bhabha for many valuable discussions I have had with them, and the Rask Oersted Fond for the opportunity to work at the Institute for Theoretical Physics at Copenhagen.

AUGUST 15, 1937 PHYSICAL REVIEW VOLUME 52

The Beta-Ray Spectrum of Radium E

JOHN S. O'CONOR, S. J.*
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received May 15, 1937)

The beta-ray spectrum of radium E was examined with a magnetic spectrometer, by using coincidence counting, under various conditions of source strength, mounting and aperture of defining slits. It is concluded that the experimental high energy end point depends on the source strength used, as well as the width of the defining slits. Data from all but very strong sources gave a K.U. plot which was linear within the limits of error set by statistical fluctuations and finite slit widths. Extrapolation of the K.U. plot gave 1.25 ± 0.03 Mev as the high energy end point.

SINCE the original work of Schmidt[1] in 1907 more than a score of workers[2–24] have made measurements on the beta-ray spectrum of radium E with none too concordant results. The disagreement between observers may be appreciated by consulting Table I wherein one finds values for the high energy end point which range from 4500 $H\rho$ up to 12,000 $H\rho$.

The importance of obtaining the true energy distribution curve for the disintegration electrons from radioactive bodies has been recently enhanced by the tentative success of the Konopinski-Uhlenbeck[25] modification of the Fermi[26] theory of beta-disintegration, especially as applied to the lighter artificially radioactivated elements. The present investigation was undertaken with the purpose of securing data from one of the naturally radioactive elements which could be used after the manner proposed by Kurie, Richardson and Paxton[27] as a further criterion in determining more conclusively the validity of the theory.

Fermi's original formula (44), reference[26] gives the probability of disintegration with the emis-

* Now at Woodstock College, Woodstock, Maryland.
[1] H. W. Schmidt, Physik. Zeit. **8**, 361 (1907).
[2] Baeyer, Hahn and Meitner, Physik. Zeits. **11**, 488 (1910); **12**, 273 (1911); **13**, 264 (1912).
[3] J. A. Gray, Proc. Roy. Soc. **87**, 487 (1912).
[4] J. Danysz, Ann. Chim. Phys. S8, **30**, 241 (1913).
[5] Kovarik and McKeehan, Phys. Rev. **8**, 574 (1916).
[6] A. V. Douglas, Proc. Roy. Soc. Canada **16**, 113 (1922); B. W. Sargent, P. C. P. S. **25**, 514 (1929).
[7] Curie and d'Espine, Comptes rendus **181**, 31 (1925).
[8] Yovanovitch and d'Espine, J. de phys. et rad. **8**, 276 (1927).
[9] E. Madgwick, P. C. P. S. **23**, 982 (1927).
[10] Gray and O'Leary, Nature **123**, 568 (1929).
[11] F. R. Terroux, Proc. Roy. Soc. **A131**, 90 (1930).
[12] N. Feather, Phys. Rev. **35**, 1559 (1930).
[13] B. W. Sargent, P. C. P. S. **28**, 538 (1932).
[14] F. C. Champion, Proc. Roy. Soc. **A134**, 672 (1932).
[15] J. A. Chalmers, P. C. P. S. **28**, 319 (1932).
[16] B. W. Sargent, Proc. Roy. Soc. **A139**, 659 (1933).
[17] H. O. W. Richardson, Proc. Roy. Soc. **A147**, 442 (1934).
[18] F. A. Scott, Phys. Rev. **48**, 391 (1935).
[19] Alichanow *et al.*, Nature **137**, 314 (1936); Physik. Zeits. Sowjetunion, Band 10, Heft. 1, p. 78 (1936).
[20] Ho and Wang, Chinese J. Phys. **2**, 1 (1936).
[21] Champion and Alexander, Nature **137**, 744 (1936).
[22] M. Lecoin, Comptes rendus **205**, 171 (1936).
[23] E. M. Lyman, Phys. Rev. **51**, 1 (1937).
[24] Langer and Whitaker, Bull. Am. Phys. Soc. **12**, No. 2, p. 32 (1937).
[25] Konopinski and Uhlenbeck, Phys. Rev. **48**, 7 (1935).
[26] E. Fermi, Zeits. f. Physik **88**, 161 (1934).
[27] Kurie, Richardson and Paxton, Phys. Rev. **48**, 167 (1935); **49**, 203 (1936).

sion of an electron having momentum between η and $\eta+\Delta\eta$, and the expression used therein is an approximation applicable to elements whose atomic number is in the neighborhood of 82.

By grouping all constants under the symbol K and writing for the probability of disintegration the number N, which is proportional to this probability, we may express Fermi's formula as follows:

$$N=K(\eta+0.355\eta^2)[(1+\eta_0^2)^{\frac{1}{2}}-(1+\eta^2)^{\frac{1}{2}}]^2. \quad (1)$$

Here $\eta=H\rho/1700$; where ρ is the radius of curvature of the electron trajectory in a magnetic field of strength H. η_0 is the maximum value of η.

The relation between the momentum η and the kinetic energy ω in mc^2 units may be expressed as follows:

$$\eta=(2\omega+\omega^2)^{\frac{1}{2}}; \quad \omega=(1+\eta^2)^{\frac{1}{2}}-1. \quad (2)$$

By substituting for $(1+\eta^2)^{\frac{1}{2}}$ its value $(\omega+1)$ and similarly for $(1+\eta_0^2)^{\frac{1}{2}}$ we can write $[\omega_0-\omega]^2$ for the last bracket of Fermi's equation; which yields:

$$c[N/(\eta+0.355\eta^2)]^{\frac{1}{2}}=\omega_0-\omega. \quad (3)$$

In the case of the heavy elements the only difference between the original Fermi formula and the modification introduced by Konopinski and Uhlenbeck is a change in the exponent of the final bracket which appears raised to the fourth instead of the second power.

If $[N/(\eta+0.355\eta^2)]^{\frac{1}{2}}$ be plotted against the energy of the disintegrating electrons a straight line should result if the Fermi theory be correct; and the intercept on the axis of abscissa will give the value of ω_0, the maximum energy

TABLE I. *Results of various observers on the high energy end point and number maximum of the continuous beta-ray spectrum of radium E.*

Investigator	Year	Number Maximum $H\rho$	High Energy End Point $H\rho$	Method	Remarks
Schmidt[1]	1907	2500	5500	Absorption and Magnetic deflection (photographic)	
Baeyer, Hahn and Meitner[2]	1911	"	"	Magnetic deflection (photographic)	
Gray[3]	1912		5150–5450	Absorption	
Danysz[4]	1913		5500	Magnetic focusing (photographic)	Found a doubtful band between 6000 and 12000 $H\rho$
Kovarik and McKeehan[5]	1916	2000	7000 (ca)	Magnetic focusing (point counter)	
Douglas[6]	1922		4990–5080	Absorption	Results converted by Sargent in 1929
Curie and d'Espine[7]	1925	2900	4500	Magnetic deflection (photographic)	Also found feeble band between 7000 and 10,000 $H\rho$
Yovanovitch and d'Espine[8]	1927		4500	Magnetic deflection, magnetic focusing (photographic)	Feeble band or group of lines between 7000 and 10,000 $H\rho$
Madgwick[9]	1927	2200	5000	Magnetic focusing (ionization chamber)	
Gray and O'Leary[10]	1929		less than 8200	Combination absorption and deflection (ionization chamber)	Estimate less than one atom in 25000, gives ray of 8200 $H\rho$ or greater; possibly none at all
Terroux[11]	1930		11,200	Cloud chamber	Probable error of 50%
Feather[12]	1930		5300	Absorption	
Sargent[13] from data of Varder and Madgwick	1932	1750	5100	Absorption	
Champion[14]	1932		5500	Cloud chamber	Estimates that beta-rays over 5500 $H\rho$ are less abundant than 1 in 2000
Chalmers[15]	1932			Absorption	Interprets "kink" in absorption curve near high energy end point as region of most rapid change of slope of distribution curve and not true end point
Sargent[16]	1932		5100 5000 5500	Absorption Magnetic Cloud chamber	Summary of all work done by various methods. Prefers value from cloud chamber
Richardson[17]	1934	850 (?)		Cloud chamber	Finds number of electrons in range between 0.01 and 0.065 Mev exceeds number attributed to usual maximum between 0.3 and 0.4 Mev
Scott[18]	1935	2480	6604±98	Magnetic focusing (Geiger-Müller counter)	
Alichanow *et al.*[19]	1935	1100 (ca)	5400	Magnetic focusing (Geiger-Müller counter)	No pronounced number maximum. Many low energy electrons
Ho and Wang[20]	1936		7500	Cloud chamber	Relatively few tracks; large probable error
Champion and Alexander[21]	1936		5975	Cloud chamber	End point value from extrapolation of K.U. plot of 1932 data[14]
Lecoin[22]	1936	1450	5100	Cloud chamber	Sharp number maximum with rapid decline on low energy side
Lyman[23]	1937	1750	5280±20 6050 (K.U.)	Magnetic focusing (G-M Counter)	
Langer and Whitaker[24]	1937		5330±70 6250 (K.U.)	Magnetic focusing (coincidence counting)	
O'Conor	1937	1650	5350±130 5650 (K.U.)	Magnetic focusing (Geiger-Müller single and coincidence counters)	

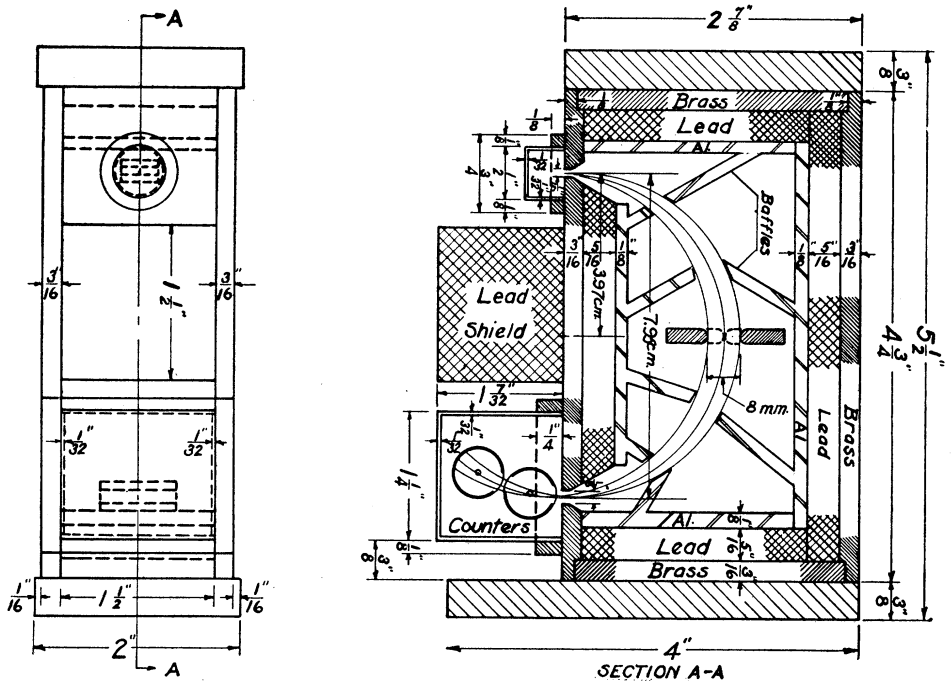

FIG. 1. Magnetic spectrometer.

possible for electrons from this particular type of disintegration.

Likewise a plot of the fourth root of the same function *versus* electron energy should show linearity if the Konopinski-Uhlenbeck modification of the Fermi theory be the correct one.

EXPERIMENTAL OBJECTIVE

The distribution in energy of electrons from radium E was observed over an extended period of time, under varying conditions of source strength, source mounting and defining slit widths so that the effect of the changes in these experimental factors might be followed and thus help to trace and eventually lead to the causes responsible for the discrepancies existing between the results of previous experimenters.

CONSTRUCTION AND OPERATION OF APPARATUS

The spectrometer proper is essentially the same as that originally used by Rutherford and Robinson.[28] The improvements which were adopted from those made by Henderson[29] and Alichanow[30] include special precautions in the form of baffles to prevent scattering as well as the use of Geiger-Müller double coincidence counters for detection purposes.

A diagram of the spectrometer is shown in Fig. 1. Electrons emerging from the source have their paths bent into the arc of a circle by a magnetic field applied at right angles to the plane of the paper.

The position of the receiving slit in front of the counters is fixed at a definite distance from the source (7.95 cm). This determines (within limits set by the finite widths of the defining slits) the radius of curvature which the electron track must possess in order that the electron itself may enter the counting chamber.

An electron of mass m and charge e, initially projected with a determined speed v, at right angles to the lines of force of a homogeneous magnetic field of strength H, will describe a circle of radius ρ.

Under such conditions the electron will be in dynamic equilibrium which may be represented

[28] Rutherford and Robinson, Phil. Mag. **26**, 717 (1913).
[29] W. J. Henderson, Proc. Roy. Soc. **A147**, 572 (1934).
[30] Alichanow *et al.*, Zeits. f. Physik **90**, 249 (1934); J. phys. et rad. S VII, Tome VII, No. 4, 163 (1936).

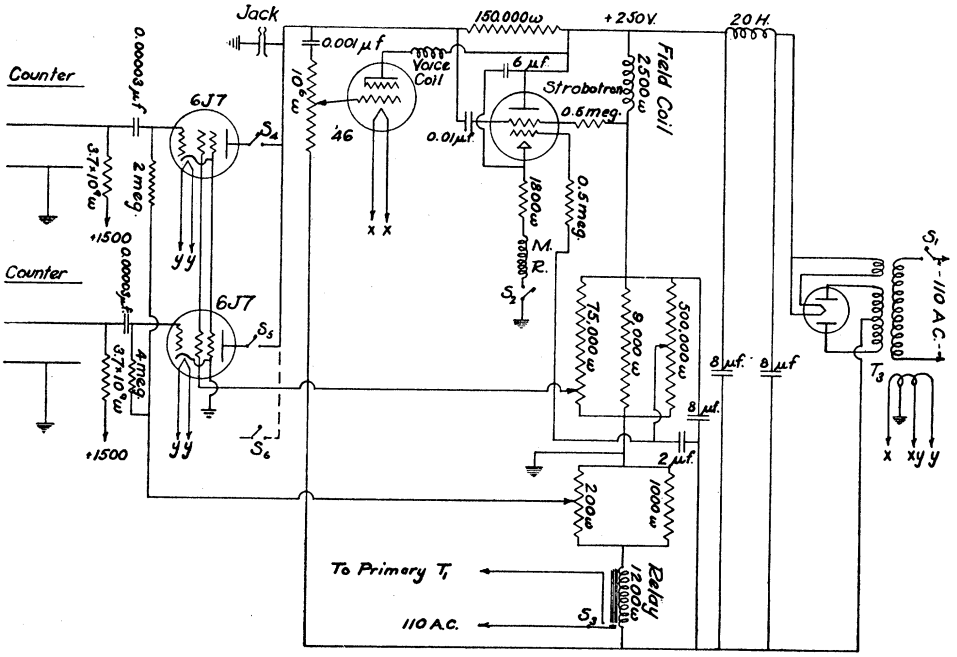

FIG. 2. Coincidence counter amplifier and power supply. S_3 acts as a time delay switch which retards the application of the potential in the high voltage circuit (not shown) until after the filament of the rectifier in that circuit is heated.

by the equation:

$$H\rho = mv/e. \tag{4}$$

If we use the relativistic form for the mass of the electron (required because of the high velocity of the beta-rays), $H\rho$ may be expressed in terms involving the kinetic energy, E, and the velocity of light c, as follows:

$$H\rho = (1/ce)(E^2+2mc^2E)^{\frac{1}{2}}. \tag{5}$$

Expressed in absolute electrostatic units (5) becomes:

$$H\rho = (1/e)(E^2+2mc^2E)^{\frac{1}{2}}. \tag{6}$$

SPREAD IN $H\rho$ DUE TO FINITE SOURCE AND SLIT WIDTHS

If in a spectrometer such as is shown in Fig. 1 we consider a receiving slit of width ϵ, a central defining slit of width δ and a source of width ξ it may be shown that to a first approximation:

$$\Delta\rho \cong \epsilon/2+\xi/2+\delta/8\rho_0. \tag{7}$$

ρ_0 is the radius of curvature corresponding to $\frac{1}{2}$ the distance from the center of the source to the outer edge (high velocity side) of the receiving slit.

The value of the above spectrometer constants as determined for computation and corrections made in this report are as follows:

ϵ, receiving slit width = 3.18 mm,
ξ, source width (Ra E on Ni) = 0.8 mm,
ξ', defining slit at source = 1.59 mm,
δ, central defining slit: variable from 0 to 8 mm
" " " Width = 1 mm for all readings except certain high energy end point readings specially noted,
$\rho_0 = 40.54$ mm,
$\rho_m = 39.95$ mm.

The mean value ρ_m is used in computing $H\rho$ for the distribution curves, and is one-half the measured distance between the outside edge of the source mounting and the center of the receiving slit.

The spread, which is determined from Eq. (7) gives the uncertainty in any mean value of $H\rho$ which will be $= \pm\frac{1}{2}H\Delta\rho$.

It is evident from Eq. (7) that the contribution to the spread in $H\rho$ due to the width of the central slit δ is small compared to that from the terms involving ϵ and ξ but, as will be shown later, the presence of this central slit system cuts down scattering between the two halves of the spectrometer chamber and for that reason all

readings, except certain special measurements on the high energy end point, were made with $\delta = 1$ mm.

Counting System

The circuit used for coincidence counting is a modification of that proposed by Rossi.[31] Type 6J7 tubes are used for the first stage of amplification. It was found that with the type '57 tubes used originally there was a complete paralysis of the tube action when the magnetic field of the spectrometer exceeded 4000 oersteds. With the metal tubes this paralyzing action was appreciably reduced, although the tubes were mounted directly over the spectrometer and magnet; and an additional cylindrical iron shield placed over the metal housing made it possible to attain a field well over 8000 oersteds without measurably disturbing the normal operation of the 6J7 tubes. The complete circuit, which uses a strobotron (neon filled, double grid, cold cathode discharge tube) to actuate the message register, is given in Fig. 2.

The high voltage supply for the Geiger-Müller counters is obtained by use of a circuit identical with that described by Gingrich.[32]

The necessary slits cut in the counter cathodes for the admission of beta-rays resulted in a distortion of the electric field and a consequent instability of the counters. This was remedied by covering the slits with aluminized Cellophane. The Cellophane used was obtained through the courtesy of the duPont Cellophane Co. and is 12.7 microns thick; equivalent to 0.00175 g/cm.^{2} Aluminum was deposited on it by evaporation to a thickness of the order of 0.1 micron. The material was then fastened over the slits with scotch tape in such a way that the metallized surface was in contact with the cathode cylinder beyond the limits of the slit.

The Cellophane with its aluminum coating is opaque to ultraviolet light and this serves the purpose of isolating the counters as well as stabilizing them.

In addition to the three aluminized windows covering the slits, a fourth (not metallized), is used to separate the counter chamber from the spectrometer proper. Such separation was necessary since the counters were operated at a pressure of 10.5 cm of Hg whereas the spectrometer was connected to a Hyvac pump which kept the pressure well under 1 mm of Hg during operation.

Correction for Stopping Power of Cellophane Windows

The correction giving the number of particles stopped by the windows for various electron energies, which must be added to the registered count, was made in a manner similar to that described by Scott.[18] Counting rates for beta-rays of various energies, in the region where this effect was known to be appreciable, were determined with three different thicknesses of Cellophane interposed between spectrometer and counter chamber. By plotting counting rate against Cellophane thickness for each definite energy a set of curves were obtained and under the assumption of linearity could be extrapolated back to give a counting rate for zero thickness of Cellophane. The correction became inappreciable above 2000 $H\rho$ and hence was applied only below that value.

An additional conclusion of importance resulting from the experimental determination of the stopping power of the Cellophane windows is the fact that there exists an apparent effective range for electrons in Cellophane practically identical with that found by Schonland[33] for aluminum. For despite certain irregularities in the counting rates (due to the difficulty of reproducing identical conditions of counter operation after changing window thickness), the cut-off at the low energy end was definitely abrupt.

This sharp cut-off at low energies is a powerful argument in favor of the absence of scattering in the spectrometer in which the measurements were made. For if electrons of energy higher than that proper to the beam being measured were being scattered onto the counter window, they would penetrate the same and be registered by the counting system,—thus rendering impossible the sharp low energy cut-off that was actually found.

Scott's Cellophane correction table does not reveal a similar cut-off despite the fact that the

[31] B. Rossi, Nature **125**, 636 (1930).
[32] N. S. Gingrich, R. S. I. **7**, 207 (1936).
[33] B. F. J. Schonland, Proc. Roy. Soc. **A104**, 235 (1923); **108**, 187 (1925).

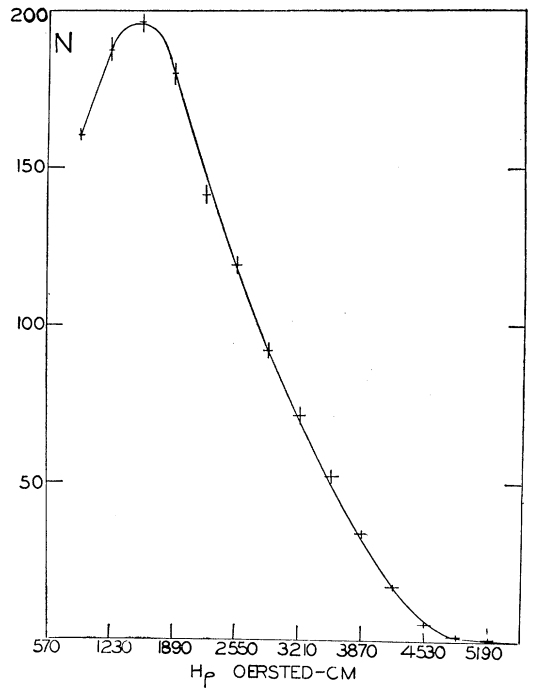

FIG. 3. Momentum ($H\rho$) distribution of electrons from radium E. Readings taken with source of intermediate strength and with central slit width of 1 mm. Corrections made for background, Cellophane absorption and finite resolving time of counters. Ordinates divided by their respective H values.

Cellophane used by him is twice as thick (linear dimension) as that used by the writer. Even allowing for the fact that there is not a linear relation between ordinary thickness and g/cm^2 for different grades of stock, an examination of Scott's tabulation of variation in counting rate with Cellophane thickness for various electron energies, would lead one to believe that electrons were getting into his counter far below the $H\rho$ cut-off value found by Schonland for the equivalent g/cm^2 of aluminum.

Finite Resolving Time of Counters

The determination of the actual number of particles reaching the counter from the number recorded by the message register requires a correction for counts missed due to the finite resolving time of the counter and associated recording apparatus.

The method for making this correction is treated by Schiff[34] and the procedure outlined by him was followed here.

[34] L. I. Schiff, Phys. Rev. **50**, 88 (1936).

The resolving time was determined by observing the maximum counting rate as a gamma-ray source was slowly brought in close proximity to the counter from a distance where the counting rate was well below this maximum. The relation $t=1/N_{max}e$ then gives the resolving time, where N_{max} is the maximum counting rate in counts per second. The value of t thus determined was 0.076 sec.

Measurement of the Magnetic Field

The magnetic field was first measured with a flip coil and flux meter. The constants of the flip coil were calculated from its dimensions, and the flux meter readings were corrected from a calibration curve made for the instrument in the department of electrical engineering. A plot of H *vs.* I was made and found to be linear throughout the region to be used for examination of the radium E spectrum.

An estimate of the inhomogeneity of the field was made from a flux plot of Dr. H. B. Dwight, professor of electrical machinery, of the department of electrical engineering, and it was found that in the region of greatest variation traversed by the electrons the inhomogeneity was less than 2 percent.

The plot of field strength against current was then checked against the four most prominent lines of the thorium B+C+C″ spectrum. A mean value for the $H\rho$ of each of these lines was taken from the recent determinations of Ellis[35] and Wang.[36] The true value of H at which these lines were recorded was determined by dividing their accepted $H\rho$ value by the radius of curvature of the spectrometer in use, and these four values of H were plotted, using the current values at which the lines came in as abscissae, and the mean value of H (obtained from the data of Ellis and Wang as explained above) for ordinates. These points coincided with the line from the flux meter data within an accuracy of 0.6 percent.

Measurements on the Radium E Spectrum

The radium E source was prepared on three different mountings; nickel, palladium and platinum. The nickel mounting consisted of a

[35] C. D. Ellis, Proc. Roy. Soc. **A138** 318, (1932).
[36] K. C. Wang, Zeits. f. Physik **87**, 633 (1933).

flattened wire 6 mm long, 3 mm wide and 0.8 mm thick. The Pd and Pt mountings were of the same dimensions in lengths and width; the thickness of the Pt being 0.5 mm and that of the Pd 0.4 mm.

The flattened face of each mounting was so oriented in the spectrometer housing that its plane made a grazing angle with the plane passing perpendicularly through the center of the source slit.

The spectrum from the source mounted on nickel was examined on two occasions separated by an interval of three months. The first test consisted of twenty-five explorations of the spectrum which were continued over the period of a month, the initial source strength being such that the total emission was approximately 10^7 beta-particles per second. Additional tests were then made with source mountings of Pd and Pt. In these latter cases the intensity of total emission was relatively weak compared to the maximum activity of the source on the nickel mounting. Several more tests, with a new source mounted on nickel of strength slightly greater than the original one, were then made as a final check.

Results

We may summarize the results as follows: Comparing source strengths of the same magnitude; no difference in the shape of the energy distribution curve resulted from changing the source mounting from nickel to platinum. The data from the source mounted on palladium are meager, but all indications were that the distribution was unaffected by the substitution of this mounting. These findings seem to eliminate any possibility of error from selective reflection due to the type of source mounting used.

The similarity of the distribution curves taken from sources mounted on the three materials extends from the high energy maximum down to 900 $H\rho$. Below 1000 $H\rho$ the error from the correction for absorption due to the cellophane windows was of such magnitude as to obscure any difference due to absorption in or selective reflection from the material of the source mounting.

In the explorations which attempted to cover the entire spectrum in an uninterrupted run, a source strength giving a total emission of less than 6×10^5 particles per second was employed.

For this limited source strength and below it all momentum (and energy) distribution curves were in agreement from the lower observable limit, 900 $H\rho$ (0.067 Mev), up to 4860 $H\rho$ (1.03 Mev). Beyond 1.03 Mev the shape of the distribution curve definitely became a function of the central defining slit width as well as of source strength.

A typical momentum ($H\rho$) distribution curve from weak and intermediate source strengths is shown in Fig. 3, while Fig. 4 shows the effect of variation in slit width on the experimental high energy end point.

In the case of the complete distribution curve, background readings have been subtracted, and necessary corrections for absorption in cellophane windows as well as for finite resolving time of counters have been made. Each counting rate has also been divided by the corresponding magnetic field strength, thus reducing the effective slit width to a common value for all parts of the spectrum.

The high energy end point readings of Fig. 4 are plotted as registered, without corrections.

When the source strength used was of the order of, or greater than, a total emission of 10^7 particles per second, only the high energy portion of the spectrum could be satisfactorily examined, because as the number maximum of the distribution curve was approached the increase in counting rate produced paralysis of the recording system. As the source strength was increased the "tail" of the distribution curve continued to extend to higher and higher energies (even with

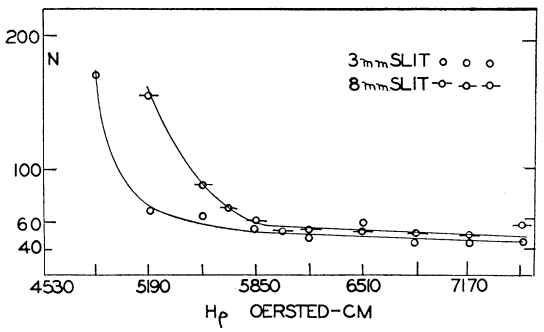

FIG. 4. End point counting rates with 3 and 8 mm central slit widths. Single G-M counter, with fairly strong source. Background included.

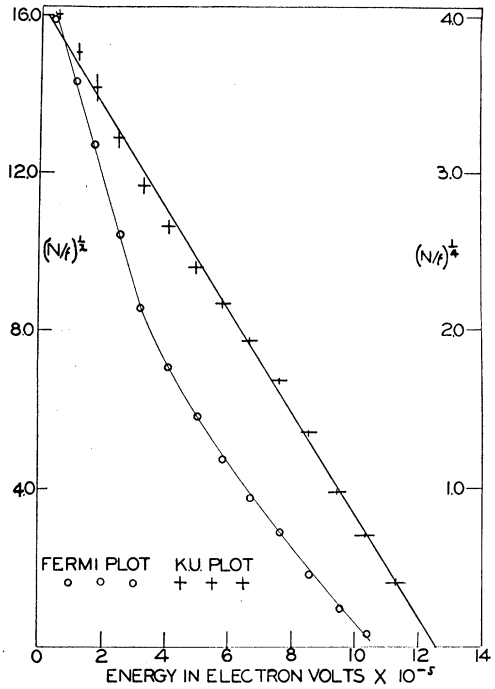

FIG. 5. Fermi and K.U. plots. Radium E. Number data averaged from four sets of readings.

minimum central slit width of 1 mm), until with the maximum strength available, counting rates unquestionably above the background could be detected (in five minute runs) out as far as 9500 $H\rho$ (2.4 Mev).

This effect is somewhat similar to that obtained by Curie and d'Espine[7] as well as by Yovanovitch and d'Espine.[8] These workers used extremely strong sources with photographic detection, and found a feeble band between 7000 and 10,000 $H\rho$ which they hesitated to interpret as an actual continuation of the main continuous spectrum. The interpretation offered by the writer is that the effect is due to multiple reflection from the walls, baffles or slit systems of the spectrometer; the geometry of these reflecting surfaces producing broken trajectories for electrons of certain preferred intermediate energies which results in an apparent but spuriously large radius of curvature in the case of photographic registration.

That this scattering is not indiscriminate seems to follow from the fact that with the present apparatus, using fixed radius of curvature and variable magnetic field, different apparent maxima can be found for each source strength, beyond which the counting rate with the source shutter entirely closed does not differ from that obtained with the shutter wide open. It is again worth noting that this scattering is entirely absent at low energies, as deduced from the sharp cut-off (previously mentioned) produced by absorption in the Cellophane windows.

It also seems reasonable to presume that this postulated selective scattering drops below the background level for weak sources and becomes negligible in the main portion of the spectrum; otherwise it would produce discrepancies between the curves from weak and intermediately strong sources, and such discrepancies were not found.

The above reasoning leads to the rejection of the high energy end points obtained with excessively strong sources as unreliable, and also to the rejection of the distribution curves made from data taken with slit widths which give a fractional resolution ($\Delta\rho/\rho$) poorer than 0.05.

Under these restrictions the best value of the experimental end point was estimated as 5350 ± 130 $H\rho$ (1.175 ± 0.04 Mev). From the previous discussion it would seem that any attempt to fix the experimental end point with greater accuracy would be unwarranted under conditions imposed by the spectrometer in use.

FERMI AND K.U. PLOTS

A plot of $[N/(\eta+0.355\eta^2)]^{\frac{1}{2}}$ against the energy of the electrons yields the curved line shown to the left of Fig. 5, indicating that the Fermi function does not correctly represent the experimental distribution in energy of electrons from radium E. In the same figure the line to the right represents the plot of $[N/(\eta+0.355\eta^2)]^{\frac{1}{4}}$ *versus* electron energy and shows a degree of linearity which appears to indicate substantial agreement between the experimental distribution

TABLE II. *Expected energy-number distribution (N calculated from values of the function $(N/f)^{\frac{1}{2}}$ as determined from the extrapolation of the straight line graph of Fig. 5).*

$(N/f)^{\frac{1}{2}}$	N	Mev
0.99	5	0.945
0.69	1.33	1.036
0.38	0.134	1.132
0.093	0.0005	1.224

and the theoretical one predicted by Konopinski and Uhlenbeck.

Data from several distribution curves obtained with both single and double coincidence counting and with both nickel and platinum source mountings all yield K.U. plots whose extrapolated high energy end points coincide at the value of 1.25 ± 0.03 Mev. This is but 6 percent higher than the most probable experimental value for the end point.

It is also of interest to note that the straight line plot of $(N/f)^{\frac{1}{4}}$ *vs.* energy continues through to fairly low energies and does not curve downward as in the plots from the data of Scott[18] and Lyman.[23]

Omission of the previously listed corrections only increases by an additional 5 percent the divergence between the experimental and extrapolated end points determined in this work.

Comparison Between Values of High Energy End Point from Experimental Distribution Curve and Extrapolation of K.U. Plot

If the Konopinski-Uhlenbeck theory have any validity, then the experimentally determined end point should always be found at a value which is considerably lower than the one determined from extrapolation of the K.U. plot. The reason for this is the fact that the K.U. theory requires a fourth order contact between the energy distribution curve and the energy axis, thus limiting the number of electrons close to the end point to a value which makes their detection by present methods most improbable.

Thus within the limits of the source strengths used we cannot expect experimental points beyond 1.13 Mev. This may be readily shown by calculating the relative expected counting rates for points on the K.U. plot beyond this energy value. This has been done and the results are tabulated in Table II.

If we take the ratios of successive pairs of predicted counting rates we notice that between 1.036–1.132 and 1.132–1.224 Mev the ratio of successive counting rates should drop from 10 : 1 to 373 : 1. Thus if the counting rate is near the limit of sensitivity of the detector at the former values, it is hopelessly below it for the latter.

Thus a counting rate of 5 per min. at 1.036 Mev would give (on K.U. predictions) a rate of $\frac{1}{2}$ count per min. at 1.132 Mev.

This latter would not be recognized above the background in a normal 5-minute run. And as a matter of fact no electrons above 1.036 Mev were found with the 1 mm slit width.

With a three mm slit width, which because of a larger solid angle gives a higher counting rate, we obtain a counting rate of between 150 and 200 a minute which would be cut to only 15 to 20 per min. at 1.132 Mev. This rate is well within

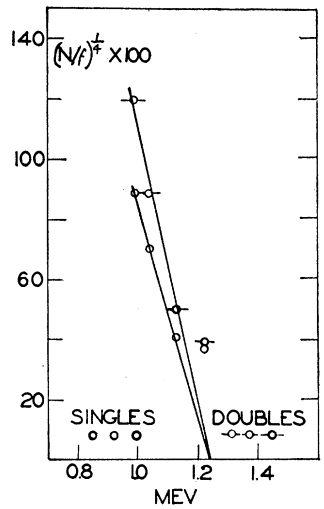

FIG. 6. K.U. plot in end point region. Number data taken with central slit width 3 mm.

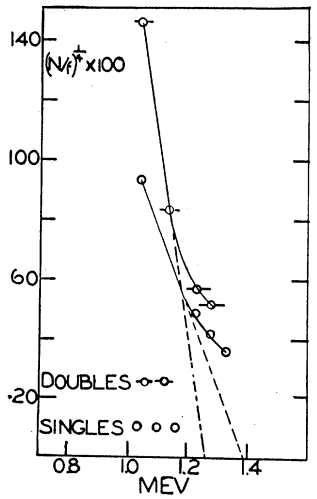

FIG. 7. K.U. plot in end point region. Number data taken with central slit width 8 mm.

the limits of detection. Electrons in the correct abundance were found at this energy and with the three mm slit.

Extending the reasoning to higher energies we find that with counting rates obtainable with a 3 mm slit width we have a predicted drop on K.U. theory from 150 counts per min. at 1.132 Mev to less than $\frac{1}{2}$ count per min. at 1.224 Mev.

This is the first energy at which there is found a disagreement between K.U. prediction and experimental count; the departure being relatively slight in the case of the 3 mm slit when the probable error in N and H is considered. Beyond 1.22 Mev there is no further count recorded above background with the 3 mm slit, and the recorded readings from the 8 mm slit above these energies depart still more from the K.U. predictions.

If the experimental counting rates near the end point be used for a plot of the K.U. function, it is found, as may be seen in Fig. 6, that for a 3 mm slit width all the points but the last one lie on the straight line which gives the same extrapolated end point as does the line which was plotted from the average of several runs (taken with 1 mm slit) over the entire range of the spectrum.

On the other hand with the 8 mm slit (Fig. 7), there is a definite curving away from linearity exhibited by the last three or four points; and this trend is such as to indicate an *excess* of electrons over the amount predicted by the K.U. theory.

The most plausible interpretation seems to be that the departure seen in Fig. 7 is due to scattering as a result of the use of a wide slit; and this effect becomes more noticeable as the ratio between scattered electrons and those of energy proper to the beam being measured increases.

We may sum up the previous discussion by saying:

(1) All counting rates at energies amenable to measurement with source strength below a total emission of 10^6 electrons per second and 1 mm slit width, are in full agreement with predictions of the K.U. theory.

(2) Only the last measurable counting rate taken with the 3 mm slit width is in disagreement with the predictions of the K.U. theory, and this discrepancy may be partially explained by the spread in $H\rho$ due to finite slit widths.

(3) Measurable counting rates taken with the 8 mm slit width and at energies higher than the maximum found with the 3 mm slit width, are in complete disagreement with the K.U. predictions, and the disagreement increases with increasing energy.

Keeping in mind the probability of scattering with wide slits, the logical conclusion from the above seems to be that the reliability of the end point measurements is in inverse proportion to the central slit width, and that the rejection of those readings taken with wide slits, which are in disagreement with the predictions of the Fermi-K.U. theory, is justified.

Position of Number Maximum

Measurements progressing in the direction of decreasing energy show that the distribution curve continues to rise until a value of 1650 ± 40 $H\rho$ (0.182 Mev) is reached. This differs considerably from the early estimates of the number maximum but agrees with the trend of more recent observers to place the number maximum at lower energies. Because of the uncertainty in the corrections for absorption, no great confidence can be placed in the way the curve behaves on the low energy side of the maximum. Errors in the corrections may exaggerate the height of the peak, but it is not displaced with respect to the energy axis; its abscissa being the same with or without the corrections.

Comparison of End Points Obtained by Various Observers

The evidence for a true experimental end point appreciably above 5500 $H\rho$ seems both dubious and untrustworthy. Terroux's[11] abnormally high value of 11,200 $H\rho$ (from cloud chamber data), according to a footnote in a subsequent article,[37] is admittedly an overestimate due to an improper criterion for the rejection of tracks whose radii of curvature had been altered by collision with nuclei.

In addition any estimate such as that of Terroux, which attributes an $H\rho$ of over 5000 to as many as 4 percent of the particles, is in disagreement with the work of Ellis and Wooster[38]

[37] F. R. Terroux, P. C. P. S. **28**, 115 (1932).
[38] Ellis and Wooster, Proc. Roy. Soc. **A117**, 109 (1927).

and of Meitner and Orthman.[39] These investigators determined the mean energy of the continuous spectrum of radium E by calorimetric methods, and its value absolutely precludes the possibility of so many high energy electrons.

The paucity of tracks and large probable error in the work of Ho and Wang[20] does not tend to convince one of the objectivity of an end point at 7500 $H\rho$.

Among the recent spectrometer measurements we have the results of Alichanow,[19] Lyman,[23] Langer and Whitaker[24] and the writer all in agreement to the extent of placing the most probable value of the experimental end point between 5300 and 5400 $H\rho$.

Scott's high energy end point of 6600 $H\rho$ is far beyond that found in the present investigation with a source strength of the same order of magnitude (280 counts per min. at the number maximum of the spectrum).

It also exceeds most other values obtained from spectrometer measurements.

The possibility of scattering being increased by the small radius of curvature (2 cm) and lack of baffle system in his spectrometer seems to receive confirmation from the lack of sharp cut-off at the low energy end of the spectrum, as mentioned previously in the paragraph on the correction for absorption in the Cellophane windows.

While all workers on beta-ray spectra agree in the rejection of the Fermi function, because of its inability to satisfy the criterion established by Kurie, Richardson and Paxton, there is no such unanimity concerning the shape of the K.U. plot and consequent validity of that theory.

Champion and Alexander[21] have found the K.U. plot linear for both radium E and thorium C″, with an extrapolated end point for the former only 9 percent above their experimental one of 5500 $H\rho$.

Scott's data, when used for making the K.U. plot, show linearity for the main body of the graph, and this line gives an extrapolated end point value almost identical with his rather high experimental one. However, there are serious departures from linearity at both low and high energy portions of this same plot; the upward curvature at the high energy end indicating an excess of electrons over the number predicted by the K.U. theory. Lyman on the contrary finds a deficiency of electrons near the end point which causes his K.U. plot to drop to the energy axis more rapidly than the theory requires.

Lyman as well as Langer and Whitaker find the K.U. extrapolated end point more than 14 percent higher that the experimental one, while the results of the present investigation indicate a difference of only 6 percent.

Table III summarizes the results just discussed.

The different values obtained for the extrapolated end point by various workers indicate that the shape, even of the main portion, of the distribution curve from which the data are taken must be seriously distorted by some of the spectrometers in use; for independently of the truth or error of the theory, plots of the identical function from the data secured by four different spectrometers from the same radioactive substance should yield curves which have the same intercept on the energy axis. Such agreement is not found in the four last cases listed above.

In view of such discrepancies it would be difficult to justify the rejection of the Konopinski-Uhlenbeck theory until there is better agreement concerning the experimental distribution with which it may fail to conform.

Some modification, such as that proposed by Richardson,[40] may be necessary if the agreement between Lyman[23] and Langer and Whitaker[41] regarding the deficiency of high energy electrons, with respect to the K.U. plot, be taken to indicate a real rather than instrumental effect. While it is possible that such a deficiency might be obscured by lack of fine resolution (in the present investigation $\Delta\rho/\rho \leqq 0.05$) the results here re-

TABLE III. *Comparison of experimental and extrapolated K.U. end points obtained by recent observers.*

INVESTIGATOR	EXPERIMENTAL END POINT $H\rho$	EXTRAPOLATED K.U. END POINT $H\rho$	PERCENTAGE DIFFERENCE
Champion and Alexander[21]	5500	5975	9%
Scott[18]	6600	6600 (?)	—
Lyman[23]	5300	6050	14%
Langer and Whitaker[24]	5330	6250	17%
O'Conor	5350	5650	6%

[39] Meitner and Orthman, Zeits. f. Physik **60**, 143 (1930).

[40] H. O. W. Richardson, Nature **139**, 505 (1937).

[41] Langer and Whitaker, Phys. Rev. **51**, 713 (1937).

corded for the major portion of the distribution curve can be conservatively interpreted as contributing limited support to the K.U. theory.

ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation for the inspiration to begin the above investigation and the encouragement to carry it to completion which was given by Professor Robley D. Evans, under whose direction the work was done.

It is also a pleasure to acknowledge the cooperation of Professor A. F. Kovarik of Yale University who was kind enough to prepare a thorium B source on two occasions. This source was used for the calibration of the magnetic field and determination of the efficiency of the spectrometer.

The radium E sources were obtained from Dr. L. R. Hafstad of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, as well as from Professor J. A. Bearden and Dr. W. R. Kanne of the Johns Hopkins University.

It was only the continued courtesy of those just named that made it possible to study the radium E beta-ray spectrum under the various conditions reported in this paper.

For data concerning the design and operation of the spectrometer as well as for information concerning the Fermi and Konopinski-Uhlenbeck theories the writer is also deeply indebted to Dr. L. I. Schiff.

A like debt of gratitude is owed to Dr. N. S. Gingrich and Dr. E. J. Schremp for advice on the electrical circuits used in the counting apparatus.

AUGUST 15, 1937 PHYSICAL REVIEW VOLUME 52

Extreme Ultraviolet Series in Cr VI, Mn VII and Fe VIII[1]

P. GERALD KRUGER AND S. G. WEISSBERG
University of Illinois, Urbana, Illinois

(Received June 14, 1937)

Lines in the extreme ultraviolet region involving the $3d\,^2D$, $4p\,^2P$, $5s\,^2S$, $6s\,^2S$, and several $nf\,^2F$ terms have been identified in the spectra of Mn VII and Fe VIII. In Cr VI the first member of the $3d\,^2D-nf\,^2F$ series has been found. These spectra have been photographed with a twenty-one foot grazing incidence vacuum spectrograph. Identification was facilitated by the use of constant second difference displaced frequency diagrams.

THE spectra of highly ionized chromuim, manganese and iron have been photographed in the extreme ultraviolet with a twenty-one foot grazing incidence vacuum spectrograph. The instrument and source of power have been described in previous reports.[2, 3]

Heretofore the identification of the extreme ultraviolet spectra of isoelectronic ions by the use of the constant second difference law, has been greatly facilitated by the fact that the lines involved were among the very strongest in the region. However, when an attempt was made to extend the K I—Cr VI isoelectronic sequence to Mn VII, an extrapolation of the displaced frequency diagram (see Fig. 1) for the resonance multiplet $3d\,^2D_{3/2,\,5/2}-4p\,^2P_{1/2,\,3/2}$ led into a thickly populated region, with no prominent lines. This was in contrast to the appearance, on our plates, of the very strong $3d\,^2D_{3/2,\,5/2}-4p\,^2P_{1/2,\,3/2}$ multiplets in V V and Cr VI.[4] That Mn VII might not be excited would indeed be curious, since spectra both from Mn VI and Mn VIII have been identified on the same plates, and lines from both of these ions are intense. One must conclude, therefore, that the above multiplet is among the lines of intermediate intensity in Mn VII.

From the identification of the multiplet $4s\,^2S_{1/2}-4p\,^2P_{1/2,\,3/2}$, Gibbs and White had already found the fine structure interval in the

[1] Preliminary report given by P. G. Kruger and S. G. Weissberg, Phys. Rev. **49**, 873A (1936).
[2] P. G. Kruger, Rev. Sci. Inst. **4**, 128 (1933).
[3] P. G. Kruger and W. E. Shoupp, Phys. Rev. **46**, 124 (1934).
[4] Previously reported by R. C. Gibbs and H. E. White, Proc. Nat. Acad. Sci. **12**, 598, 675 (1926).