On the Capture of Thermal Neutrons by Deuterons

Recent experiments[1] on the radiative capture of thermal neutrons by deuterons indicate as an upper limit for the cross section 3×10^{-26} cm^2. The calculations for this process are readily carried through with the results of a recent paper[2] on the elastic scattering of neutrons by deuterons. The notation, coordinate system, and numerical values used here are the same as those used in reference 2.

The initial state (including the exchange wave) antisymmetric in the neutrons (particles 1 and 2) is

$$\psi^i=(1/\surd 2)[\chi(r_{13})\phi_0(r_{23})S(123)-\chi(r_{23})\phi_0(r_{13})S(213)];$$
$$\chi(r)=1+(\Delta/r)(1-e^{-\epsilon_0 r});\quad \phi_0(r)=(a^3/\pi)^{\frac{1}{2}}e^{-ar};$$

where the S's are the quartet and doublet spin functions given in Eq. (19) of reference 2. We have (in nuclear units) $a=3.236$, $\epsilon_0=5.701$, $\Delta=-0.916$ for the quartet state, and $\Delta=-0.381$ for the doublet state. The final H^3 bound states are

$$\psi_{\pm\frac{1}{2}}{}^b=uR_{\pm\frac{1}{2}}+vR'_{\pm\frac{1}{2}};\quad R_{\frac{1}{2}}=(1/\surd 2)[(+-+)-(-++)];$$
$$R_{-\frac{1}{2}}=(1/\surd 2)[(+--)-(-+-)];$$
$$R_{\frac{1}{2}}'=(1/\sqrt{6})[(+-+)+(-++)-2(++-)];$$
$$R'_{-\frac{1}{2}}=(1/\sqrt{6})[2(--+)-(-+-)-(+--)];$$

the space functions u and v are, respectively, symmetric and antisymmetric in the neutrons. The initial quartet and doublet states can be written

$$\psi_q{}^i=(1/\surd 2)I^-S_q(123);\quad \psi_d{}^i=(1/2\surd 2)[\sqrt{3}I^+R_{\frac{1}{2}}-I^-R_{\frac{1}{2}}'];$$
$$I^\pm=\chi(r_{13})\phi_0(r_{23})\pm\chi(r_{23})\phi_0(r_{13}).$$

The orthogonality condition requires that $\sqrt{3}\ (u,I^+)-(v,I^-)=0$; also $(u,I^-)=(v,I^+)=0$.

At thermal energies, the transition occurs under the influence of the magnetic dipole $\mathbf{G}=g_n(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)+g_p\boldsymbol{\sigma}_3$. The mean transition probability is given in terms of

$$\bar{G}^2=\tfrac{1}{3}\sum_m\sum_{xyz}\{2|(\psi_m{}^b,\ G_x\psi_q{}^i)|^2+|(\psi_m{}^b,\ G_x\psi_d{}^i)|^2\},$$

and the inverse mean life of a thermal neutron before capture is[3]

$$(1/\tau)=n(m_e/m_p)^{3/2}(W_r{}^3e^2h/6\pi m_p{}^2c^2)\bar{G}^2;$$

here, n is the number of deuterons per cc of absorber, W_r is the energy in nuclear units ($12.2\ m_ec^2$) given up to the photon, and $\bar{G}^2$ is in nuclear units. With the help of the orthogonality condition, it can be shown that

$$\bar{G}^2=(1/18)(g_n-g_p)^2\{2|(v,I_q{}^-)|^2+|(v,I_d{}^-)|^2\}.$$

We take for u the normalized function of the form[4] $e^{-\mu(r_{13}+r_{23}+\sigma r_{12})}$ with $\mu=2.604$ and $\sigma=0.8$. To estimate v, which is probably small, we write it as $v=(K\mu/2)(r_{13}-r_{23})u$, where K is a numerical factor. The calculations of Rarita and Present[5] indicate that $K=0.013$; although this value certainly will not be exact for our case, it serves to give an order of magnitude.

The integrals (v,I^-) can be estimated; with $g_n=-4.0$ and $g_p=5.7$ we obtain $(1/\tau)\simeq 50$ sec.$^{-1}$ for D_2O ($n=6.7\times 10^{22}$ deuterons per cc). This estimate is probably an upper limit to the inverse mean life. For thermal neutrons (mean velocity 250,000 cm/sec.), this corresponds to a cross section for capture of 0.3×10^{-26} cm^2. This is well within the observed[1] upper limit of 3×10^{-26} cm^2, and indicates that the capture process is likely to be too improbable to be observed experimentally.

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[1] Kikuchi, Aoki and Takeda, Tokyo Institute of Physical and Chemical Research **31**, 195 (1937).

[2] Schiff, Phys. Rev. **52**, 149 (1937).

[3] Morse, Fisk and Schiff, Phys. Rev. **50**, 748 (1936).

[4] Fisk, Schiff and Shockley, Phys. Rev. **50**, 1090 and 1191 (1936).

[5] Rarita and Present, Phys. Rev. **51**, 788 (1937).

Erratum: Some Lattice Sums Involved in the Calculation of Elastic Constants

(Phys. Rev. **50**, 99 (1936))

In our letter of May 26, 1936 entitled "Some Lattice Sums Involved in the Calculation of Elastic Constants" an error occurred in the calculation of the electrostatic part of C_{12} for the CsCl type lattice from C_{11} and Madelung's constant. The contribution to C_{11} remains unchanged

$$A^{(e)}=(e^2/\delta^4)\times 2.1253$$

whereas the revised value of C_{12} is

$$B^{(e)}=-(e^2/\delta^4)\times 2.0803.$$

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