AUGUST 1, 1937 PHYSICAL REVIEW VOLUME 52

The Intensity and Polarization of the Light Diffracted by Supersonic Waves in Solids

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(Received May 7, 1937)

The intensity and polarization of the light diffracted by supersonic waves in solids are calculated by considering the photoelastic effect due to the strains created by the elastic waves. For natural incident light the diffracted light is partially polarized. In glasses and cubic crystals the diffracted light is partially depolarized if the incident light is polarized. The intensities are evaluated with the help of the theory of Raman and Nath and it is shown that this theory explains the measurements better than Brillouin's approximation. The calculated intensity distributions for natural and polarized incident light agree with the observations of Schaefer and Bergmann on glasses, quartz and calcite. This agreement justifies Fues' and Ludloff's assumption that all forced vibrations of the same frequency oscillate with the same amplitude.

SCHAEFER and Bergmann[1] have shown that supersonic waves in crystals act as diffraction gratings for light. Their results justify the assumption that the piezoelectric oscillator excites over a narrow frequency range all the standing waves which can exist in a crystal according to the theory of specific heats. Fues and Ludloff[2] have given a theory of the diffraction pattern. They calculate the wave-lengths of the infinite number of progressing waves of a given frequency which can travel in an infinite crystal, and then consider the diffracted light as due to Bragg reflections on the wave fronts. Since the positions of the interference maxima are the same for standing and traveling waves this theory gives the locus of all diffraction spots produced by the finite number of standing waves in a finite crystal.

In some diffraction patterns all spots appear to be of nearly equal intensity. This observation led Fues and Ludloff to conclude that the coupling between the different modes of vibration of a crystal results in an equipartition of energy for all sound waves of the same frequency. Since this conclusion, if correct, is of considerable theoretical importance, and since no theoretical proof of its validity has been given, it seems worth while to see whether it really is justified by the experimental evidence. Many of the published photographs of the diffraction patterns, particularly those taken with polarized light, reveal a systematic variation of the light intensities along the diffraction curves. Some parts of the calculated curves are not observed. A test of Fues and Ludloff's hypothesis calls therefore for an investigation of the intensities of the interference maxima. We will show below that a large part[3] of the observed intensity variations can be accounted for by assuming that all waves oscillate with approximately the same amplitude. Hence the proposed principle of equipartition appears to be validated by the observations.

The intensity of the diffracted light depends primarily on the periodic variations of the refractive index which are produced by the sound waves. The calculation of these index variations is a problem of photoelasticity. An elastic wave creates periodically varying strains which in turn introduce local periodic alterations of the optical index ellipsoid. These pulsations of the index ellipsoid determine the fluctuations of the index of refraction for light polarized in any direction. As was first shown by Brillouin[4] the angle of deviation of the diffracted light is given by Bragg's law, but the intensities must be calculated by using the theories developed by Raman and Nath[5] and Estermann and

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[1] C. Schaefer and L. Bergmann, Naturwiss. **22**, 685 (1934).

[2] E. Fues and H. Ludloff, Sitzungsber. d. preuss. Akad., Math. phys. Kl. **14**, 222 (1935).

[3] Some diffraction patterns, for instance those for glasses (C. Schaefer and L. Bergmann, Naturwiss. **23**, 799 (1935)), show also intensity variations which apparently are due to the method of excitation of the sound waves. The waves which are directly transmitted from the piezo quartz to the glass or crystal are much stronger than all others.

[4] L. Brillouin, "La diffraction de la Lumière par des Ultra-sons," Act. Sci. et Ind. (1933).

[5] C. V. Raman and N. S. N. Nath, Proc. Ind. Acad. Sci. **2**, 406 (1935).

Wannier[6] for the Debye-Sears effect in liquids. Since these theories give the intensities as a function of the amplitude of the index variations and since their results are applicable to solids without any modifications we have to discuss here only the photoelastic problem.

The first part of the calculation involves the determination of the wave-lengths and directions of oscillation of all sound waves of a given frequency which can travel in an infinite crystal. Fues and Ludloff have given the solution of this problem. The three differential equations of a sound wave in a crystal

$$\rho\frac{\partial^2 u_x}{\partial t^2}=\sum_y\sum_{x'}\sum_{y'}c_{xx',\,y'y}\frac{\partial^2 u_y}{\partial x'\partial y'} \tag{1}$$

have the solutions

$$u=a\exp i[\omega t-(kr)] \tag{2}$$

provided the components of the amplitude vector satisfy the three linear homogeneous equations

$$\begin{aligned} a_x(k^2Q_{xx}-\rho\omega^2)+a_yk^2Q_{xy}+a_zk^2Q_{xz}&=0,\\ a_xk^2Q_{yx}+a_y(k^2Q_{yy}-\rho\omega^2)+a_zk^2Q_{yz}&=0,\\ a_xk^2Q_{zx}+a_yk^2Q_{yz}+a_z(k^2Q_{zz}-\rho\omega^2)&=0. \end{aligned} \tag{3}$$

The wave-length λ_s of the sound wave is determined by the condition that the determinant of Eqs. (3) must vanish.

$$|k^2Q_{xy}-\rho\omega^2\delta_{xy}|=0. \tag{4}$$

The wave vector $\mathbf{k}$ has the length $2\pi/\lambda_s$ and points in the direction of propagation. The Q_{xy}'s are quadratic forms

$$Q_{xy}=Q_{yx}=\sum_{x'}\sum_{y'}c_{xx',\,y'y}\alpha_{x'}\alpha_{y'} \tag{5}$$

where α_x, α_y, α_z are the direction cosines of $\mathbf{k}$, and $c_{xx',\,y'y}=c_{ik}$ are the elastic constants of the crystal.[7] To every direction of propagation there exist three solutions of Eq. (4). In an isotropic solid these correspond to one longitudinal and two transversal waves. In crystals the waves are usually neither longitudinal nor transversal, but the amplitude vectors of the three waves traveling in the same direction are always mutually perpendicular.

The solutions $\mathbf{k}(\alpha_x, \alpha_y, \alpha_z)$ of Eq. (4) determine the so-called "form-frequency surface." Fues and Ludloff have shown that the diffraction pattern is a curve geometrically similar to the intersection of this surface with a plane normal to the direction of the incident light. This means that only those waves which travel in directions normal to the light beam are effective in the diffraction.

By introducing the solutions of Eq. (4) in Eqs. (3) one finds the direction of oscillation of each wave. The absolute value of the amplitude remains of course undetermined. In accordance with the proposed principle of equipartition of energy we assume that the amplitude has the same value for all directions of oscillation and propagation.

After all possible sound waves in a crystal have thus been found, the next step is to calculate their photoelastic effects. Differentiation of Eq. (2) gives the strains due to the sound waves

$$x_x=\frac{\partial u_x}{\partial x},\quad z_y=y_z=\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}. \tag{6}$$

Let us assume that the axes x, y, z have been chosen to coincide with the axes of the crystal's optical index ellipsoid:

$$x^2/n_1^2+y^2/n_2^2+z^2/n_3^2=1,$$

where n_1, n_2, n_3 are the principal indices of refraction. The local strains will rotate and deform this ellipsoid and its equation takes the general form

$$a_{11}x^2+a_{22}y^2+a_{33}z^2+2a_{23}yz+2a_{31}zx+2a_{12}xy=1. \tag{7}$$

The polarization constants a_{ik} in the deformed state differ from those in the undeformed state by quantities which are to a first approximation linear functions of the local strains. Hence

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[6] R. Estermann and G. Wannier, Helv. Phys. Acta 9, 337, 520 (1936).

[7] Using Voigt's notation for the elastic constants the coefficients of the Q_{xy}'s are given by the following array:

	α_x^2	α_y^2	α_z^2	$\alpha_y\alpha_z$	$\alpha_z\alpha_x$	$\alpha_x\alpha_y$
Q_{xx}	c_{11}	c_{66}	c_{55}	$2c_{56}$	$2c_{15}$	$2c_{16}$
Q_{yy}	c_{66}	c_{22}	c_{44}	$2c_{24}$	$2c_{46}$	$2c_{26}$
Q_{zz}	c_{55}	c_{44}	c_{33}	$2c_{34}$	$2c_{35}$	$2c_{45}$
Q_{yz}	c_{56}	c_{24}	c_{34}	$(c_{23}+c_{44})$	$(c_{45}+c_{36})$	$(c_{25}+c_{46})$
Q_{zx}	c_{15}	c_{46}	c_{35}	$(c_{36}+c_{45})$	$(c_{13}+c_{55})$	$(c_{56}+c_{14})$
Q_{xy}	c_{16}	c_{26}	c_{45}	$(c_{25}+c_{46})$	$(c_{14}+c_{56})$	$(c_{12}+c_{66})$

$$
\begin{aligned}
a_{11}-1/n_1{}^2 &= p_{11}x_x+p_{12}y_y+p_{13}z_z+p_{14}z_y \\
&\quad +p_{15}x_z+p_{16}y_x, \\
a_{23} &= p_{41}x_x+p_{42}y_y+p_{43}z_z \\
&\quad +p_{44}z_y+p_{45}x_z+p_{46}y_x.
\end{aligned}
\tag{8}
$$

The 36 constants $p_{ik}+p_{ki}$ are Pockels' elasto-optical constants.[8] They are known for a large number of glasses, where their number is reduced to two, for the cubic crystals NaCl, KCl and CaF_2 (3 constants) and for quartz and calcite (8 constants).

The first three equations of (8) determine the fluctuations of the lengths of the axes of the index ellipsoid, the last three give the change of orientation of these axes. The variations of the indices of refraction of the two plane polarized components of a light beam are determined, of course, by the variations in lengths of the axes of the ellipse which is formed by the intersection of the index ellipsoid with a plane normal to the direction of propagation of the beam.

For a triclinic crystal these calculations would be very laborious. They are, however, fairly simple for crystals of the rhombic, rhombohedric, hexagonal and cubic systems provided the light is incident parallel to one of the crystallographic axes. If, for instance, the light enters in the x direction and oscillates in the y direction the amplitude of the variation of the index of refraction is

$$\Delta n=\tfrac{1}{2}n_2{}^3(a_{22}-1/n_2{}^2), \tag{9}$$

where $(a_{22}-1/n_2{}^2)$ is obtained by introducing in Eq. (8) the amplitudes of the strains given by Eq. (6). This value of Δn determines the intensity of the diffracted light component which has the same polarization as the incident light. Due to the periodic change of the orientation of the index ellipsoid the diffracted beam will also have a component oscillating in the z direction. If the crystal is naturally birefringent for light traveling in the given x direction this "depolarizing" component along z is very small and can be neglected.

The situation is, however, entirely different if the light travels in the direction of an optical axis or in any arbitrary direction through a cubic crystal or an amorphous solid.[9] In this case the intersect of a plane normal to the light beam with the index ellipsoid is a circle for the undeformed state. The deformations change this circle to an ellipse whose axes have a *fixed* direction. For instance for light traveling in the x direction the angle θ between one axis of the ellipse and the y axis is given by

$$\tan 2\theta=2a_{23}/(a_{22}-a_{33}). \tag{10}$$

Hence one resolves the incident light into two components oscillating parallel to these axes. The amplitudes of their index variations are

$$
\begin{aligned}
\Delta n=\tfrac{1}{4}n^3[&(a_{22}-1/n^2)+(a_{33}-1/n^2) \\
&\pm((a_{22}-a_{33})^2+4a_{23}{}^2)^{\frac{1}{2}}].
\end{aligned}
\tag{11}
$$

Consequently for plane polarized incident light the diffracted beam is usually partly depolarized, and for natural incident light the diffracted beam is partially polarized.

As mentioned previously the relation between Δn and the intensity of the diffracted light has been investigated by Raman and Nath and Estermann and Wannier. If Δn is large the relation is very complicated and higher order diffraction images appear. If Δn is very small only the first order is visible and its intensity is proportional to $(\Delta n)^2$. In the experiments of Schaefer and Bergmann only the first order is observed, but this fact is not sufficient justification for the use of Brillouin's approximation $I=c(\Delta n)^2$. According to Raman and Nath[10] the best approximation for the intensity of the first order diffraction images due to standing supersonic waves is

$$I(v)=c\int_0^{2\pi}J_1{}^2(v\sin\psi)d\psi, \tag{12}$$

where J_1 is a Bessel function, $v=2\pi\Delta nL/\lambda$ and L/λ is the length of the optical path in the

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[8] F. Pockels, *Lehrbuch der Kristalloptik* (Leipzig, 1906) p. 460. For amorphous solids the p_{ik} can be replaced by Neumann's strain-optical constants $q=\frac{1}{2}np_{11}$, $p=\frac{1}{2}np_{12}$, $p_{44}=\frac{1}{2}(p_{11}-p_{12})$.

[9] The intensity ratio of the "depolarizing" and polarized components is proportional to $a_{23}{}^2/(a_{22}-a_{33})^2$. For birefringent crystals the denominator differs very little from $(1/n_2{}^2-1/n_3{}^2)^2$. Hence the ratio is proportional to the square of the small strains x_x, y_z etc. For cubic crystals, however, where $n_1=n_2=n_3$ the denominator is proportional to the square of the strains and hence the above ratio is of the order of magnitude 1.

[10] C. V. Raman and N. S. N. Nath, Proc. Ind. Acad. Sci. 3, 75 (1936).

crystal. For $v<0.2$ this relation is identical with Brillouin's approximation. For $v>2$ the higher orders have an appreciable intensity. Since they do not appear in the experiments we are only interested in the range where $v<2$. In this range $I(v)$ increases continuously (a maximum is reached at about $v=2.2$). Hence we conclude that the observed intensities must be a monotonic function of Δn and that the intensity variations are analogous to the variations of Δn.

At present only a qualitative verification of the theory is possible because no intensity measurements are available. We will show below that all intensity variations observed in the diffraction patterns of glasses, cubic crystals, quartz and calcite agree qualitatively with the variations of $i=2\Delta n/n^3$. The variation of this quantity can be calculated with the help of Eq. (9) or (11) from the elastic and elasto-optical constants of the solid. The absolute value of i is proportional to $a\omega\sqrt{\rho}$, where a is the amplitude of the sound wave, $\omega=2\pi\nu$ its frequency and ρ the density of the solid. The data given below are calculated[11] with $a\omega\sqrt{\rho}=10^7$.

Isotropic Solids

For isotropic solids the diffraction pattern consists of two concentric circles. The ratio of their radii is $k_1/k_2=[\frac{1}{2}(1-2\sigma)/(1-\sigma)]^{\frac{1}{2}}$, where σ is Poisson's ratio. Let x be the direction of the incident beam, y the direction of oscillation of the plane polarized light, and consider a diffraction spot on the diameter which forms an angle φ with the y axis.

The inner circle is due to the longitudinal waves. The Eqs. (8), (10) and (11) furnish $\theta=\varphi$, $i_1=p_{11}=2q/n$, $i_2=p_{12}=2p/n$, where i_1 refers to the light component which oscillates in the direction θ and i_2 to the component normal thereto.

The outer circle is a double solution since it corresponds to the two transversal sound waves. For those transversal waves which oscillate in the x direction we get $i_1=i_2=0$. Hence they do not contribute to the diffraction pattern. The outer circle is produced exclusively by those shear waves which oscillate in the yz plane.[12] For these waves $\theta=\varphi+\pi/4$, $i_1=i_2=p_{44}=(q-p)/n$.

It is now a simple matter to find the intensity distributions for any experimental condition. If we denote with A, B, C the values of the intensity function (12) for the arguments $v_A=4\pi n^2 k_1 aqL/\lambda$, $v_B=4\pi n^2 k_1 apL/\lambda$ and $v_C=2\pi n^2 k_2 a(q-p)L/\lambda$, where k_1 and k_2 are the lengths of the wave vectors for the longitudinal and transversal waves and p and q are Neumann's strain-optical constants, we have:

For observations with crossed Nicols

Inner ring $I=\frac{1}{4}(A+B)\sin^2 2\varphi$,
Outer ring $I=\frac{1}{2}C\cos^2 2\varphi$.

For observations with parallel Nicols

Inner ring $I=A\cos^4\varphi+B\sin^4\varphi$,
Outer ring $I=\frac{1}{2}C(1+\sin^2 2\varphi)$.

For observations with a single Nicol the results are the same whether it is inserted in the incident or transmitted beam

Inner ring $I=A\cos^2\varphi+B\sin^2\varphi$,
Outer ring $I=C$.

For observations with natural light all spots on each ring have of course the same intensity. The intensity ratio between spots on the inner and outer circle is $(A+B)/2C$.

The experimental results on glasses are in good agreement with the calculations. For glasses $(q-p)$ is always much smaller than either q or p and it may even vanish. Hence the outer circle appears always much weaker than the inner one.[13] The photographs made with crossed Nicols[14] show clearly that the intensity vanishes for $\varphi=0°$ and $90°$ on the inner circle, and for $\varphi=\pm 45°$ on the outer circle, in agreement with the above equations. For parallel Nicols the intensity variations are less pronounced but they follow the theoretical predictions.

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[11] In the experiments ($\rho\omega^2=10^{16}$, $a^2=10^{-14}$) $a\omega\sqrt{\rho}$ is probably about 10, hence i is of the order of magnitude 10^{-5} (see Table I and the figures). For a crystal 1 cm thick $v=\pi n^3 i/\lambda$ is therefore approximately 1 and our assumption $v<2$ appears tenable.

[12] The discovery of diffraction patterns produced by transversal waves is due to E. Hiedemann and K. H. Hoesch, Zeits. f. Physik **98**, 141 (1935). The statement of Fues and Ludloff that transversal waves should not produce a diffraction pattern, since they create no density fluctuations, is of course incorrect. In solids a shear creates index variations without changing the density. On the other hand the diffraction patterns from the longitudinal waves are not solely due to density variations. The values of p and q depend also on other effects, as was shown by the author, Physics **6**, 179 (1935).

[13] $(q-p)$ has the largest values for quartz glass and light crown glasses and is very small for the heaviest flints. All published photographs were made with light glasses. We suspect that it is very difficult to observe the outer ring if heavy glasses are used.

[14] C. Schaefer and L. Bergmann, Naturwiss. **23**, 799 (1935).

Recently Bergmann[15] has measured the ratio A/B for a series of different glasses. His data show definitely that A/B is not proportional to $(q/p)^2$, as Fues[15] assumes. Fues' result would be correct if Brillouin's approximation were applicable. The fact that it is inadequate shows that the Raman-Nath theory is of importance not only for liquids but also for solids. The value of A/B depends on the amplitude of the soundwave and if p and q were known Bergmann's data could be used to determine this amplitude.

Cubic Crystals

The calculations have been carried out for NaCl, KCl and CaF_2 using the elastic constants of Voigt and the elasto-optical constants of Pockels.[16] If the light travels in a direction x normal to a cube face the observed diffraction patterns consist of two rings, which are not circular but have the symmetry 4 mm (C_{4v}). A third ring, a circle, which should be present according to the theory of Fues and Ludloff, does not appear. It can easily be shown that the intensity of this circle is zero, because it would be due to transversal waves vibrating in the x direction. The strains created by such a wave do not alter the cross section of the index ellipsoid normal to the x axis and hence $i_1 = i_2 = 0$. The vibrations which give rise to the inner ring are "quasi-longitudinal," i.e., their direction of oscillation forms a small angle with the direction of propagation. Similarly the outer ring is due to "quasi-transversal" waves oscillating in the yz plane. In Table I we give the quantities θ, i_1, i_2, defined in the previous sections, as functions of the angle φ which determines the position of the diffractions spots on the rings. Due to symmetry conditions φ has to be varied only between 0 and 45°. The intensities of the diffracted light are given by the formulas: $I(\varphi) = \frac{1}{4}(A+B) \sin^2 2\theta$ for crossed Nicols, $I(\varphi) = A \cos^4 \theta + B \sin^4 \theta$ for parallel Nicols, the Nicols being parallel to the crystal axes. For natural incident light the diffracted light is partially polarized and has the intensity $(A+B)$. A and B are the values of the intensity function (12) for the arguments

$$v_A = \pi n^3 i_1 L/\lambda, \qquad v_B = \pi n^3 i_2 L/\lambda.$$

Bergmann[17] has published photographs of the diffraction patterns for all three crystals. They were made with natural light and show very little intensity variations. This agrees with the results in Table I because wherever i_1 is small i_2 is large and *vice versa*, and hence the sum of the intensities of both components varies but little. The observations also agree with the theory insofar as the outer ring appears less intense than the inner one. No observations with polarized light have been published. They should show pronounced intensity variations. Owing to the fact that NaCl, KCl and CaF_2 belong to three different photoelastic groups[18] these variations must have a different character for the three crystals. *Vice versa* it will be possible to decide to which photoelestic group other cubic crystals belong by studying the diffraction patterns with parallel and crossed Nicols.

TABLE I. *The orientation and amplitudes of vibration of the index ellipse due to supersonic waves in cubic crystals.*

φ	NaCl			KCl			CaF_2		
	θ	i_1	i_2	θ	i_1	i_2	θ	i_1	i_2
Inner ring									
0	0	62.8	81.6	90	86.0	122.0	90	55.8	13.5
13.4	6	63.5	81.0	81	87.7	118.1	87	55.3	15.4
19.6	10	65.2	81.6	76	89.7	118.8	86	54.2	17.4
25.4	15	66.5	81.5	71	91.8	121.2	83	52.7	20.1
31.7	22	68.4	81.0	64	96.3	126.0	79	50.0	24.0
45	45	70.8	81.2	45	100.6	133.4	45	44.6	30.8
Outer ring									
0	45	9.5	9.5	45	34.4	34.4	45	12.7	12.7
13.4	65	17.4	4.5	35	1.2	61.2	17	13.4	26.1
19.6	72	19.9	4.7	30	9.0	66.7	12	16.5	32.1
25.4	77	21.6	5.3	25	13.5	67.0	8	20.2	36.1
31.7	84	20.4	8.6	18	13.3	63.3	5	24.7	38.5
45	90	15.6	15.6	0	23.4	23.4	0	35.2	35.2

Quartz and Calcite

Both crystals belong to the rhombohedric class. Hence the calculations involve 14 parameters

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[15] L. Bergmann and E. Fues, Naturwiss. **24**, 492 (1936). Although the strain-optical constants of the glasses used by Bergmann are not known, their values can be estimated by using the formula of H. Mueller, Physics **6**, 179 (1935).

[16] Except for KCl where the elastic constants have been corrected to satisfy Cauchy's relation. The diffraction pattern of KCl offers a very simple method for the verification of this relation.

[17] L. Bergmann, Physik. Zeits. **37**, 867 (1936).

[18] Hans Mueller, Phys. Rev. **47**, 947 (1935).

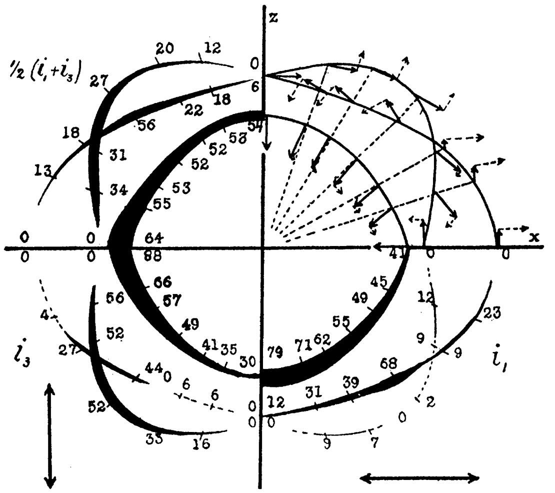

FIG. 1.

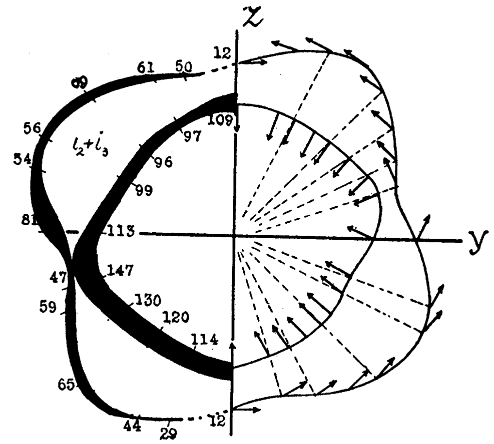

FIG. 2.

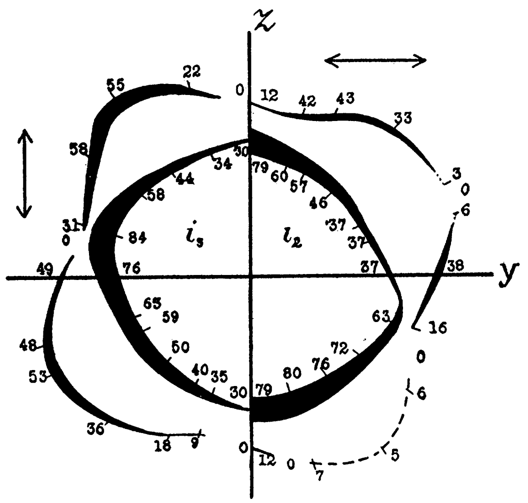

FIG. 3.

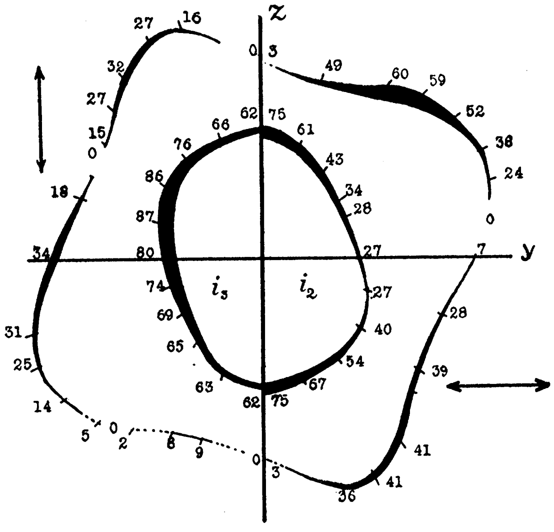

FIG. 4.

FIG. 1. Quartz, light passing in y direction. The arrows in the first quadrant give the directions of oscillation of the sound waves traveling normal to the y axis. The dotted vectors are the components of the amplitude vectors in the y direction. Second quadrant: the intensity distribution for natural incident light. Third quadrant: for polarized light oscillating in z direction. Fourth quadrant: for polarized light oscillating in x direction.

FIG. 2. Quartz, light passing in x direction. Right side: directions of oscillation of the sound waves traveling normal to x axis. Left side: intensities for natural incident light.

FIG. 3. Quartz, light traveling in $-x$ direction. Right side: intensities for light polarized in z direction. Left side: for light polarized in y direction.

FIG. 4. Calcite, light passing in x direction. Right side: intensities for polarized light oscillating in y direction. Left side: for light oscillating in z direction.

(6 elastic constants and 8 elasto-optical constants). Since the calculations, as outlined, are based on well-known principles of classical crystal physics they are of little interest and can be omitted. We give therefore only the results for those cases where a comparison with published data is possible.

For light passing through quartz in the direction of the optical axis the observed intensity distribution is practically uniform and is not changed appreciably by using polarized light. While the theory predicts a noticeable variation for polarized light, it does not take into account the optical activity of quartz, and this smoothes out any intensity variation.

For light traveling in the y direction the diffraction pattern for a rhombohedric crystal is symmetrical with respect to the x and z axes. Hence it is sufficient to give the results for one quadrant only (Fig. 1). If the incident light is plane polarized in either the x or z direction the diffracted light has the same polarization. For natural incident light the diffracted light is partially polarized and its intensity is the sum of the intensities of the two components. The vectors in the first quadrant of Fig. 1 give the directions of oscillation of the supersonic waves. The inner ring is due to quasi-longitudinal waves, the outer rings are produced by quasi-transversal waves. The numbers in the second, third and fourth quadrants are the absolute values of $\frac{1}{2}(i_1+i_3)$, $i_3=a_{33}-1/n_3^2$ (Eq. (9)) and $i_1=a_{11}-1/n_1^2$, respectively. Guided by the form of the intensity function (12) we have sketched the expected intensities for natural and for polarized light.

Figures 2, 3 and 4 give the results for light traveling in the x direction through quartz and calcite. Since x is a polar axis the diffraction pattern changes to its mirror image if the light direction is reversed (Figs. 2 and 3). The patterns consist of two rings which have central symmetry. Hence in all figures only half the pattern is sketched for each case. The theory of Fues and Ludloff predicts the existence of a third ring, an ellipse, which is due to transversal waves oscillating in the x direction. This ellipse has everywhere the intensity zero.

When comparing these theoretical curves with the observations[1] one must take into account that in the experiments the patterns consist of a finite number of spots, corresponding to the finite number of standing waves in a finite crystal. It may therefore happen that no spots appear in places where the calculation predicts a large intensity. The results must be judged by the fact that strong spots appear only where the calculated intensity is large and that no spots are visible where the theory gives a very small intensity. In this respect the agreement is very satisfactory indeed. All the peculiarities of the complicated intensity distributions agree in every detail with the published photographs. We believe that this is sufficient proof that both the theory and the proposed principle of equipartition of energy are correct. A rigorous test, of course, involves intensity measurements.

The agreement between theory and observation may be considered a verification of Pockels' values of the elasto-optical constants. Bergmann and Fues[15] have pointed out that the intensities of the diffracted light might be used to determine these constants. This method is, however, only feasible after the intensity function (12) has been verified, or if the experiments are carried out with small enough sound amplitudes for which Brillouin's approximation is valid.

Since, as we have shown, the light diffracted by forced oscillations of solids is usually depolarized, it is natural to expect that also the light diffracted by the standing waves representing the temperature motion will be depolarized. This suggests a new explanation of the results of Krishnan[19] on the light scattering in glasses, which would not involve his assumption of crystalline groupings.

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[19] R. S. Krishnan, Proc. Ind. Acad. Sci. **3**, 211 (1936). The variation of the depolarization $1/\rho_h$ with the chemical composition is similar to the behavior of the photoelastic constant $(q-p)$.