TABLE V. *Variation of the reversible susceptibility with magnetization and temperature.*

T°C \ J/J_s	Reversible susceptibility, χ_r. 0.0*	0.1	0.2	0.4	0.6	0.7	0.8	0.9	0.95
21.6	24.4	24.0	23.0	19.6	13.9	7.6	2.4	0.5	0.4
99.0	46.0	44.2	43.2	37.1	27.1	17.2	7.3	2.0	1.3
171.5	111	110	104	86	63.7	46.7	28.0	9.7	4.1
213.0	205	199	186	150	104	78.0	49.1	18.6	6.8
252.2	152	150	143	121	90.5	71.2	50.0	26.0	9.2
278.4	164	162	156	130	94.0	72.5	50.2	24.0	8.9
305.8	216	209	192	150	106	82.5	54.5	24.0	9.1
341.8	278	264	244	189	128	96.7	60.0	24.8	7.9

* Extrapolated values

with magnetization and temperature is given in Table V. The values corresponding to $J=0$ are obtained by extrapolation.

An interpretation of the data presented in this report, in accordance with the domain theory of ferromagnetism, will appear in a paper shortly to be submitted for publication in this journal by Dr. W. F. Brown, Jr. In conclusion the writer desires to acknowledge his indebtedness to Dr. S. L. Quimby, who suggested this research and followed its progress with helpful counsel and encouragement.

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Magnetic Field of a Symmetrical Bundle of Parallel Wires Carrying Equal Currents

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The radial and circumferential components of the magnetic field outside and inside a bundle of parallel current filaments equally spaced on an infinitely long cylinder are derived in convergent series form. The assumption usually made that the mean circumferential component is a sufficiently good approximation to the actual field at radii of interest is justified by calculation at 12 mm radius from the axis of a bundle containing 36 wires, 6 at 3 mm radius, 12 at 6 mm, 18 at 9 mm. The maximum deviation of the circumferential field from its mean value is here found to be less than one-half percent. A quickly applied primary winding of 36 turns for ring specimens is described. It consists of a central bundle, flat end connecting blocks and external cage. The region of good circular field lies between radii of 12 mm and 60 mm, and is 70 cm in axial length.

THE type of magnetic field produced by a single straight wire infinite in length and carrying an electric current is well known. A compact group of wires would produce a similar field except for the region near the wires where the discontinuous nature of the group would produce fluctuations in the field. By considering a symmetrical group it is possible to determine the amount of the periodic variation in the field, and it turns out to be expressible in terms of a rapidly converging series. The actual details of the derivation are simplified by using complex variables and properties of the roots of unity.

Consider n infinitely long wires each carrying a current I amperes and placed so that the axis of each wire lies on a circle of radius a. The angular separation of the wires will be $2\pi/n$ radians. The average magnetic field at a distance r from the center of the group is

$$\bar{H}=\int Hdl \Big/ \int dl = 0.4\pi nI/2\pi r = 0.2nI/r.$$

The actual magnetic field, however, depends upon the position of the point P on the circle and will be found by adding together the fields obtained by considering each wire separately. Referring to Fig 1, it is seen that wire number 1 gives a magnetic field $0.2I/R$ with components $0.2I\cos\beta/R$ perpendicular to r and $0.2I\sin\beta/R$ parallel to r. It follows that

$$(H_1)_\theta=\frac{0.2I}{r}\frac{1-\rho\cos\alpha}{1-2\rho\cos\alpha+\rho^2}$$

and

$$(H_1)_r=\frac{0.2I}{r}\frac{\rho\sin\alpha}{1-2\rho\cos\alpha+\rho^2},$$

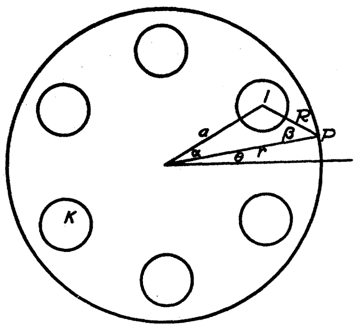

FIG. 1. A symmetrical group of six wires.

where $\rho = a/r$. By writing $H_1 = (H_1)_\theta + i(H_2)_r$ in complex variable form the following is obtained:

$$H_1 = \frac{0.2I}{r}[1 + \rho e^{i\alpha} + \rho^2 e^{2i\alpha} + \cdots + \rho^m e^{mi\alpha} + \cdots].$$

In general the angular position of the $(k+1)$th wire is

$$\alpha + 2\pi k/\text{n}, \quad k = 0, 1, 2, \cdots n-1.$$

Therefore

$$H_{k+1} = \frac{0.2I}{r}[1 + \rho e^{i\alpha} e^{2\pi k i/n} + \rho^2 e^{2i\alpha} e^{2\pi 2k i/n} + \cdots + \rho^m e^{mi\alpha} e^{2\pi m k i/n} + \cdots].$$

The total field H will be

$$\sum_{k=0}^{n-1} H_{k+1}$$

and will appear in complex variable form. Therefore

$$H = \frac{0.2I}{r}\left[\sum_{k=0}^{n-1} 1 + \rho e^{i\alpha} \sum_{k=0}^{n-1} e^{2\pi k i/n} + \cdots + \rho^m e^{mi\alpha} \sum_{k=0}^{n-1} e^{2\pi m k i/n} + \cdots\right].$$

The terms of the sum

$$\sum_{k=0}^{n-1} e^{2\pi k i/n}$$

are the nnth roots of unity; so

$$\sum_{k=0}^{n-1} e^{2\pi k i/n} = 0.$$

Consider the general term $e^{2\pi m k i/n}$. If m and n have no common factor such as 2, 3, 4, $\cdots$, then the sum is again composed of the nnth roots of unity. If m and n have a factor of 2, one obtains a repeated subgroup of the nnth roots of unity with only $n/2$ roots present. However, these are just the ones that correspond to the $n/2$ roots of unity so

$$\sum_{k=0}^{n-1} e^{2\pi m k i/n} = 0.$$

If m and n have a common factor of 3, 4, etc., the arguments are similar. However, if $m/n = 1$, 2, 3, etc., then

$$\sum_{k=0}^{n-1} e^{2\pi m k i/n} = n.$$

Consequently, most of the powers of ρ drop out of the series. The result is

$$H = \frac{0.2nI}{r} + \frac{0.2nI}{r}(\rho^n e^{ni\alpha} + \rho^{2n} e^{2ni\alpha} + \cdots)$$
$$= \bar{H} + \bar{H}(\rho^n e^{ni\alpha} + \rho^{2n} e^{2ni\alpha} + \cdots).$$

The component of H perpendicular to r is

$$H_\theta = \bar{H} + \bar{H}(\rho^n \cos n\alpha + \rho^{2n} \cos 2n\alpha + \cdots).$$

The imaginary part gives the other component H_r. Since $\rho < 1$, $(r > a)$, it is seen that the series will tend to converge very rapidly for n fairly large and furthermore the second term gives the variation of H_θ with α.

If a circular path is taken within the group ($r < a$ and $\rho = r/a$), then the average field is zero. The addition of the fields obtained by considering each wire separately gives directly the variation in the field. The procedure is the same as that used above. The following expression is obtained:

$$H_1 = (0.2I/\rho a)(\rho e^{i\alpha} + \rho^2 e^{2i\alpha} + \cdots).$$

Adding all the fields gives

$$H = (0.2nI/a)(\rho^{n-1} e^{ni\alpha} + \rho^{2n-1} e^{2ni\alpha} + \cdots).$$

The real part gives

$$H_\theta = (0.2nI/a)(\rho^{n-1} \cos n\alpha + \rho^{2n-1} \cos 2n\alpha + \cdots).$$

Application

In a previous paper[1] one of the authors described a form of primary winding that was being used in testing ring specimens. This type of winding has been improved from a two dimensional form to a symmetrical three dimensional form which produces a good circular field along the entire central region. The fundamental principle of the device is not essentially new. A prototype is described by Mollinger[2] and various "bird-cage" windings have been in use. Kelsall[3] devised a one-turn winding for high frequency a.c. which was an improvement over earlier models.

The frame consists of two hard rubber ends and four brass rods. The magnetizing coil is made of copper wire of B and S gauge No. 12. Bakelite guides are used in the closely packed central group to form three symmetrical layers of six, twelve, and eighteen wires respectively. The No. 12 wire also enables one to make socket type connections in such a way as not to interfere with the use of small specimens. Rings of inside diameters of twenty-four millimeters may be slipped onto the central group after first compressing the brush-like group at the top and sliding the brass ring well up towards the end. Symmetry arrangements and slight differences in the lengths of the wires makes it possible to correctly connect the thirty-six wires in series. If only eighteen wires are desired, a shift in one lead removes a symmetrical group of eighteen. Thus, the primary winding can be applied to one or many specimens in just a few minutes. Suitable specimen holders may be placed anywhere along the central region.

In regard to the uniformity of the magnetic field, it is fairly easy to see that the main variation in H near the center is produced by the central group—the ends and sides producing negligible effects. On a circle of radius r taken anywhere along the central group the average magnetic field $\bar{H}$ is $0.2\times 36I/r$. To compute the variation in H, it is sufficiently accurate to consider these wires as infinite in length. Hence,

$$H_\theta=\bar{H}+\bar{H}(\rho^n \cos n\alpha+\rho^{2n}\cos 2n\alpha+\cdots),$$

which is a maximum when $H_\theta=\bar{H}(1+\rho^n+\cdots)$. For the three layers of wires the maximum value of ρ is $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ respectively and n is 6, 12, and 18, respectively. (The three layers of wire have mean radii of 3, 6, and 9 mm and r is equal to or greater than 12 mm.) Therefore,

$$(H_\theta)_6=(\bar{H}/6)[1+(1/4)^6+\cdots]$$
$$=\bar{H}[1/6+1/6(1/4)^6+\cdots$$

and $$(H_\theta)_{12}=(\bar{H}/3)[1+(1/2)^{12}+\cdots]$$
$$=\bar{H}[1/3+1/3(1/2)^{12}+\cdots$$

and $$(H_\theta)_{18}=(\bar{H}/2)[1+(3/4)^{18}+\cdots]$$
$$=\bar{H}[1/2+1/2(3/4)^{18}+\cdots.$$

If all three were to become a maximum together, then

$$(H_\theta)_{\max}=(H_\theta)_6+(H_\theta)_{12}+(H_\theta)_{18}=\bar{H}[1.003].$$

Consequently, the variation of H_θ is less than one-half of one percent. With respect to the apparatus described above, it may be stated that a good circular field occurs over a region of about seventy centimeters along the central part and radially outward to a distance well beyond six centimeters.

The authors are indebted to Dr. D. G. Bourgin, department of mathematics and Dr. J. Kunz, department of physics, for valuable advice.

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[2] Mollinger, *Gumlich—Leitfaden der magnetischen Messungen* (1918), p. 149.
[3] Kelsall, J. O. S. A. **8**, 329 (1924).