But U is a matrix and the sum of the indicated elements is merely the diagonal element $(U^2)_{0,0}$ of the product matrix. The integration is readily performed, giving $(U^2)_{0,0}=[(u-2)/(u+2)]^{3/2}B^2$. Hence this series is

$$\sum_{l,\,m,\,n=0}^{\infty}{}' \frac{(2l)!(2m)!(2n)!}{l!^2m!^2n!^2}u^{-2(l+m+n)}=\frac{u^3}{(u^2-4)^{3/2}}-1. \quad (2)$$

In order to obtain the series (1) for $\Delta E^{(2)}$ from this series just summed, we multiply each term by $2du/u$ and integrate from u to ∞. Applying the same process to the sum, Eq. (2), we obtain

$$-\frac{Ma^2B^2}{2\hbar^2\sigma}\left(\frac{u-2}{u}\right)^3\int_u^\infty \frac{2}{u}\left(\frac{u^3}{(u^2-4)^{3/2}}-1\right)du=-\frac{Ma^2B^2}{2\hbar^2\sigma}\left(\frac{u-2}{u}\right)^3 \times\left[\log 2-1+\frac{u}{(u^2-4)^{1/2}}-\log\left(1+\frac{(u^2-4)^{1/2}}{u}\right)\right].$$

To this expression must be added, first the contribution from the second term in V, which turns out to be

$$-\frac{3}{8}\frac{\hbar^2\sigma}{Ma^2}+\frac{3B}{4(\sigma+1)}\left(\frac{\sigma}{\sigma+1}\right)^{3/2},$$

second the result of the first Schrödinger approximation:

$$\frac{3}{2}\frac{\hbar^2\sigma}{Ma^2}-B\left(\frac{\sigma}{\sigma+1}\right)^{3/2}.$$

When expressed in terms of σ, this gives the final result

$$E^{(1+2)}=\frac{9}{8}\frac{\hbar^2\sigma}{Ma^2}-B\left[1-\frac{3}{4(\sigma+1)}\right]\left[1-\frac{1}{\sigma+1}\right]^{3/2} -\frac{Ma^2B^2}{2\hbar^2}\frac{\sigma^2}{(\sigma+1)^3}\left\{\frac{\sigma+1}{[\sigma(\sigma+2)]^{1/2}} -\log\left[1+\frac{\{\sigma(\sigma+2)\}^{1/2}}{\sigma+1}\right]-0.30685\right\}.$$

The dependence of this expression on σ is shown in the last row of Table I.

NOVEMBER 15, 1937 PHYSICAL REVIEW VOLUME 52

The Disintegration of Beryllium and the Masses of the Beryllium Isotopes

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The following disintegrations have been studied up to bombarding energies of 250 Kv.

$$Be^9+H^1\rightarrow\begin{pmatrix}Be^8+H^2\\ Li^6+He^4\end{pmatrix}, \quad (1)$$

$$Be^9+H^2\rightarrow Li^7+He^4+Q_2, \quad (2)$$

$$Be^9+H^2\rightarrow Be^8+H^3+Q_3, \quad (3)$$

$$Be^9+H^2\rightarrow Be^{10}+H^1+Q_4. \quad (4)$$

Efficiency curves for the four reactions are smooth and regular. Absolute yields from a thick target are given. Range measurements of the disintegration products determine the values of Q_2, Q_3, Q_4 as 6.95, 4.32, and 4.44 Mev respectively and the masses of Be^8, Be^9, Be^{10} as 8.0081, 9.0150 and 10.0168.

Introduction

Previous work[1, 2] in this laboratory on the disintegration of light elements by protons and deuterons has been extended to include beryllium. In view of the anamolous scattering of 130 to 190 Kv protons by beryllium found by Dymond,[3] the efficiency curves for the proton-beryllium disintegration in this energy range should be of some interest. Furthermore, a consistent set of range measurements of the products of disintegrations by deuterons provides an opportunity to evaluate the masses of the beryllium isotopes.

We have investigated the following reactions with the use of the 250 Kv apparatus and the experimental method previously described.[2]

$$Be^9+H^1\rightarrow\begin{pmatrix}Be^8+H^2\\ Li^6+He^4\end{pmatrix}, \quad (1)$$

$$Be^9+H^2\rightarrow Li^7+He^4+Q_2, \quad (2)$$

$$Be^9+H^2\rightarrow Be^8+H^3+Q_3, \quad (3)$$

$$Be^9+H^2\rightarrow Be^{10}+H^1+Q_4. \quad (4)$$

Oliphant, Kempton and Rutherford[4] have studied these disintegrations at 550 Kv and

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[1] Williams, Wells, Tate and Hill, Phys. Rev. **51**, 434 (1937).

[2] Williams, Shepherd and Haxby, Phys. Rev. **52**, 390 (1937).

[3] Dymond, Proc. Roy. Soc. **A157**, 302 (1936).

[4] Oliphant, Kempton and Rutherford, Proc. Roy. Soc. **A150**, 241 (1935).

Allen[5] has investigated the efficiency curve for (1) up to 125 Kv.

Efficiency Measurements

Efficiency curves for the four disintegrations were obtained from thick targets by comparing, with the technique previously described,[2] the yield at a given voltage to the yield at an arbitrarily chosen standard voltage, 212 Kv. The results are shown in Fig. 1. It is seen that there is a marked similarity between these curves and that they are quite smooth and regular, showing no signs of any resonance process such as had been found for boron.[1]

The thick target yield curves are also compared by integrated Gamow plots[6] in Fig. 2. The experimental points are well fitted by straight

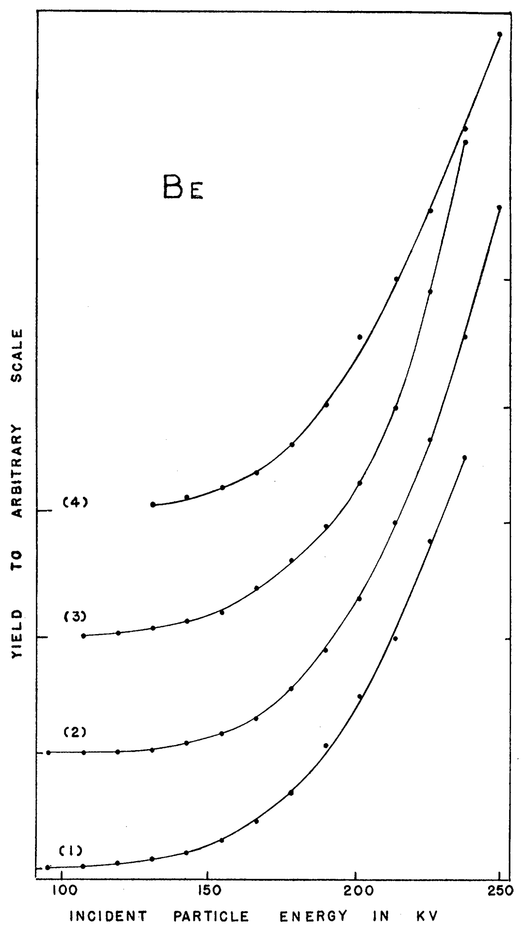

Fig. 1. Thick target yield curves for Be. The curves are displaced and plotted to the scale of unit yield at 212 Kv.

[5] Allen, Phys. Rev. **51**, 182 (1937).
[6] Henderson, Phys. Rev. **43**, 98 (1933).

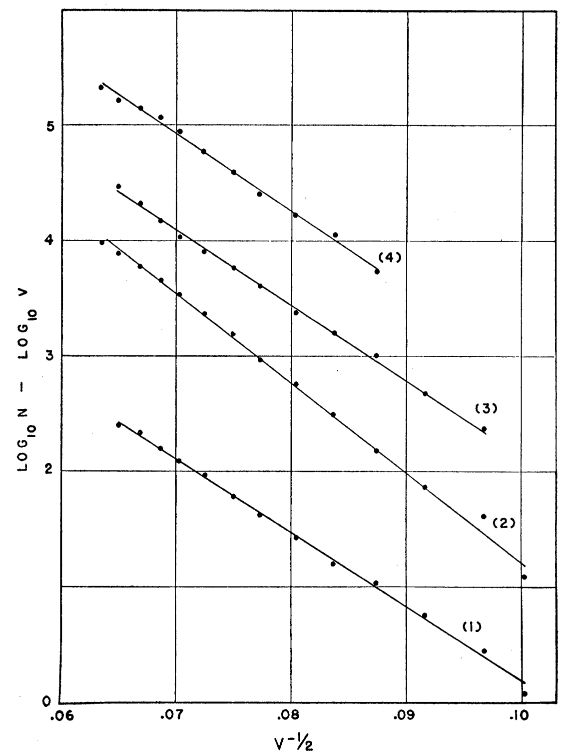

Fig. 2. Integrated Gamow plots from the thick target data shown in Fig. 1. The curves are displaced vertically.

lines and indicate the similarity between the excitation functions. It may be concluded, as in the case of lithium,[2] that for these deuteron energies the Oppenheimer-Phillips process does not contribute appreciably to the disintegration probability of (4).

The absolute yields from a thick target of beryllium per incident ion of 212 Kv energy are: (1) 4×10^{-7}, (2) 1×10^{-9}, (3) 2×10^{-10}, (4) 2×10^{-11}. These yields are based on reasonable assumptions of the solid angle employed, the deuteron concentration in the mass two spot and isotropic angular distribution of the disintegration particles. The estimated accuracy is ±50 percent. At 125 Kv our measured yields are approximately four times those of Allen. Oliphant, Kempton and Rutherford give a ratio of 10 : 45 : 45 for (2) : (3) : (4) at 550 Kv deuteron energy. Extrapolating the straight lines of Fig. 2 to this voltage leads to a value for the ratio of 90 : 9 : 1.

Range Measurements

In order to study the distribution in range of the particles from the deuteron reactions we have

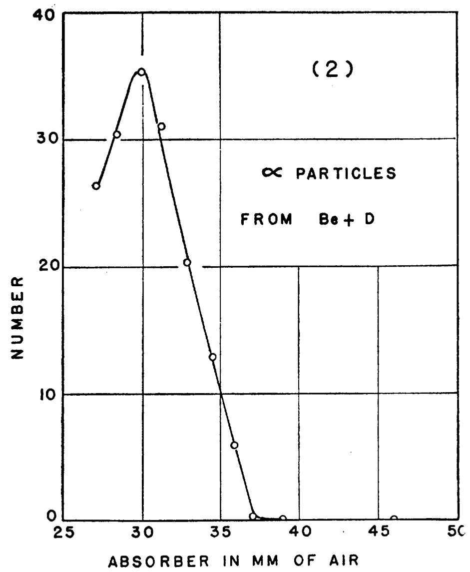

FIG. 3. Differential range number curve of the alpha-particles from reaction (2). The range scale shown should be increased by 0.6 mm.

made use of a differential ionization chamber and a variable bias counter so that only particle ionization kicks greater than a selected magnitude would be recorded. To vary the amount of absorbing material in the path of the particles we have used mica windows and an adjustable pressure air absorption tube which limited the particles observed to $90\pm5°$ with respect to the direction of the incident deuteron beam. The residual range of the mica windows and air absorption tube was calibrated with alpha-particles from a clean Th C+C′ source replacing the beryllium target.

A typical curve of the number of alpha-particles from (2) *vs.* absorbing material in cm of air at 15°C is shown in Fig. 3. By comparison with the Th C+C′ alpha-particles under identical detecting conditions we conclude that the mean range is 2.92 ± 0.02 cm at zero deuteron energy. This value agrees with the range of 3.0 cm given by Oliphant, Kempton and Rutherford.

When we substitute Bonner's values for the masses of Li^7, He^4, and H^2, we obtain for Be^9 a mass of 9.0150 which agrees within the accuracy of the mass data with Bonner's value of 9.0149 and with Bainbridge and Jordan's value of 9.01516 ± 0.0002.

The process of calibrating the range of the H^3 particles from (3) by comparison with alpha-particles from Th C′ is open to question. Since the variation of the specific ionization with residual range of an alpha-particle and a H^3 nucleus may be quite different we have adopted the following procedure. Differential numbers *vs.* range curves of approximately the same width were obtained by adjusting the detecting apparatus to record only those particles whose specific ionization was more than an arbitrarily large amount. Such a curve is shown for the H^3 particles from (3) in Fig. 4.

A direct comparison of the specific ionization curves of protons and alpha-particles has been made by Schmidt and Stetter and Stetter and Jentschke.[7] They showed that the alpha-particles had a maximum specific ionization when 4.6 mm from end of range and protons at 3.9 mm from end of range. If one assumes that the variation of specific ionization of a singly charged particle is a function of velocity alone, one can construct a Bragg curve for H^3 from the H^1 curve of Schmidt and Stetter. The peak of this Bragg curve should then be three times as far removed from the end

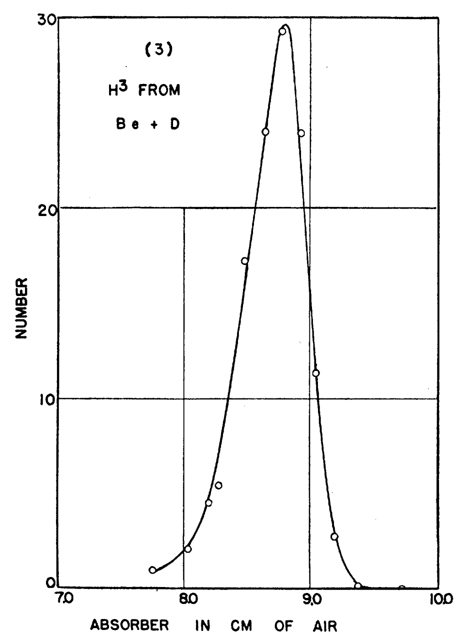

FIG. 4. Differential range number curve of the H^3 particles from reaction (3). The range scale shown should be increased by 1.4 mm.

[7] Schmidt and Stetter, Wiener Ber. **139**, 123 (1930). Stetter and Jentschke, Physik. Zeits. **36**, 441 (1935).

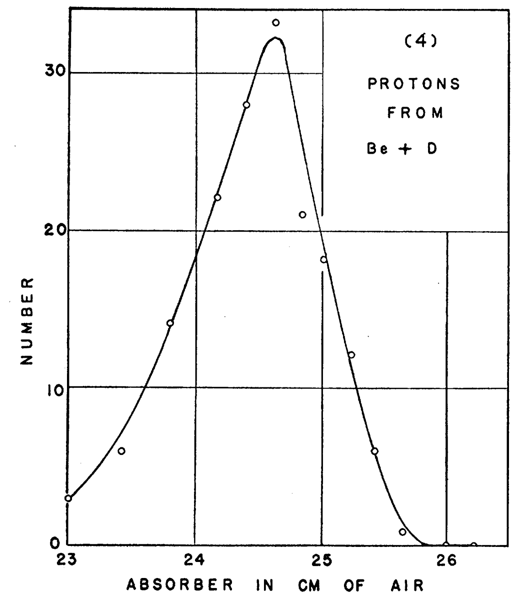

FIG. 5. Differential range number curve of the protons from reaction (4). The range scale shown should be increased by 8.0 mm.

of range, i.e., 11.7 mm, as in the case of protons, since the peak occurs at a range position corresponding to equal velocities of the particles. On the basis of these measurements we should calibrate our range scale by adding the difference between the alpha-particle and H^3 particle residuals to our direct comparison of the differential peaks.

On the other hand the recent measurements of Parkinson, Herb, Bellamy and Hudson[8] shown in curve VI of their Fig. 5 indicate that the maximum of the specific ionization curve for protons is not more than 10 Kv from the end of range, corresponding to about 0.2 mm range. One is therefore inclined to question the validity of the measurements of Schmidt and Stetter where the ionization from recoil protons is measured in a deep ionization chamber. Measurements on alpha-particles which may be compared directly to the above results[8] for protons are not available. It therefore seemed only possible to assume that the H^3 and alpha-particle differential peaks of approximately the same width may be compared directly to establish the range scale for the former and to realize that future experiments with artificially accelerated alpha-particles, deutrons and protons will make possible a method of correcting range calibrations with alpha-particles.

On the basis of these considerations, the H^3 particles from (3) shown in Fig. 4 have a mean range of 8.94 ± 0.10 cm at 225 Kv deuteron energy. The stated error is purely experimental and makes no allowance for uncertainty in the above assumptions. If we set $Be^9=9.0150$ and use Bonner's masses for H^2 and H^3 we obtain for the mass of Be^8 a value of 8.0081 with an error of 0.00005 from the *experimental* range determination. This is to be compared to the mass of two alpha-particles, 8.0080, and Bonner's value of 8.0078. The range determination is to be compared to Oliphant, Kempton and Rutherford's value of 8 cm.

For the case of the protons from reaction (4), differential range *vs.* number curves were taken with the same air absorption tube. A typical curve is shown in Fig. 5. Additional mica foils were interposed in the path of the protons. These foils were of approximately 5 cm air equivalent and were separately calibrated with Th C′ alpha-particles which first passed through the mica and then through an adjustable distance of air. By comparison with the data of Mano[9] on the relative stopping power of mica and air for protons, we conclude that the mean range of the protons from (4) is 25.4 ± 0.3 cm of air. Oliphant, Kempton and Rutherford's value is 26 cm. Substituting the mass-energy data in (4) we obtain for the mass of Be^{10}, 10.0168 ± 0.0001, compared to Bonner's value of 10.0163. The nucleus should then be unstable with respect to B^{10} of mass 10.0160.

The range and mass data are shown in Table I.

TABLE I.

REACTION	PARTICLE	Q IN MEV	MASS OF BE ISOTOPE
(2)	He^4	6.95	$Be^9=$ 9.0150
(3)	H^3	4.32	$Be^8=$ 8.0081
(4)	H^1	4.44	$Be^{10}=10.0168$

We wish to express our sincere thanks to Professor E. L. Hill for his discussion of range calibrations and to Professor John T. Tate for his continued interest and support. This research was supported in part by a grant from the Graduate School.

[8] Parkinson, Herb, Bellamy and Hudson, Phys. Rev. **52**, 75 (1937).

[9] Mano, J. de phys. et rad. **5**, 628 (1934).