On the Relation of Proton-Deuteron to Neutron-Deuteron Scattering

H. PRIMAKOFF
Department of Physics, New York University, University Heights, New York, N. Y.
(Received September 15, 1937)

It is shown that the scattering cross section per unit solid angle at large scattering angles, found by Tuve in his experiments on the collisions of 0.83 Mev protons with deuterons, is so much greater than the Rutherford cross section, that it cannot be solely due to scattering of s protons, by the nuclear forces. It follows that the scattering observed by Tuve is resonance scattering of protons of orbital angular momenta (relative to the deuterons) greater than zero. Due to the symmetry of nuclear forces in protons and neutrons, a similar resonance scattering of 0.83 Mev neutrons incident on deuterons is to be expected. The total cross section for this neutron-deuteron resonance scattering should be between 20×10^{-24} and 40×10^{-24} cm^2.

THE equality of the specific nuclear forces between like and unlike elementary heavy particles[1] leads to a simple relationship between proton-deuteron and neutron-deuteron scattering.[2] In certain limiting cases, it is even possible to see what this relationship is, without any calculation: thus, if proton energy and scattering angle are so great, that the classical distance of approach of proton to deuteron is less than the extension of the nuclear forces, then the Coulomb force is negligible compared to the nuclear forces, and the proton-deuteron and neutron-deuteron scattering cross sections are just equal. In the general case, as well, it is possible, from a knowledge of the angular dependence and magnitude at a given energy, of one of the cross sections, to draw conclusions about the angular dependence, and magnitude at the same energy, of the other. Now experimental data for the cross section per unit solid angle (per u.s.a.) of 0.83 Mev protons on deuterons, have been given by Tuve, Heydenburg and Hafstad,[3] and it is our purpose here, to compare Tuve's results, with the cross section per u.s.a., of 0.83 Mev protons on deuterons which we calculate, using the relationship between proton-deuteron and neutron-deuteron scattering discussed above, and taking the cross section per u.s.a. of 0.83 Mev neutrons on deuterons, from a theoretical curve of Schiff's.[4]

Schiff gives the variation of the neutron-deuteron cross section, with energy, from zero to 3 Mev, assuming an angular dependence, spherically symmetric in the center of gravity system.[5] His curve shows that the cross section decreases monotonically as the energy increases; further, that above a neutron energy $\cong 0.75$ Mev, "exchange" scattering (scattering in which the incident neutron changes places with the initially bound neutron which is then observed), is unimportant. Thus Schiff's calculations indicate that for energies $\cong 0.83$ Mev, and greater, the effective neutron-deuteron interaction may be represented by a space-varying potential. Schiff's theoretical cross section exceeds Dunning's experimental one,[6] by 25 percent at 2.5 Mev and by a factor of 2 or 3 at thermal energies. No experimental results for 0.83 Mev neutrons are available, and so, for this energy, we take for the cross section per u.s.a., Schiff's theoretical value namely: $1/4\pi\times2.6\times10^{-24}$ cm^2.

Using the "one-body" theory[7] formula for the proton-deuteron cross section per u.s.a.: $\sigma_{p-d}(\vartheta)$, supplemented by the equality of the proton and

[1] Breit, Condon and Present, Phys. Rev. **50**, 825 (1936); Breit and Feenberg, Phys. Rev. **50**, 850 (1936).

[2] The above-mentioned equality of the nuclear forces, is the cause of other characteristic phenomena, among which may be mentioned the approximate equality of the binding energies of H^3 and He^3; cf., for instance, Present and Rarita, Phys. Rev. **51**, 788 (1937); the approximate equality of the yields of protons and neutrons in the D^2 on D^2 reaction; cf discussion by Johnson, Phys. Rev. **51**, 799 (1937).

[3] Tuve, Heydenburg and Hafstad, Phys. Rev. **50**, 806 (1936), Fig. 13.

[4] Schiff, Phys. Rev. **52**, 149 (1937) especially Fig. 2. Schiff's treatment neglects polarization of the deuteron by the neutron, and the possible effect of the formation of an intermediate three-body system with an energy equal to that of any one of the excited states of H^3 (resonance).

[5] Bethe, Rev. Mod. Phys. **9**, 69 (1937) estimates (P. 180) that the spherical symmetry should be valid up to 2 Mev.

[6] Dunning, Pegram, Fink and Mitchell, Phys. Rev. **48**, 265 (1935).

[7] Since capture and inelastic scattering of the proton are both small, and an effective proton-deuteron potential appears to exist, the "one-body" theory should not be too incorrect. Cf Bethe, reference 5, §73. Our Eq. (1) is Bethe's Eq. (636).

neutron phase-shifts[8] ($\delta_l^{p-d}=\delta_l^{n-d}=\delta_l$), demanded by the symmetry of the nuclear forces in like and unlike particles, we have:

$$\sigma_{p-d}(\vartheta)=\left|\frac{e^2}{2Mv^2\sin^2\frac{1}{2}\vartheta}e^{-i\alpha\log\sin^2\frac{1}{2}\vartheta}+\frac{1}{2}i\frac{\hbar}{Mv}\sum_l(2l+1)(e^{2i\delta_l}-1)e^{2i(\eta_l-\eta_0)}P_l(\cos\vartheta)\right|^2$$

$$=\left[\left(\frac{e^2}{2Mv^2\sin^2\frac{1}{2}\vartheta}\right)^2\right]+\left[\frac{-e^2}{Mv^2\sin^2\frac{1}{2}\vartheta}\frac{\hbar}{Mv}\sum_l(2l+1)\sin\delta_l\cos(\delta_l+2\eta_l-2\eta_0+\alpha\log\sin^2\tfrac{1}{2}\vartheta)P_l(\cos\vartheta)\right]$$

$$+\left[\left(\frac{\hbar}{Mv}\right)^2\sum_l\sum_{l'}(2l+1)(2l'+1)\sin\delta_l\sin\delta_{l'}\cos(\delta_l-\delta_{l'}+2\eta_l-2\eta_{l'})P_l(\cos\vartheta)P_{l'}(\cos\vartheta)\right]$$

$$\equiv\sigma_{\text{Ruth.}}(\vartheta)+\sigma_{\text{intf.}}(\vartheta)+\sigma_{n-d}(\vartheta).\quad(1)$$

Here ϑ is the proton's scattering angle in the center of gravity system; v is the proton's velocity in the laboratory system; $M=2/3\times$proton mass, is the reduced mass; η_l is the phase shift introduced by the Coulomb field; (see Bethe, reference 5, Eqs. (613b), (612c)) $\alpha=e^2/\hbar v$; the first bracket$\equiv\sigma_{\text{Ruth.}}(\vartheta)$ is the Rutherford-Coulomb cross section; the second bracket $\equiv\sigma_{\text{intf.}}(\vartheta)$ is due to interference between Coulomb and nuclear scattering; the third bracket $\equiv\sigma_{n-d}(\vartheta)$ is just the neutron-deuteron cross section per u.s.a., since $\alpha\ll 1$ and thus $\eta_l\ll\delta_l$.

Now, if the neutron-deuteron cross section is spherically symmetric, only δ_0 is appreciable. Moreover, in this case, using Schiff's value for $\sigma_{n-d}(\vartheta)$ we can determine $\sin\delta_0$ and thus calculate $\sigma_{\text{intf.}}(\vartheta)$ and finally $\sigma_{p-d}(\vartheta)$.

Our results are given in Table I, which indicates that at large angles, the observed proton-deuteron cross section, far exceeds the calculated one. Further, the observed $\sigma_{p-d}(\vartheta)$ for large ϑ, which is of the order:

$$\left(\frac{\hbar}{Mv}\right)^2\sum_l\sum_{l'}(2l+1)(2l'+1)\sin\delta_l\sin\delta_{l'}\times\cos(\delta_l-\delta_{l'})P_l(\cos\vartheta)P_{l'}(\cos\vartheta)$$

is so great, that it cannot be due to s scattering alone. For, even if we assume s resonance (i.e. $\sin^2\delta_l=0$, if $l>0$; $\sin^2\delta_0=1$, and not 0.39, the value previously obtained from Schiff's neutron-deuteron cross section), then $\left\{\frac{\sigma_{p-d}(\vartheta)}{\sigma_{\text{Ruth.}}(\vartheta)}\right\}_{\text{calculated}}$ is only multiplied by 2.5, whereas at $\vartheta=150°$, a factor $\cong 35$ is required for agreement with Tuve's data. Nor can the use of the "many body" theory help, for it will merely lead, in the case of resonance scattering, to the same formula as the

[8] This has been pointed out by Bethe, reference 5, §74B.

TABLE I. *Comparison of observed and calculated proton-deuteron cross sections (per u.s.a.).*

Θ[1]	ϑ	$\sigma_{\text{Ruth.}}(\vartheta)\times 10^{26}$	$\sigma_{\text{intf.}}(\vartheta)\times 10^{26}$	$\sigma_{n-d}(\vartheta)\times 10^{26}$	$\left\{\frac{\sigma_{p-d}(\vartheta)}{\sigma_{\text{Ruth.}}(\vartheta)}\right\}$ calculated from eq. (1)	$\left\{\frac{\sigma_{p-d}(\vartheta)}{\sigma_{\text{Ruth.}}(\vartheta)}\right\}$ observed by Tuve
126°	150°	0.43	4.8	21.0	61	2275
90	120	0.70	6.0	21.0	40	124
80	110	0.82	6.7	21.0	35	70
75	104	1.02	7.3	21.0	29	23
70	98	1.22	8.0	21.0	25	18
65	92	1.48	8.8	21.0	21	15
60	86	1.81	9.7	21.0	18	15
40	59	6.60	18.5	21.0	7	8
30	45	18.7	31.3	21.0	4	5
20	30	86.6	67.0	21.0	2	2

[1] Θ is the scattering angle in the laboratory system.

"one-body" theory with the exception that $\sin^2\delta_0$ is replaced by[9]

$$\left(\frac{\Gamma_P{}^J}{\Gamma^J}\right)^2\frac{2J+1}{(2i+1)(2s+1)},$$

a quantity always less than unity. Thus it appears necessary, if we are to explain Tuve's results, to invoke resonance scattering[10] of p protons of energy 0.83 Mev, due to a level of the intermediate He^3 nucleus at an excitation energy $\cong 6$ Mev.

Further, if we wish to retain even approximately, the symmetry of nuclear forces in like and unlike particles, then, since with this assumption: $\sigma_{p-d}(\vartheta)\cong\sigma_{n-d}(\vartheta)$, for large ϑ, it follows that there should exist a resonance scattering of p neutrons of energy 0.83 Mev, incident on deuterons, and due to an excited level of the intermediate H^3 nucleus. The resultant total cross section should be about 25/3 times that given by Schiff, i.e. a total cross section $\cong 25/3\times 2.6\times 10^{-24}$ cm$^2\cong 20\times 10^{-24}$ cm^2. It would be interesting to search for this experimentally. Again, Schiff's neutron-deuteron cross section due to s scattering alone, agrees with experiment at energies $\cong 2.5$ Mev,[11] thus, we would expect (since at large ϑ, $\sigma_{p-d}(\vartheta)\cong\sigma_{n-d}(\vartheta)$), a cross section per u.s.a., for 2.5 Mev protons and large ϑ, which is

$$\cong\frac{((\hbar/Mv)\sin\delta_0)^2}{(e^2/2Mv^2)^2}\times\sigma_{\text{Ruth.}}(\vartheta)\cong 160\times\sigma_{\text{Ruth.}}(\vartheta).$$

Note 1. The assumption of p resonance scattering (i.e. $\sin^2\delta_1\cong 1$; $\sin^2\delta_0\cong\sin^2\delta_2\cong\cdots\cong 0$) for the 0.83 Mev protons, leads to a cross section per u.s.a., which at large ϑ, is about $\frac{1}{2}$ of that observed by Tuve; the assumption of d resonance scattering (i.e. $\sin^2\delta_2\cong 1$; $\sin^2\delta_0\cong\sin^2\delta_1\cong\sin^2\delta_3\cong\cdots\cong 0$) to a cross section $\frac{3}{2}$ that observed by Tuve. This is seen as follows. We have:

$$\sigma_{\text{Ruth.}}(\vartheta)\cong\left(\frac{e^2}{2Mv^2}\right)^2\cong 0.4\times 10^{-26}\text{ cm}^2$$

$$\{\sigma_{p-d}(\vartheta)\}_{p\text{ resonance}}\cong\left(\frac{\hbar}{Mv}\right)^2(2\times 1+1)^2\sin^2\delta_1\cos^2\vartheta$$
$$\cong 55\times 10^{-26}\times 9\times 1\times 1\text{ cm}^2\cong 500\times 10^{-26}\text{ cm}^2\cong 1250\times\sigma_{\text{Ruth.}}(\vartheta)$$

$$\{\sigma_{p-d}(\vartheta)\}_{d\text{ resonance}}\cong\left(\frac{\hbar}{Mv}\right)^2(2\times 2+1)^2\sin^2\delta_2\times(\tfrac{3}{2}\cos^2\vartheta-\tfrac{1}{2})^2\cong 55\times 10^{-26}\times 25\times 1\times 1\text{ cm}^2$$
$$\cong 1375\times 10^{-26}\text{ cm}^2\cong 3400\times\sigma_{\text{Ruth.}}(\vartheta)$$

$$\{\sigma_{p-d}(\vartheta)\}_{\text{observed by Tuve}}\cong 2300\times\sigma_{\text{Ruth.}}(\vartheta).$$

Further, if we assume $\sin^2\delta_1\cong 1$; $\sin^2\delta_0$ appreciable; other phases $=0$, then the resulting cross section will be less than the one calculated above, on the assumption of $\sin^2\delta_1\cong 1$; $\sin^2\delta_0\cong 0$, because of the destructive interference, at large angles, of the scattered s and p waves. Thus if Tuve's experimental data is correct to within a factor of 2, even p resonance is not quite sufficient. However as pointed out to me, by Professor Breit, the protons scattered at large angles were observed only indirectly through the recoil deuterons, and there is therefore some chance of a large error in the experiments.

Note 2. Through the kindness of Professors Mitchell and Oppenheimer, a paper by Yukawa and Sakata (Proc. Phys. Math. Soc. Jap.; June 1937) on neutron-deuteron scattering, has been brought to my attention. These authors show that upon taking the breadth of the neutron-deuteron effective potential well as 4.5×10^{-13} cm, with a correspondingly suitable depth, there appears a resonance scattering of p neutrons of energy $\cong 0.7$ Mev and due to a P level of the intermediate H^3. The magnitude of their total cross section at resonance is $\cong 10\times 10^{-24}$ cm^2, which may be compared with the cross section of magnitude $\cong\frac{1}{3}\times 4\pi\times 500\times 10^{-26}$ cm$^2\cong 20\times 10^{-24}$ cm^2 obtained by assuming complete p resonance, at this energy (i.e. $\sin^2\delta_1=1$; $\sin^2\delta_0=0$; $\sin^2\delta_2=0$, etc.), and with the cross section of magnitude $\cong 40\times 10^{-24}$ cm^2, to be expected from Tuve's proton scattering data, and the symmetry of the nuclear forces in protons and neutrons. Except for this resonance, the variation of their cross section with energy, as well as its absolute magnitude, is approximately like that of Schiff, who, it seems, missed predicting the resonance by choosing his effective neutron-deuteron potential well too narrow.

In conclusion, I think it is therefore not too unlikely to suppose that there is some sort of resonance scattering for protons and neutrons of energies $\cong 0.8$ Mev, incident on deuterons, even if Tuve's results are somewhat in error.

---

[9] Cf. Bethe, reference 5, Eq. (625). J is the angular momentum of the resonance level; i, s, the spins of the deuteron and proton; $\Gamma_P{}^J$, Γ^J the proton and total widths of the resonance level.

[10] For p resonance scattering, we have in the one-body theory: $\sin^2\delta_1\cong 1$; in the many body theory: $\Gamma_P{}^J\cong\Gamma^J$.

[11] It follows from this, that the resonance level of H^3, if it exists, cannot have a width more than 2 Mev. However, it is probably much narrower.