On the Magnetic Scattering of Neutrons. II

Some time ago[1] we have shown that the scattering of slow neutrons in magnetized matter should depend noticeably on their magnetic moment. In deriving a formula for this effect, it was assumed that the neutron could be treated as a true magnetic dipole; at present the interior properties of heavy particles are not known well enough to decide whether this assumption, or any other, is justified; but it is interesting to notice that the physical nature of the neutron moment has an influence on the magnetic scattering, marked enough to be decided experimentally.

To obtain a unique theoretical answer it is in fact not sufficient to treat the neutron as a mass point, carrying an angular momentum and a magnetic moment, since this would lead to a magnetic field of the neutron, singular at the position point in such a way as to make the result on that basis ambiguous. To illustrate the situation, let us consider a case in which the electron currents of the scattering atom are flowing only outside a closed surface S, surrounding the neutron. We shall see that the geometrical shape of this surface matters for the result even in the limit in which its linear dimensions are small compared to the wave-length of the neutron and the radius of the atom, both entering in a first-order treatment of the scattering process. This exclusion of the interior of S is, of course, not meant as a physical model; in fact with our present knowledge it is impossible to give such a model as soon as the linear dimensions of S become comparable to those of the neutron itself or to the electron radius. The exclusion is merely introduced as a device to separate the origin of any ambiguities from well-established facts.

In this case the force acting on a neutron at a position $\mathbf{r}_n$ is the negative gradient of a potential

$$U(\mathbf{r}_n) = -\int (\mathbf{H}_n(\mathbf{r}-\mathbf{r}_n)\cdot \mathbf{m}_e(\mathbf{r}))d\tau, \tag{1}$$

where $\mathbf{H}_n(\mathbf{r}) = \text{curl } (\boldsymbol{\mu}_n \times \mathbf{r})/r^3$

is the magnetic field of the neutron with magnetic moment $\boldsymbol{\mu}_n$ and where the integral is to be extended over the volume outside the surface S. $\mathbf{m}_e$ is the well-known polarization vector of Maxwell's theory, connected with the stationary current density $\mathbf{i}_e$ of the atomic electrons by the relation $\mathbf{i}_e = c \text{ curl } \mathbf{m}_e$. Accepting the law of action and reaction between neutron and electrons the expression (1) follows immediately from the force $1/c(\mathbf{i}_e \times \mathbf{H}_n)d\tau$ acting on the volume element $d\tau$ of the atom. We notice further that the total magnetic moment of the atom is given by

$$(1/2c)\int (\mathbf{i}_e \times \mathbf{r})d\tau = \int \mathbf{m}_e d\tau. \tag{2}$$

The values of $\mathbf{i}_e$ and $\mathbf{m}_e$ are to be taken as the expectation values of the corresponding operators in a stationary state of the atom; the above equations are then just as valid as in classical theory.[2]

Treating the scattering of the neutron, due to the potential (1) by the usual Born approximation and assuming that the nuclear cross section σ_ω per unit solid angle is small compared to the square of the neutron wave-length the total cross section per unit solid angle is now easily found to be

$$\phi_\omega = \sigma_\omega \left| 1 \pm \frac{\gamma_n \gamma_e}{2(\sigma_\omega)^{\frac{1}{2}}} \frac{e^2}{mc^2} \left(\frac{q_z^2}{q^2} - C \right) \right|^2 \tag{3}$$

with $$C = (1/4\pi)\int (z ds_z/r^3). \tag{4}$$

Here we have arbitrarily chosen as z axis the direction of the vector

$$\boldsymbol{\mu}_e(\mathbf{q}) = \int \mathbf{m}_e(\mathbf{r}) e^{i(\mathbf{q}\mathbf{r})} d\tau.$$

γ_e and γ_n are the magnitudes of $\boldsymbol{\mu}_e$ measured in Bohr magnetons and $\boldsymbol{\mu}_n$ measured in nuclear magnetons, respectively; $\mathbf{q}$ is the difference between the vectors of propagation of the incident and scattered neutron wave, and the plus or minus sign in (3) is valid according to the two possible orientations of the neutron moment with respect to $\boldsymbol{\mu}_z$. The integral (4) is to be taken around the origin over the surface S, which is supposed here to extend over linear dimensions, small compared to $1/q$; ds_z is the z component of a vector in the direction of the external normal to S with magnitude equal to the surface element.

In the limit, where the neutron wave-length is large compared to the linear dimensions of the atom the quantity γ_e in (3) becomes simply the magnetic moment of the atom in Bohr magnetons and the z axis points in the direction of magnetization. The characteristic constant C depends then solely on the shape of the surface S: If, for example, one chooses S to be a sphere, one finds $C = \frac{1}{3}$;[3] choosing S as a cylinder with height h and radius R around the z axis, one has the two limiting cases

(a) $R \ll h$. This corresponds to treating the neutron as a true dipole and gives $C = 0$, i.e., our result, obtained in I.

(b) $R \gg h$. This corresponds to treating the neutron as a little amperian current and gives $C = 1$, i.e., Schwinger's result.[2]

An assumption about the interaction between nuclear magnetic moments and atomic electrons, analogous to the choice (b) underlies the usual calculations of hyperfine structures and is experimentally supported for the proton and the deuteron by the agreement of the results for their magnetic moments obtained by the Stern-Estermann and Rabi methods. What choice has to be made for the neutron is not *a priori* certain and therefore the value of the constant C should be left open. Since it appears in (3) together with a strongly angle dependent term, one may hope to determine this constant experimentally by directly measuring the angular distribution of slow neutrons, scattered in their passage through magnetized matter.

F. Bloch

Department of Physics,
Stanford University, California,
April 28, 1937.

[1] Bloch, Phys. Rev. **50**, 259 (1936). Here referred to as "I".

[2] If in the light of these considerations one examines a recent paper by J. S. Schwinger (Phys. Rev. **51**, 544 (1937)) one finds that his result for the magnetic scattering is not essentially based on the use of the Dirac operator for the electron current but on the assumption, implicit in the form chosen for the energy of the neutron in the magnetic field of the atomic electrons, that the neutron can be treated as a little amperian current.

[3] This result has been kindly communicated to me by Mr. M. Bronstein, who obtained it by introducing polar coordinates around the neutron and first integrating over the angles.