A New Method for the Measurement of the Bohr Magneton

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A molecular ray method for the measurement of forces acting on molecules is discussed in which these forces are compensated by the force of gravity (molecular balance).

IN the following paper a method is discussed in which, by employing a molecular ray, the acceleration given to a molecule by an external field (magnetic, electric) is compared directly with the acceleration produced by gravity. The experiments now under way in this laboratory attempt to employ this method for an exact determination of the Bohr magneton.[1] However, the method should be useful also for many other problems.

THE MEASUREMENT OF THE FREE FALL OF MOLECULES

The free fall of molecules in the gravitational field of the earth could be easily observed by the following experiment with molecular rays.

A molecular ray, Cs in our case, is produced by the ovenslit A (Fig. 1) and the collimating slit B. The detector C is a heated tungsten wire. Both slits and the detecting wire are horizontal. The Cs atoms striking the surface of the wire are ionized. The ion current between the wire and a negatively charged cylinder gives directly the number of impinging atoms per second (Langmuir-Taylor method[2]). The dotted lines in Fig. 1 give the paths of some Cs atoms with different velocities. We shall find a deflected beam with an intensity distribution corresponding to Maxwell's law.

NUMERICAL EXAMPLE

We assume the distance $AB=BC=l$. Then in our arrangement the distance of free fall s_α for the atoms with the most probable velocity α is

$$s_\alpha=gl^2/\alpha^2=gl^2M/2RT \quad (\text{since } \tfrac{1}{2}M\alpha^2=RT). \qquad (1)$$

With $l=100$ cm we have

$$s_\alpha=\tfrac{3}{5}\times(M/T) \text{ mm}. \qquad (1a)$$

For Cs ($M=132.9$; $T=450°$K): $s_\alpha=0.177$ mm. Fig. 3 gives the distribution of the intensity in the vertical direction for a beam of 0.04 mm width (beam without half-shadow, detecting wire very thin). s is the distance from the center of the beam, i/i_0 the ratio of the ion current i at the position s to i_0 for the undeflected beam, that is, the straight beam of atoms not influenced by any force.

The available intensity J_0 is in a very rough approximation given by

$$J_0=\frac{2\times10^{-5}}{(MT)^{\frac{1}{2}}}\frac{h}{r^2}\frac{\text{mol}}{\text{cm}^2\text{ sec.}},^3$$

where $r=2l$ is the length of the beam and h the height of the ovenslit (in this case h is horizontal).[4] With $M=132.9$; $T=450°$K; $2l=r=2\times10^2$ cm, $h=0.2$ cm:

$$J_0=4\times10^{-13}(\text{mol/cm}^2\text{ sec.}).$$

If the diameter of the detecting wire is 4×10^{-3} cm and the effective length 2×10^{-1} cm, J_0 corresponds to an ion current $i_0=3\times10^{-11}$ amp.

[1] Specially interesting with regard to the present inconsistencies in the numerical values of the fundamental constants. By measuring the Bohr magneton per mole we get essentially a numerical value for h/m. h/m combined with the Rydberg constant gives directly (after a remark of Niels Bohr) the fine structure constant α and a check on Eddington's hypothesis $\alpha=1/137$.

[2] John B. Taylor, Zeits. f. Physik **57**, 242 (1929); U.z.M. 14 (U.z.M., Untersuchungen zur Molekularstrahlmethode, refers to a series of papers concerning the molecular ray method.)

[3] O. Stern, Zeits. f. Physik **39**, 755 (1926); U.z.M. 1.

[4] J_0 depends also on the product of the width b of the ovenslit and the pressure p in the oven. But because of the condition that the mean free path λ in the oven should not be smaller than b, this product is constant. In the above equation it is assumed that for all substances in the first approximation $\lambda=1/10$ mm for $p=1/10$ mm.

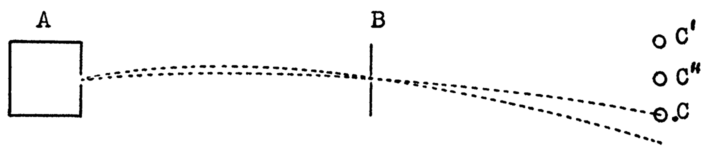

Fig. 1.

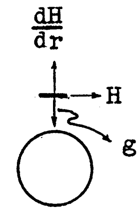

Fig. 2.

Compensation of the Force of Gravity by a Magnetic Field

The magnetic field may be produced by a current I flowing through a wire underneath and parallel to the beam. Led d be the distance between the center of the beam and the center of the wire. Then at the place of the beam the field strength H is $2I/d$ and the inhomogeneity $dH/dr = -2I/d^2$. H is horizontally, dH/dr vertically directed (Fig. 2). The magnetic force $F_m = \mu(dH/dr)$ exerted on a magnetic dipole has also the vertical direction. Thereby μ is the component of the magnetic moment of the dipole in the direction of H (horizontal in our case).[5] For alkali atoms in a strong field μ has only the two values $+\mu_0$ and $-\mu_0$ (μ_0 Bohr magneton). In our case we have to deal with a very weak field where we have many more components. But this does not make any difference in the essential point as we shall see later. So let us assume for the moment that we have only the two components $+\mu_0$ and $-\mu_0$.[6] Then for one-half of the atoms the magnetic force has the same direction as the force of gravity, for the other half of the atoms the opposite direction. For these atoms it will be possible to choose $|dH/dr| = 2I_0/d^2$ so that the magnetic force just cancels the force of gravity. These atoms will get no acceleration at all and move strictly in straight lines. I_0 is determined by the equation

$$mg = \mu_0 |dH/dr| = \mu_0(2I_0/d^2). \qquad (2)$$

To find I_0 we can employ different methods. The most straightforward procedure seems to be the following one: We place the detecting wire a short distance above the straight beam (Fig. 1, C') and let I increase. As long as $I < I_0$ all atoms are deflected downward, no atom strikes the wire and we have no ion current. The instant I becomes larger than I_0, half of the atoms regardless of their velocity are deflected upwards and some atoms strike the wire. Since the amount of the deflection depends on the velocity, the slowest atoms strike the wire first, then with increasing $I - I_0$ the faster ones. No matter how far above the beam we set the detecting wire, we shall get an ion current as soon as I becomes larger than I_0. The intensity of this ion current, however, depends of course on the distance between the beam and the detector. It can be easily calculated as a function of $I - I_0$ by using Maxwell's law of the velocity distribution.

At this point we can see at once why the splitting of the beam into many components, 16 for Cs, by a weak field does not matter. The component with the largest value of μ has always

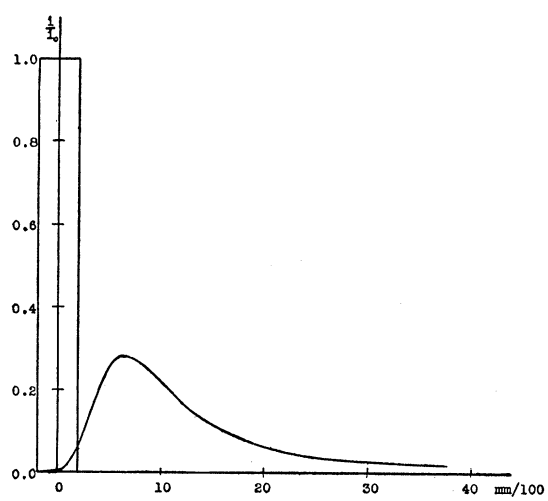

Fig. 3.

[5] In the usual arrangement H and dH/dr are parallel. The validity of $F_m = \mu(dH/dr)$ for the present case follows directly from the consideration of the energy or from considering the forces and taking into account curl $H = 0$.

[6] This case could be realized experimentally by superimposing a strong homogeneous field.

the moment μ_0.[7] But this component is the only one we are concerned with because only for this one the deflection has an upward direction as long as $I-I_0$ does not become too large (till about $I-I_0<\frac{1}{3}I_0$).

Another method to determine I_0 would be to place the detecting wire directly in the path of the straight beam (Fig. 1, C'') and measure i as a function of I. Then i should have a maximum for $I=I_0$ because if I is larger or smaller than I_0 we deflect atoms upward or downward and diminish the intensity.[8] The other components do not disturb us in this case either because they give no maximum of i for $I=I_0$ but only a monotonic increase of i with I. Of course, also here i can be easily calculated as a function of I.

It seems that I_0 could be determined very accurately by either one of these methods. This should make possible a very exact measurement of $N\mu_0$. Eq. (2) gives:

$$\mu_0=mgd^2/2I_0 \quad \text{or} \quad N\mu_0=\mathrm{M}_0=Mgd^2/2I_0$$

(N Avogadro's number, M molecular weight).

Since M and g are well known the accuracy of the result will probably depend mainly on the accuracy of d, that is of the alignment of the arrangement.

To calculate numerical values we write (2) in the form

$$|dH/dr|=(\mathrm{M}/\mathrm{M}_0)g=2I_0/d^2. \tag{2a}$$

For Cs we have

$$|dH/dr|=(132.9/5550)\times 980=23.5 \quad \text{gauss/cm}$$

and for $d=1$ cm

$$I_0=\tfrac{1}{2}\times 23.5 \text{ e.m.u.}=117.5 \text{ amp.}$$

Corrections for the finite height h of the beam and the magnetic field of the earth are small (quadratic terms) and can easily be taken into account. Furthermore, the beam must be placed in the north-south direction. In this case the Coriolis force produced by the rotation of the earth has no vertical component. Otherwise this force amounts to some tenths of one percent of the force of gravity even for the atoms with the velocity α.

Nuclear Moments

It is quite interesting to consider the numerical values for a similar experiment with H_2 molecules. For the deflection by gravity Eq. (1a) gives

$$s_\alpha=\frac{3}{5}\times\frac{M}{T}=\frac{3}{5}\times\frac{2}{60}=\frac{1}{50}\text{ mm}$$

if we take $T=60°$K. For the compensating inhomogeneity we get from Eq. (2a) taking $N\mu$ equal to 5 nuclear magnetons per mole

$$\left|\frac{dH}{dr}\right|=\frac{M}{N\mu}g=\frac{2}{15}\times 980=131\,\frac{\text{gauss}}{\text{cm}},$$

still quite a convenient value for a wire field.

But in this case it will be necessary to take into account the diffraction of the de Broglie waves for the interpretation of the measurements. The wave-length λ_α of a molecule with the velocity α is

$$\lambda_\alpha=\frac{h}{m\alpha}=\frac{Nh}{(2RTM)^{\frac{1}{2}}}=\frac{30.7}{(TM)^{\frac{1}{2}}}10^{-8}\text{ cm.}$$

For this wave-length the distance s_d of the first diffraction maximum from the beam at the place of the detector is

$$s_d=l\frac{\lambda_\alpha}{b}=\frac{l}{b}\times\frac{30.7}{(MT)^{\frac{1}{2}}}\times 10^{-8}\text{ cm,}$$

where b is the width of the collimating slit and l the distance between the collimating slit and the detector. For H_2 at 60°K we get:

$\lambda_\alpha=2\ 8\times 10^{-8}$ cm and with $b=1/100$ mm,
$s_d=2.8\times 10^{-3}$ cm

compared with $s_\alpha=2\times 10^{-3}$ cm. For Cs, however, we have

$\lambda_\alpha=0.125\times 10^{-8}$ cm and with $b=2\times 10^{-3}$ cm,

$s_d=0.62\times 10^{-4}$ cm. Consequently the diffraction will require at most a small correction.

It is self-evident that, employing the same method, we can use also other forces to compensate the force of gravity.

[7] Exactly, $\mu_0\pm$magnetic moment of the nucleus. Since this moment is of the order of magnitude $10^{-3}\mu_0$ it has to be known only very roughly. On the other hand it may be possible in the future to determine nuclear moments in this way.

[8] This is analogous to the method used by Rabi and his fellow-workers (see for instance, Phys. Rev. **50**, 472 (1930)) compensating deflections by sending the beam through a weak and afterwards a strong field. They also were the first ones to employ wire fields in actual experiments. On the other hand the whole method has a certain analogy with Millikan's experiments for the determination of e.