On the Capture of Orbital Electrons by Nuclei

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According to the theory of β decay put forward by Fermi[1] a nucleus of charge Z can, provided the process is energetically possible, be transformed into a nucleus of charge $Z-1$ in two different ways: by the emission of a positron or by the absorption of an orbital electron. In a paper to appear shortly.[2] I have calculated the probability of the latter process and have found that for heavy elements the capture of a K electron is in most cases much more probable than the emission of a positron. The total probability per second for the emission of a positron is in the case of "allowed" transitions given by

$$\lambda^+ = |M|^2 G^2 \kappa F(Z, W_0), \qquad \kappa = \frac{4(2\rho)^{2S} mc^2}{\Gamma(3+2S)^2 \hbar};$$
$$S = (1-(Z\alpha)^2)^{\frac{1}{2}} - 1. \quad (1)$$

Here $\alpha(=1/137)$ is the fine structure constant, ρ is the radius of the nucleus divided by $\hbar/mc$, m is the mass of the electron and W_0 is the energy difference between the initial nucleus of charge Z and the final nucleus of charge $Z-1$, mc^2 being taken as unit energy. $F(Z, W_0)$ is a function of Z and W_0, whose form depends on what assumption is made about the interaction between heavy and light particles. If we take the (0, 0) interaction proposed in Fermi's original paper and put $Z=82.2$ we get for different values of W_0 the F values given in the second column of Table I. The corresponding F values for the case of the (0, 1) interaction proposed by Konopinski and Uhlenbeck[3] are given in the fourth column. G in (1) is a dimensionless constant related to the universal constant g in Fermi's theory. To account for the decay periods found experimentally of the ordinary β-ray emitters we must put

$$G = 1.1\times10^{-13} \text{ in the case of (0, 0) interactions,} \quad (2)$$
$$G = 0.1\times10^{-13} \text{ in the case of (0, 1) interactions.}$$

$|M|$ in (1) is the matrix element of the Fermi theory, a quantity which by definition cannot be larger than unity. The experimental data on the decay constants and on the upper limits of the ordinary β-ray spectra show that the matrix elements $|M|$ for heavy nuclei are in general several times smaller than the matrix elements of the light elements.

For the probability per second of a K electron being absorbed by the nucleus we found the expression

$$\lambda_K = |M|^2 G^2 \kappa F_K(Z, W_0) \quad (3)$$

for "allowed" transitions, where F_K in the case of (0, 0) interactions is given by

$$F_K = \pi\Gamma(3+2S)(Z\alpha)^{3+2S}(W_0+1)^2.$$

For (0, 1) interaction the factor $(W_0+1)^2$ is replaced by $(W_0+1)^4$. The values of F_K for $Z=82.2$ and for the two kinds of interactions are given in the third and fifth columns of Table I. It can be seen that F_K is certainly considerably larger than F so that the probability of the capture of a K electron is much larger than the probability of the emission of a positron.

This result seems to be essential for the interpretation of the experiments of Cork and Lawrence[4] who bombarded platinum with deuterons and found that among other things a radioactive substance was found which emitted positrons. They assumed that $_{78}Pt^{192}$ is trans-

TABLE I. *Values of F (Z, W_0) and of F_K (Z, W_0).*

	(0, 0) INTERACTION		(0, 1) INTERACTION	
W_0	$F(Z=82.2)$	F_k	$F(Z=82.2)$	F_k
0	0	1.2	0	1.19
1	0	4.8	0	19.0
1.40	$<6\times10^{-4}$	6.8	$<10^{-5}$	39.4
2.29	0.02	12.9	0.008	139.3
3.20	0.25	21.0	0.38	369.9
4.11	1.2	31.0	3.8	810.6
5.03	3.9	43.2	20.7	1572
7.09	26.5	77.8	318.2	5092

[1] E. Fermi, Zeits. f. Physik **88**, 161 (1934).
[2] Physik. Zeits. Sowjetunion.
[3] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **48**, 7 (1935).
[4] J. M. Cork and E. O. Lawrence, Phys. Rev. **49**, 788 (1936).

formed by the bombardment into $_{78}Pt^{193}$, which can emit a positron and go over to $_{77}Ir^{193}$. The upper limit of the positron spectrum lies in the neighborhood of 2.1 MV, which corresponds to $W_0=5.1$. The decay period is 49 min. corresponding to a decay constant $\lambda=3.4\times10^{-4}$ sec.$^{-1}$.

For $Z=78$ and $W_0=5.1$ we get the following values for F and F_K (in the neighborhood of $Z=82.2$ F_K varies approximately as $Z^{3-2\times0.2}$ while F is practically constant)

$$\begin{aligned} F&=\ 4.2, & F_k&=\ 37.8 & &\text{for (0, 0) interaction}\\ F&=30.8, & F_k&=1440 & &\text{for (0, 1) interaction.}\end{aligned} \qquad (4)$$

From (1) we get, using (2)

$$\begin{aligned} \lambda^+&=2.6\times10^{-4}|M|^2, & &\text{(0, 0) interaction,}\\ \lambda^+&=3.1\times10^{-5}|M|^2, & &\text{(0, 1) interaction.}\end{aligned}$$

Even if we give $|M|^2$ its maximum value of unity these values are smaller than the experimental decay constant $\lambda=3.4\times10^{-4}$, which shows that there must exist some other process, in addition to the positron emission, whereby $_{78}Pt^{193}$ can be transformed into $_{77}Ir^{193}$. If we take into account the possibility of a K electron being captured, we have $\lambda=\lambda^++\lambda_K$. Using (1), (2) and (3) we get for the two different types of interaction

$$\lambda=\lambda^++\lambda_K=\begin{cases}26.2\times10^{-4}|M|^2\\ 14.8\times10^{-4}|M|^2\end{cases}$$

which can be brought into agreement with the experimental value by choosing a suitable value for $|M|^2$. This gives $|M|^2\sim\frac{1}{8}$ which is of the same order of magnitude as has been found for the matrix elements of other heavy elements.

While the total decay constant $\lambda^++\lambda_k$ is about the same for both types of interaction, the ratio $\lambda_k/\lambda^+=F_K/F$ between the probabilities of the two processes depends very much on the assumption made about the interaction. As can be seen from (4) we have

$$\frac{\lambda_K}{\lambda^+}=\frac{F_K}{F}=\begin{cases}9 & \text{for (0, 0) interaction,}\\ 47 & \text{for (0, 1) interaction.}\end{cases}$$

From a theoretical point of view it would therefore be of great value if this ratio could be determined experimentally. Since the capture of a K electron will always be followed by the emission of a quantum belonging to the characteristic x-ray spectrum of the element formed by the process, the ratio λ_K/λ^+ is equal to the ratio between the number of x-rays and the number of positrons emitted in a given time interval.

Finally in using the output of positrons after saturation has been obtained, to calculate the cross section for the formation of $_{78}Pt^{193}$ by the bombardment of $_{78}Pt^{192}$ with deuterons, we must take into account the fact that the number of active nuclei formed per second is $(F+F_K)/F$ times the number of positrons emitted per second. Allowing for the abundance of $_{78}Pt^{192}$ in platinum being smaller than 3 percent[5] we find from the data on the output of positrons given in the paper of Cork and Lawrence that this cross section is of the order of at least 10^{-26} cm^2 on the assumption of (0, 0) interaction. On the (0, 1) interaction assumption this cross section is even larger, being in fact of the order of 7×10^{-26} cm^2 at the lowest.

[5] B. Fuchs und H. Kopfermann, Naturwiss. **23**, 372 (1935).