Ranges and Straggling Coefficients of Alpha-Particles[1]

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(Received February 3, 1937)

A review and evaluation of the analytical and graphical methods customarily applied in range determinations reveals several types of errors that frequently have been overlooked. Of particular importance are the area and error effects. The extrapolated range is shown to be subject to other errors and it is concluded, therefore, that the *mean range* is much the best measure of range. A method of analysis which takes into account these errors is developed and applied to a set of polonium data. The experimental straggling coefficient is only 13 percent greater than the Bethe-Williams theoretical value and 20 percent greater than the Bohr value.

THE ranges of the α-particles from a given radioactive source, as quoted by different observers, frequently exhibit serious discrepancies. Since range values are important not only for the identification of the emitting atoms; but also as a check on theoretical investigations in the fields of natural and induced radioactivity, we believe it desirable to review and evaluate the analytical and graphical methods customarily applied in range determinations. In the course of this discussion we shall point out distinctions and corrections which apply to the concept and measurement of ranges and to the related subject of α-particle straggling.

The range of a homogeneous group of α-particles we define as the most probable distance which an α-particle of the group travels in a given medium before it loses sufficient energy so that it no longer can ionize atoms of that medium. This definition implies that the α-particle emerges from the radioactive atom into the second medium directly without being affected by any neighboring electrons and nuclei of the radioactive source. Strictly, this condition is not always satisfied, so that any residual errors in the final result should be due partly to the presence of source fields. The *mean range* will be taken as the one which most nearly satisfies this definition.

As a group of α-particles penetrates into the surrounding medium, it gradually slows down due to collisions, both near and distant, primarily with electrons of the medium. Since the number of encounters which occur for a single α-particle in a group may differ from the average for all the particles, the path lengths of the particles in a homogeneous group are distributed about a mean value in a manner represented very closely by the Gaussian error function. This effect Darwin[2] termed "straggling," and the ratio of the shape parameter for the function to the mean range was called the *straggling coefficient.*

These two numbers, range and straggling coefficient, which characterize the slowing down process of an α-particle in a given medium, depend upon the homogeneity of the α-ray group; the initial energy associated with the group; the thickness of the source; the atomic density of the medium; the number of electrons per atom; and a characteristic potential for these atoms. We shall confine our attention to a homogeneous bundle of particles which enter into a gaseous medium from a thin source. For thick sources a correction must be applied which increases the uncertainty in the final result.

METHODS OF INVESTIGATION

The earliest range investigations were made with scintillation screens. Counts of the scintillations produced on a zinc sulfide screen, when placed at various positions between the source and observer, were plotted as number-distance distribution curves from which values for the range were obtained. Ranges found in this manner generally are less than those from other methods. One explanation suggests that this result is due to an inability of the observer to

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[1] To be read in conjunction with the preceding paper "A Wilson Cloud Chamber Investigation of the Alpha-Particles from Uranium."

[2] Darwin, Phil. Mag. 23, 901 (1912).

count faint flashes in the presence of bright ones. However, in order to obtain a flash the screen must be placed at some distance from the source less than that at which the particle stops ionizing; and this distance will depend upon the energy required to produce detectable triboluminescence. As long as this energy is greater than that necessary to ionize the gaseous medium, any method involving a recording mechanism of this kind will yield ranges less than the true values.

Of the electrical methods, that first used by Bragg has been employed extensively for range measurements. Two parallel metal screens with a potential difference between them are placed at known distances from the source; the ionization produced by the α-particles passing through the space between them is measured. These ionization currents when plotted against distances, yield the well-known Bragg curve which has a maximum near the end and a straight line portion with a negative slope. Marsden and Perkins[3] suggested that the intercept of this linear portion extrapolated to the distance axis would be a reliable and reproducible measure of range. Other investigators, notably Henderson,[4] have improved this apparatus and made many observations over the linear portion and toe of the Bragg curve for various radioactive substances. Since the exact variation of α-particle ionization with distance traveled really is not known, it is difficult to discuss the effect of errors on this extrapolated range. A more serious objection to the use of the extrapolated range is the fact that straightforward analytical methods cannot be used to establish relations between it and related phenomena. Geiger and Nuttall[5] and, later, Geiger[6] used a total ionization method which yielded range values comparable to it. In Fig. 1 are shown several possible measures of range associated with the Bragg ionization curve.

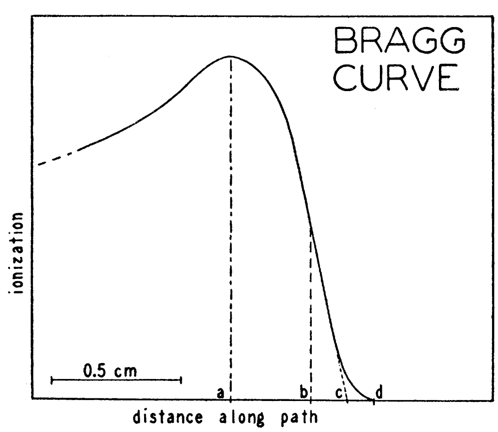

FIG. 1. The Bragg curve near path terminus showing; (*a*) the peak range; (*b*) a characteristic range half-way between maximum and zero ionization; (*c*) the extrapolated ionization range; and (*d*) the maximum range.

The most recent electrical method to be used is that of the differential ionization chamber developed at the Cavendish laboratories.[7] Essentially it measures the position of the maximum rate of change of ionization along the paths of α-particles. The maximum response determines a characteristic range, and the ratio of two such characteristic ranges is assumed to be the same as that of the corresponding true mean ranges. The validity of this assumption should be carefully investigated. Effects of straggling can be followed rather well, so that results from this method are comparable with those obtained by means of the Wilson chamber. It should be noted, however, that the Wilson chamber measures the most probable distance a particle will travel before it stops producing cloud-forming ions.

The Wilson cloud chamber is one of the most direct methods for getting values of ranges and straggling coefficients. A pair of stereoscopic pictures are taken simultaneously of the cloud tracks at a definite instant during the cycle of the chamber. When the developed negatives are projected together on a screen, one may easily measure the lengths of the track images. From the analysis of a number-distance plot of several hundred such tracks one may readily find a mean range and straggling coefficient. In the discussion below, the methods developed will be applied particularly to Wilson chamber data obtained for polonium.

The fine-grained photographic emulsion provides another medium for getting values of mean range. Of course this photographic mean

[3] Marsden and Perkins, Phil. Mag. **27**, 690 (1914).
[4] Henderson, Phil. Mag. **42**, 538 (1921).
[5] Geiger and Nuttall, Phil. Mag. **22**, 613 (1911); **23**, 445 (1912).
[6] Geiger, Zeits. f. Physik **8**, 45 (1921).
[7] Proc. Roy. Soc. **A129**, 211 (1930); **A131**, 684, 391 (1931); **A133**, 351 (1931); **A136**, 349 (1932).

range need not be equal to any of the above ones. Errors in the measurement of the track grains are large enough to mask straggling effects so that this medium is useful primarily as a convenient and simple check on range results from other methods.

The magnetic deflection method is used to measure velocities of α-particles and velocity straggling coefficients. This method proves very useful for straggling investigations. Range values, on the other hand, which are determined from the measured velocities are inexact; since the relation between range and velocity is not known.

In view of the wide variety of possible measures of range, it seems best that one should fix upon a definite standard to which all measurements may be reduced and a standard method by which mean ranges may be compared. In order to satisfy this demand, all measurements should be made relative to a homogeneous standard source, such as freshly prepared polonium films, and then reduced to normal temperature and pressure in dry air. Since the Wilson chamber affords a most direct means of measuring ranges, which very nearly satisfy the above definition, we suggest that it be used wherever possible as a standard method. A careful and detailed investigation of relative mean range values found by each of the above methods should be made in order to find out how such relative values may be reduced to the Wilson chamber standard. In this way, one may find some justification for quoting ranges to *four* significant figures.

Error Effect and Area Effect[8]

As indicated above, a straggling curve for ranges is represented very closely by the Gaussian function. If y is the fraction of the total number of tracks which have path lengths between z and $z+dz$, then

$$y=(1/\pi^{\frac{1}{2}}\alpha)e^{-(z-l)^2/\alpha^2}dz. \tag{1}$$

Here l is the mean range and α is the shape parameter; so that $\rho=\alpha/l$ is the straggling coefficient. We shall discuss the methods by which values of l and α may be determined from observed data.

Actually, due to instrumental and observational errors, the number of tracks of true length z is spread over a small region of observed length values in accordance with the Gaussian error function. That is, superimposed upon the straggling curve is an infinite array of error curves given by the relation

$$y_1=(y/\pi^{\frac{1}{2}}\alpha_0)e^{-(x-z)^2/\alpha_0^2}dx, \tag{2}$$

in which we have assumed the precision parameter, α_0, the same for all values of z. Combining (1) and (2) and integrating over all values of z from $-\infty$ to $+\infty$ one finds that

$$y_1=(1/\pi^{\frac{1}{2}}\alpha_1)e^{-(x-l)^2/\alpha_1^2}dx, \tag{3}$$

in which
$$\alpha_1^2=\alpha^2+\alpha_0^2. \tag{4}$$

It should be noted that the mean range, l, is unaltered by these errors, but, as shown later, that the extrapolated range depends markedly upon them.

In almost all range experiments measurements are made over finite intervals of path length. Thus the ionization chamber cuts out a definite portion of the space through which α-particles are passing; and, on the other hand, the stereoscopic images of fog paths from a Wilson chamber are measured to the nearest half-millimeter.

Experimental points, therefore, should lie on a curve found by plotting

$$1/\pi^{\frac{1}{2}}\alpha_1\int_{x-\Delta x/2}^{x+\Delta x/2}e^{-(x-l)^2/\alpha_1^2}dx \tag{5}$$

against x. The interval of path length, Δx, ordinarily is small enough so that the experimental points may be represented by the function

$$y_2=(\Delta x/\pi^{\frac{1}{2}}\alpha_2)e^{-(x-l)^2/\alpha_2^2}. \tag{6}$$

The relation between α_1 and α_2 is now found by equating (5) and (6). Since the relation is assumed to be good for all values of x, it must be valid for $x=l$ in particular; then

$$P\left(\frac{\Delta x}{2\alpha_1}\right)=\frac{2}{\pi^{\frac{1}{2}}}\int_0^{\Delta x/2\alpha_1}e^{-z^2}dz=\frac{\Delta x}{\pi^{\frac{1}{2}}\alpha_2}. \tag{7}$$

[8] I. Curie, J. de phys. **3**, 299 (1925). Mme. Curie discussed the area effect in detail and supplied tables from which correction factors may be obtained. In her analysis, however, Δx is equal to *one-half* the interval used for plotting.

Values of $P(\Delta x/2\alpha_1)$ may be found in tables of values of the probability integral.

As discussed below, this area effect changes the value of the straggling coefficient by approximately 2 percent whereas the error effect described above changes it by nearly 30 percent. Ordinarily the former is neglected and the latter is not even considered.

Evaluation of l and α

Usually the value of l is obtained by making $\sum y_x(x-l)^2$ a minimum;[9] so that

$$l=\sum xy_x, \tag{8}$$

where $\sum y_x=1$. Since the mean square value of $x-l$ is equal to $\alpha^2/2$ one assumes that

$$\alpha^2=2\sum y_x(x-l)^2. \tag{9}$$

These values of l and α will be designated by l_0, α_0 and will be used as zero-order approximations to the best value for a given set of data.

Because the abscissa values are fixed at finite intervals, it would seem preferable to adjust the parameters l and α so as to make $\sum(\Delta y_x)^2$ a minimum. To do this one assumes that $l=l_0+h$ and $\alpha=\alpha_0+k$ in which h and k are small correcting terms to be determined from the minimizing condition. Then the Gaussian function is expanded into a Taylor series about the point l_0, α_0 and the first two terms of the expansion are retained. Thus

$$y=\left\{1+\frac{2z}{\alpha_0}h+\frac{1}{\alpha_0}(2z^2-1)k\right\}pe^{-z^2}, \tag{10}$$

in which $z=(x-l_0)/\alpha_0$ and $p=\Delta x/\pi^{\frac{1}{2}}\alpha_0$. The partial derivatives of $\sum(y_x'-y_x)^2$ with respect to h and k are assumed to vanish; from which one readily finds that

$$2h=\frac{\alpha_0}{p\Delta}\begin{vmatrix}\sum BC & \sum y_x'B-p\sum AB\\ \sum C^2 & \sum y_x'C-p\sum AC\end{vmatrix}, \tag{11a}$$

$$k=\frac{\alpha_0}{p\Delta}\begin{vmatrix}\sum y_x'B-p\sum AB & \sum B^2\\ \sum y_x'C-p\sum AC & \sum BC\end{vmatrix}, \tag{11b}$$

in which $\Delta=(\sum BC)^2-\sum B^2\sum C^2$,

$A=e^{-z^2}$, $B=ze^{-z^2}$, $C=(2z^2-1)e^{-z^2}$, and y_x' represents the experimental values of the ordinates. By use of these relations the correcting terms h, k for l and α are determined. The corrected values of l and α may now be used in place of l_0 and α_0; thus a new set of correcting terms may be found. This operation is repeated as often as desired for higher approximations; but usually one or two applications of it are sufficient. The method may be extended for the case of a more complicated distribution.

Application

The number-distance curve obtained by Rayton and Wilkins for the polonium alpha-rays as a reference standard in the study of the alpha-ray isotopes of uranium,[10] has been analyzed by the procedure developed above. Values of the zero, first, and second order approximations of the best values for l and α_2 are given in Table I. The large mean range value is due to the fact that tracks were formed in a mixture of helium and air. We observe that an application of the method of the preceding section to Wilson chamber data does not lead to a significant correction of the values for l and α as obtained by the usual method.

To obtain a value for the precision parameter, α_0',[10a] two artificial tracks, 7.3 cm in length were made from small drill rod and photographed in various angular positions with respect to a plate. The developed negatives were projected on a viewing screen in the same manner as used for the polonium track negatives, and were measured in the same way. An analysis of the data yielded the result $\alpha_0'=0.101$. Finally

$$\alpha=(\alpha_2^2-\alpha_0'^2)^{\frac{1}{2}}. \tag{12}$$

The area effect is automatically taken into account when this equation is applied; so that α is the shape parameter for the straggling curve given by Eq. (1). Then $\rho=\alpha/l=0.0134$ is the straggling coefficient.

If α_2 is corrected for the area effect alone, the value 0.0188 instead of 0.0191 is found for ρ.

[9] The summation, $\sum$, occurs over all values of x.

[10] W. M. Rayton and T. R. Wilkins, Phys. Rev. this issue.

[10a] α_0' involves the area effect and is related to α_0 by the equation

$$\frac{\Delta x}{\pi^{\frac{1}{2}}\alpha_0'}=P\left(\frac{\Delta x}{2\alpha_0}\right).$$

TABLE I.

	0	1	2
Mean range l	7.337	7.336	7.336
Straggling Coefficient α_2	0.140	0.141	0.141

On the other hand, the error effect reduces it to 0.0134. Thus the area effect changes the value by only 1.4 percent whereas the error effect reduces it by 30 percent. This last value for ρ, although 1.2 times greater than the Bohr theoretical value,[11] is still much less than any previous value obtained with a Wilson chamber. Bennett,[12] from investigations of straggling in mica, obtained a ratio 1.1–1.2. These low values for the ratio indicate that the differences between experiment and theory need not be interpreted in the manner suggested by Briggs.[13]

In Fig. 2 is shown the curve which best fits the experimental points and the curve which is corrected for the area and error effects.

THE INTEGRAL PLOT AND EXTRAPOLATED NUMBER-DISTANCE RANGE

In order to find the integral plot of the number-distance curve, Eq. (3) is integrated from x to $+\infty$, and the results are plotted against the corresponding values of x. Thus there is obtained a curve, as shown in Fig. 3, which is very similar to the end of the Bragg curve. As suggested by Marsden and Perkins, the linear portion may be extrapolated to the distance axis to get a measure of range. This intercept value is related to the mean range by the equation

$$I = l + \pi^{\frac{1}{2}}\alpha_1/2. \quad (13)$$

When one introduces the area effect, the only change produced is that α_1 is replaced by α_2.

The measured extrapolated range is subject to two errors. In the first place, because of the error effect, it depends seriously upon the experimental errors through α_2. Secondly, any small errors in the ordinate in the neighborhood of the mean range are considerably magnified by the extrapolation so that relatively large errors may occur in the intercept value. Thus in Fig. 3 the intercept occurs at 7.471 cm from which one obtains $\alpha' = 0.152$. Since $\alpha_2 = 0.141$, the difference must be ascribed to the second type of error.

A third error affects many such range values found in the literature. It is due to the fact that the number of tracks with lengths greater than x are plotted against x rather than against $x + \Delta x/2$;[14] or else the number greater than and equal to x are plotted against x rather than $x - \Delta x/2$.[15] In other words, since for Wilson chamber data Δx is usually one-half millimeter, a systematic error of ± 0.025 cm has been introduced into several of the extrapolated number-distance range values.

Because of these various errors, results of different observers are generally in disagreement. On the whole, the mean range, l, seems much the better measure of range.

SUMMARY

(1) The various methods of measuring ranges and straggling coefficients are briefly described

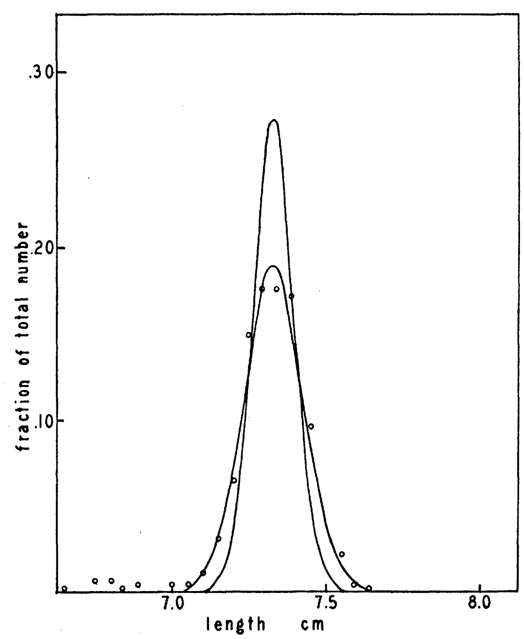

FIG. 2. Track length distribution curve for a set of experimental points, and the corrected curve.

[11] The measured ρ is only 1.1 times greater than the Bethe-Williams theoretical value (see reference 10).

[12] Bennett, Proc. Roy. Soc. **A155**, 419 (1936).

[13] Briggs, Proc. Roy. Soc. **A114**, 313 (1927).

[14] Laurence, Phil. Mag. **5**, 1027 (1928); Kurie, Phys. Rev. **41**, 701 (1932).

[15] Meitner and Freitag, Zeits. f. Physik **37**, 484 (1926); Philipp and Donat, Zeits. f. Physik **52**, 759 (1929).

and the results from these methods are shown to differ from one another.

(2) Range is defined as the most probable distance an α-particle of a homogeneous group will travel in a given medium before it ceases to ionize atoms of the medium.

(3) The mean range from Wilson chamber data is shown to satisfy the above definition, and it is suggested that this method of measurement be used wherever possible. Also all range measurements should be referred to the range of α-particles from a standard homogeneous source such as polonium.

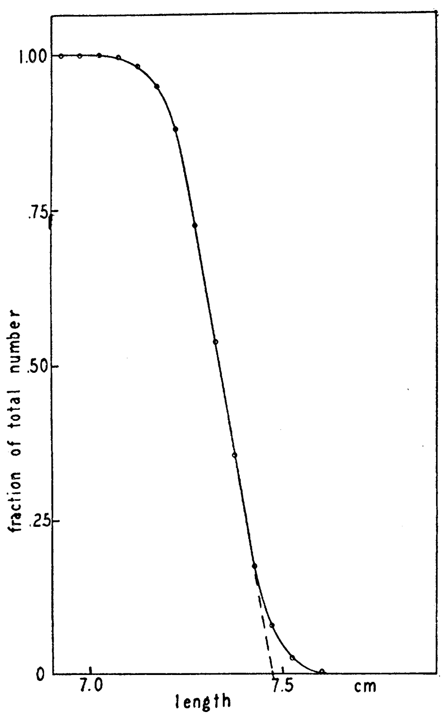

FIG. 3. Integrated number-distance distribution from the data of Fig. 2.

(4) All observations on the straggling of α-particles are shown to be subject to two disturbing factors; (a) the error effect which depends upon the presence of experimental errors and is readily eliminated from Wilson chamber data by measuring artificial track lengths; and (b) the area effect, first analyzed by Irene Curie, is easily taken into account.

(5) Values of l and α may be computed by making $\sum(\Delta y)^2$ rather than $\sum(x-l)^2$ a minimum. Equations are given from which the correction terms may be calculated.

(6) This method of analysis is applied to polonium data and yields a result for the straggling coefficient ρ, which is much less than that from any previously published Wilson chamber data. It is 1.2 times greater than the Bohr theoretical value, a result which agrees with that recently obtained by Bennett.

(7) Finally, the extrapolated number-distance range is shown to be subject to two errors; namely, that due to experimental errors in α, and that due to the magnification of small errors which occur in the neighborhood of $x=l$. A third error appears in much published data due to incorrect plotting.