Paramagnetism at Radiofrequencies

With the heterodyne beat method small changes in the inductivity of a coil at radiofrequencies can be detected with good accuracy. This has been used in order to study the magnetic susceptibility at radiofrequencies of a paramagnetic specimen inserted into the coil. It has been found that this susceptibility in some cases decreases considerably if simultaneously a strong constant magnetic field is applied in the direction of the alternating field.

Preliminary experiments have been performed with several paramagnetic alums at temperatures of 77°K, with frequencies of about 2 megacycles and constant magnetic fields up to 4 kilo-oersteds. The magnetic susceptibility of iron alum for the high frequency field decreases in strong fields to about 25 percent of its normal value. Chromic alum shows a similar behavior. Vanadium alum on the contrary retains at least 80 percent of its normal susceptibility. These figures are in a first approximation independent of the frequency.

The explanation of this phenomenon is not difficult. In previous experiments[1] the relaxation time of the magnetic moment in these alums has been shown to be of the order of 10^{-10} seconds in the absence of a constant field. According to the theory[2] this relaxation time is determined primarily by the interaction energy ΔE between the magnetic ions, which is of course of the order of $(h/2\pi)10^{-10}$. If, however, the Zeeman separations introduced by the constant magnetic field are larger than ΔE, changes in the distribution over the Zeeman levels cannot be "paid" out of the interaction energy. Such changes only can result from the feeble coupling between the magnetic moments and the heat motion of the crystal lattice. The experiments thus demonstrate that this coupling involves a relaxation time of more than 10^{-7} seconds.[3]

The different behavior of V alum indicates that the main part of its susceptibility is due to nondiagonal elements of the magnetic moment,[4] which is connected with the fact that the number of electrons is even.

Further experimental data as well as a theoretical discussion will appear in *Physica*.

C. J. GORTER

Natuurkundig Laboratorium der Rijks-Universiteit te Groningen
Holland,
March 22, 1937.

[1] C. J. Gorter, Physica **2**, 503 (1936) and 1006 (1936).
[2] I. Waller, Zeits. f. Physik **79**, 370 (1932); C. J. Gorter and R. de L. Kronig, Physica **3**, 1009 (1936).
[3] In agreement with: W. Heitler and E. Teller, Proc. Roy. Soc. **A155**, 629 (1936).
[4] J. H. Van Vleck, *Electric and Magnetic Susceptibilities*, Chapter VI.

Approximately Relativistic Equations for Nuclear Particles. Addendum. Proof of Approximate Invariance

Possible forms of approximate relativistic equations have been discussed in a recent paper.[1] The following points should be noted in connection with the proof of approximate invariance of Eq. (16.1) with Q given by Eq. (16.3).

Variational integrals [Eq. (15)] are used. The time t is the same for every particle. The integral in K extends over a region of space time corresponding to $t_i<t<t_f$. This region differs from any similar region $t_i'<t'<t_f'$ in K'. In order to avoid dealing with different regions, the published calculation was made by keeping fixed the portion of space time corresponding to particle 1. The points x_2', y_2', z_2', $t'=B$ covered in the integral used in K' are contained between t_i' and t_f'. The region of space time covered by them does not coincide with the region covered by the points A which were used for the integration in K. It is thus natural to expect the difference of the integrals in K and K' to be an unsymmetrical expression in 1 and 2, as is seen in the second line of Eq. (16.7).

The variational integral can be used by supposing χ to be known and varying φ. This is analogous to fixing the motion of 2 and looking for the equations of motion of 1 in the classical theory. The integrals over V_2 are just like potentials determining the forces acting on 1. The region of integration for 1 is not contained between two planes corresponding to fixed values of t. This does not prevent one from using the variational integral in determining φ because for a single particle the variational integral can be used for any portion of space time. For Dirac's electron subjected to external forces the integral is invariant.

The calculation leading to Eq. (16.5) shows that in the present case the integral is not invariant. The extra term is equivalent to the addition of a four potential to the field acting on 1. To within a constant factor, which is of no interest, the four potential, in Dirac's notation, is

$$A_0=-\int(\varphi^*\boldsymbol{\alpha}_2\varphi)\frac{\partial(\mathbf{r}J)}{\partial x}dV_2,\quad \mathbf{A}=\int(\varphi^*\varphi)\frac{\partial(\mathbf{r}J)}{\partial x}dV_2.$$

This can be expressed as:

$$A_0=-\frac{\partial G}{c\partial t},\quad A_x=\frac{\partial G}{\partial x_1};$$
$$G=-\int(\varphi^*\varphi)\frac{\partial K}{\partial x}dV_2;\quad K=\int J(r)rdr.$$

Being derivable from the gauge G, it can be removed by a suitable transformation of ψ. The extra term may be thus disregarded and the last line of Eq. (16.7) follows. It would have been better to use a new letter instead of ψ in the last line in order to indicate the function obtained by the gauge transformation. The need of it in this argument is analogous to the presence of a term in $d[(\mathbf{rv})J]/dt$ [Eq. (13″)] in the classical Lagrangian. The formal analogy is more striking if the transformation is performed in the configuration space of the two particles by setting

$$\psi=S\psi^{(1)};\quad S=1+ivxJ/(2c^2\hbar).$$

The addition to $\mathfrak{L}$ required by the transformation is then

$$\mathfrak{L}^{(1)}-\mathfrak{L}=S^{-1}(\mathfrak{L}S-S\mathfrak{L})\cong\mathfrak{L}S-S\mathfrak{L}$$

and this compensates the extra term. The transformation S leaves the density, current expressions invariant. The physical conditions are clearer, however, using G. This discussion does not prove that the equation is invariant for arbitrarily large J. It only shows that there is no need of terms linear in J in addition to those included.

G. BREIT

Department of Physics,
University of Wisconsin,
Madison, Wisconsin,
April, 1937.

[1] G. Breit, Phys. Rev. **51**, 248 (1937).