In our experiments there have thus appeared no minima of the type reported by Allison which are attributable to changes of the total intensity of the light. The author wishes to thank Professor J. Barton Hoag for his continued guidance in this work.

GEORGE C. COMSTOCK

Ryerson Physical Laboratory,
University of Chicago,
April 5, 1937.

[1] J. Beams and F. Allison, Phys. Rev. **29**, 161 (1927).
[2] S. S. Cooper and T. R. Ball, J. Chem. Ed. **13**, 210, 278, 326 (1936).
[3] G. Hughes, J. Am. Chem. Soc. **58**, 1924 (1936).

## On the Saturation Property of Nuclear Forces

The symmetrical interaction operator[1]

$$V=\sum_{i<j}\{(1-g-g_1-g_2)P_{ij}+gP_{ij}Q_{ij}+g_1 1+g_2Q_{ij}\}J(r_{ij}) \quad (1)$$

possesses the saturation property if the several different types of exchange forces are present in proportions satisfying the inequality[1]

$$G\equiv 1+g-5g_1-3g_2\geqq 0. \quad (2)$$

Recent calculations by Inglis[2] on the binding energy of $Li^6$ show that the most satisfactory theory based on Eq. (1) is obtained by requiring that the saturation parameter $G$ vanish. The artificial appearance of the condition $G=0$ is only apparent. It is the purpose of this note to point out that there exists an alternative formulation of the theory in which the vanishing of the saturation parameter $G$ is an entirely natural restriction without any trace of artificiality.

In terms of the Pauli spin matrices,[3] $\sigma=(\sigma_x, \sigma_y, \sigma_z)$, and the isotopic spin matrices,[4] $\tau=(\tau_\xi, \tau_\eta, \tau_\zeta)$, the exchange operators in Eq. (1) can be expressed in the form[5]

$$\begin{aligned} Q_{ij}&=\tfrac{1}{2}(1+\sigma_i\cdot\sigma_j),\\ P_{ij}Q_{ij}&=-\tfrac{1}{2}(1+\tau_i\cdot\tau_j),\\ P_{ij}&=-\tfrac{1}{4}(1+\sigma_i\cdot\sigma_j)(1+\tau_i\cdot\tau_j).\end{aligned} \quad (3)$$

The operator $V$ now contains terms linear in $\sigma_i\cdot\sigma_j$, $\tau_i\cdot\tau_j$, $\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j$ and also the term $G\,\Sigma J(r_{ij})$. If this last term is dropped by setting $G=0$ there remains an interaction operator which is a linear function of the three operators

$$\Sigma\sigma_i\cdot\sigma_j J(r_{ij}), \quad \Sigma\tau_i\cdot\tau_j J(r_{ij}), \quad \Sigma\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j J(r_{ij}), \quad (4)$$

each of which separately possesses the saturation property. The omission of the term which does not involve the spin and isotopic spin matrices is quite natural here because there is no need to balance carefully terms of different types which have the saturation property only when combined in the correct proportions.

The two formulations which are connected by Eq. (3) are equivalent only because the nuclear wave functions are antisymmetric with respect to the interchange of the Cartesian, spin and isotopic spin coordinates of any two heavy particles.[5, 6] For this reason arguments purporting to show that *symmetrical* interactions of the Majorana exchange type cannot be obtained from any form of the neutrino-electron field theory are not conclusive against the formulation in terms of the spin and isotopic spin matrices.[7] However, interaction terms of the form $\tau_i\cdot\tau_j$, $\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j$ between two heavy particles can be obtained from a field theory only if the description of the field includes an "isotopic spin" variable in addition to the ordinary spin variable which is usually ascribed to the light particles making up the field.

To obtain a good fit with the experimental binding energies and cross sections it may be necessary to associate different potential functions $J(r)$ in (4) with each of the different modes of dependence on the spin and isotopic spin matrices. For actual calculations an interaction operator based on (4) is most conveniently expressed as a linear combination of the three saturation type operators

$$\begin{aligned} V_a&=\tfrac{1}{5}\sum_{i<j}(4P_{ij}+1)J_a(r_{ij}),\\ V_b&=\tfrac{1}{2}\sum_{i<j}(P_{ij}Q_{ij}+Q_{ij})J_b(r_{ij}),\\ V_c&=\tfrac{1}{2}\sum_{i<j}(1-P_{ij})(4-5Q_{ij})J_c(r_{ij}).\end{aligned} \quad (5)$$

Each of these is a special case under Eq. (1) satisfying the condition $G=0$. The most important term is $V_a$ which determines the binding energy and the excitation energies of the low terms in the light stable nuclei belonging to the mass series $4n$, $4n+1$, $4n+3$; the effective depth and range of the potential function $J_a(r)$ are known from calculations on the binding energies of the three- and four-particle systems.[8] The singlet-triplet splitting in nuclei of the $4n+2$ type determines $V_b$; the effective range of $J_b(r)$ must differ from that of $J_a(r)$ according to the calculations of Present and Rarita[9] who have shown that the model in which the two ranges are equal cannot account for the observed properties of the two-, three- and four-particle systems within the limits of experimental error. The remaining term $V_c$ enters into the $p$ scattering of fast neutrons and protons in hydrogen and also is involved in the order of the singlet and triplet levels[10] in $Li^8$ and $B^{12}$ and in the capture of fast neutrons in hydrogen.

In addition to explaining the essentially linear dependence of binding energy on the number of particles a model of nuclear forces must also account for the strong dependence of binding energy on the symmetry properties of the space and spin wave functions.[10, 11] The latter requirement would seem to rule out the nuclear model in which the forces are all of the ordinary type with the saturation property resulting from strong forces of repulsion between particles when they approach closely.

EUGENE FEENBERG

Institute for Advanced Study,
Princeton, New Jersey,
April 7, 1937.

[1] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936). The operators $P$ and $Q$ exchange Cartesian and spin coordinates, respectively.
[2] D. R. Inglis, Phys. Rev. **51**, 531 (1937).
[3] P. A. M. Dirac, *Quantum Mechanics* (Oxford, 1935), pp. 69 and 225.
[4] These operators are formally identical with the spin matrices, but operate on the variables which determine whether the particle is a neutron or a proton (first introduced by W. Heisenberg, Zeits. f. Physik **77**, 1 (1932)).
[5] B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936).
[6] J. H. Bartlett, Phys. Rev. **49**, 102 (1936).
[7] G. Gamow and E. Teller, Phys. Rev. **51**, 289 (1937).
[8] E. Feenberg and S. S. Share, Phys. Rev. **50**, 253 (1936).
[9] R. D. Present and W. Rarita, to appear in Phys. Rev.
[10] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).
[11] E. Wigner, Phys. Rev. **51**, 106 (1937).