negligible except just above the threshold energy. It is interesting to notice that the curve is similar to the one calculated by Breit and Condon[7] for the "square hole" and ordinary force, rather than their curve for Majorana force. The difference can again be ascribed to the effect on the $l=1$ wave function of the presence or absence of a small amount of V beyond $r=r_0$.

In addition to the total cross section for disintegration, we have computed the differential cross section

$$f^2(\theta)=\frac{1}{4\pi}\sigma_m+\frac{3}{4\pi}\sigma_e\cos^2\theta,$$

the distribution in angle of the protons produced in the disintegration, where θ is the angle between the direction of the incident photon and that of the ejected proton. This distribution is plotted in Fig. 6 for different values of E, the difference between $h\nu$ and the threshold energy. Only the curves for $h\nu$ just above the threshold energy show any great departure from the simple $\cos^2\theta$ behavior.

A measurement of the absolute magnitudes of this disintegration cross section for photon energies between 2 and 6 Mev will be of great use in determining more precisely the form of the neutron-proton binding force. For the higher energies, this cross section is due predominantly to the electric dipole transition, involving a transition to a triplet p state of the outgoing wave from the normal triplet s state. Therefore, it depends on the $l=1$ wave function, which is more sensitive to the shape of the triplet potential hole than is the s function. The absolute magnitude of the cross section depends on the width of the hole, reduction in r_0 causing a reduction in σ. The position of the maximum in the curve, however, depends on the details of the shape of the potential; the more sharply V falls to zero beyond $r=r_0$, the higher will be the energy for maximum σ.

The angular dependence of ejected protons for photon energies close to the threshold gives information as to the form of the singlet potential, since this measures the ratio of magnetic to electric dipole transition, and the magnetic cross section depends on the singlet state. In general, the deeper or wider this singlet potential, the larger will be the range of photon energy over which this angle dependence differs markedly from the simple $\cos^2\theta$ law.

[7] Breit and Condon, Phys. Rev. **49**, 904 (1936). The correspondence is also close with the curves calculated by Way, Phys. Rev. **51**, 552 (1937).

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The Distribution in Time of Counts Due to a Constant Source and Its Daughter in Equilibrium

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An expression is obtained for the size distribution function of the time intervals between counts caused by a constant source and its daughter substance in equilibrium. The efficiency of the detector and its finite recovery time are taken into account. The results apply also to counts caused by bombarding particles and artificially radioactive atoms which they produce.

I. INTRODUCTION

EXPERIMENTS in which the time distribution of rays or particles from a radioactive source is used to obtain information about the source require for their interpretation a theory of the distribution to be expected. We have recently discussed[1] the fluctuations in the stocks of substances present in a complex source, and

[1] Ruark and Devol, Phys. Rev. **49**, 355 (1936); see also Peierls, Proc. Roy. Soc. **A149**, 467 (1935); Adams, Phys. Rev. **44**, 651 (1933); and Schiff, Phys. Rev. **50**, 88 (1936).

fluctuations in the disintegration of its atoms; but the distribution of disintegration times is not the same as that of the counts they produce in a detecting device of efficiency less than unity, and of limited time resolving power. Here we shall present a formula describing the distribution of counts produced by a long-lived substance and its short-lived daughter, of decay constant λ, in equilibrium. The formula applies also to any experiment in which a detector receives bombarding particles from a constant source and particles from a short-lived artificial radioactive source produced by that bombardment. For convenience, our discussion will be so worded as to apply to this situation. The word "rays" will be used to denote discrete entitities of any kind.

Consider a counter which is struck by F primary rays per second, on the average. Background rays are included in this category, as are also secondaries of any origin, *except* those due to the breakdown of activated atoms. A ray is included even if it does not enter the active space of the counter, provided it produces an activated atom which may later cause a count, by ejecting an ionizing particle into the active space. The chance that n primaries arrive in any interval t is given by Bateman's formula $(Ft)^n e^{-Ft}/n!$

Suppose that on the average, in one second, F_1 primaries arrive and do not produce activated atoms, while F_2 primaries do produce them. We refer to these two classes of primaries as class I and class II, respectively. Each class is distributed in time according to Bateman's formula, because a random selection from a Bateman distribution is itself a Bateman distribution. Similarly, the counts produced by each class form a Bateman distribution; for example, the chance that n class I primaries produce counts in time t is

$$(F_1 h_1 t)^n e^{-F_1 h_1 t}/n! \qquad (1)$$

Here h_1 means the probability that a particular class I primary will produce a count; h_2 and h_3 will denote similar probabilities for class II primaries and activated atoms, respectively. These quantities include the influence of solid-angle factors and of detector efficiency. The recovery time of the detector will be neglected for the present. We shall need two formulas from our previous paper.[1]

(1) The chance of having a stock of j activated atoms is

$$S_j = x^j e^{-x}/j! \qquad (2)$$

where x is the expected stock, F_2/λ.

(2) The chance that r activated atoms decay in time t is the same as the chance that r are produced, or that r class II primaries arrive.

$$(F_2 t)^r e^{-F_2 t}/r! \qquad (3)$$

Now, we desire the probability that no count occurs in an interval of length t, and two functions of this kind must be considered. The first, p, is the probability that the interval *from an arbitrary instant to the next count* will exceed t; the second, P, is the probability that the *interval between two counts* will exceed t. These two functions are the same for a simple source obeying Bateman's formula, but in the present problem they are distinct. P is the function usually needed for interpretation of experiments, but we first obtain p, because it is useful in getting P.

II. Chance that the Interval from an Arbitrary Instant to the Next Count Will Exceed t

During an interval $(0, t)$, counts are in general produced in several ways which may be classified as follows. (1) Class I primaries arrive; (2) activated atoms present at time zero disintegrate; (3) class II primaries produce activated atoms, some of which disintegrate before time t, and, of course, some class II primaries produce counts directly. Thus p contains three factors:

(1) *The chance p_1 that no class I primary causes a count;*

$$p_1 = \exp(-F_1 h_1 t). \qquad (4)$$

(2) *The chance p_2 that no activated atom present at time zero causes a count.* The chance that the stock of activated atoms is j at time zero, and that no atom of this stock causes a count in the interval $(0, t)$ is

$$S_j e^{-j\lambda t}[1+(1-h_3)(e^{\lambda t}-1)]^j, \qquad (5)$$

by Eq. (16) of our earlier paper. Summing over all values of j, we get

$$p_2 = \exp[-F_2 h_3 (1-e^{-\lambda t})/\lambda]. \qquad (6)$$

(3) *The chance p_3 that no count is caused by class II primaries, or by the atoms which they*

activate. We first work out the chance that (a) r activated atoms are produced, that (b) none of the r primaries produce counts directly, and that (c) none of the r activated atoms produce counts. Summing the product of these three chances over all values of r, we shall get p_3. Using Eq. (3), the product in question is

$$[(F_2t)^r e^{-F_2t}/r!]\cdot(1-h_2)^r\cdot u^r, \tag{7}$$

where u is the chance that a given atom activated during the interval in question does not produce a count before time t. Let us consider this given atom. Since we have specified that it is activated during the interval $(0, t)$, the chance it is activated in a particular time element, t_1 to t_1+dt_1, is dt_1/t.

If it is produced in this time element, the probability it will *not* cause a count before time t is $1-[1-e^{-\lambda(t-t_1)}]h_3$. Averaging this over all positions of the time of activation we have

$$u=1-h_3+h_3(1-e^{-\lambda t})/\lambda t. \tag{8}$$

Putting this in Eq. (7) and summing the result over r we obtain

$$p_3=\exp[-F_2t(h_2+h_3-h_2h_3)+F_2(1-h_2)h_3(1-e^{-\lambda t})/\lambda]. \tag{9}$$

Finally, by Eqs. (4), (6) and (9),

$$p=\exp[-At-B(1-e^{-\lambda t})/\lambda], \tag{10}$$

where $A\equiv F_1h_1+F_2(h_2+h_3-h_2h_3)$; and $B\equiv F_2h_3(2-h_2)$.

III. The Distribution of Intervals Between Counts

We now seek the probability P that no count will occur in an interval t, following an initial count at time zero. The initial count may be caused by a class I primary, a class II primary, or an activated atom. The probabilities of these three causes are F_1h_1/D, F_2h_2/D, and F_2h_3/D, where D is the sum of the numerators of these fractions. Let P_A, P_B, P_C denote the probabilities of no count in time t when the initial count is due to a class I primary, a class II primary, or an activated atom, respectively. Then

$$P=(F_1h_1P_A+F_2h_2P_B+F_2h_3P_C)/D. \tag{11}$$

P_A is identical with p, for the occurrence of a class I primary at time zero does not affect the probability that others will arrive in the interval $(0, t)$, or the stock of metastable atoms. P_B is computed in the same way as p, with one exception; the production of an activated atom at time zero changes the probability that no count will be produced by the activated atoms present just after time zero; therefore, to get P_B the factor p_2 occurring in p must be replaced by another, p_{B2}. The chance that the stock of activated atoms is j just after the initial count is the same as the chance that the stock is $j-1$ just before it. Therefore, in expression (5) we must replace S_j by S_{j-1}; summing over j from 1 to infinity, we find that

$$p_{B2}=[1-h_3(1-e^{-\lambda t})]p_2 \tag{12}$$

and

$$P_B=[1-h_3(1-e^{-\lambda t})]p. \tag{13}$$

Furthermore, the computation of P_C differs from that of p in only one way. Our knowledge that in the case before us the initial count was produced by decay of an activated atom alters the chance that there were j activated atoms *before* the initial count.

To obtain this chance, $S(j)_c$, we proceed as follows. Let $U_j(c)$ be the chance of a count due to an activated atom in the time element $(0, dt)$ when it is known that the stock of activated atoms was j *before* the count; and let $U(c)$ be the unconditional chance of a count due to an activated atom in this time element. Now $U(c)=\Sigma_jS_jU_j(c)$, and according to Bayes' theorem

$$S_jU_j(c)=U(c)S(j)_c. \tag{14}$$

Also $U_j(c)=h_3j\lambda dt$, and using Eq. (2) we finally get

$$S(j)_c=S_{j-1}. \tag{15}$$

That is, S_{j-1} is the chance that the stock was j *before* the initial count, so it is also the chance that the stock is $j-1$ just after the count. Therefore the computation of P_C is *identical* with that of p, and substituting the values of P_A, P_B and P_C into (11) we obtain

$$P=p[1-F_2h_2h_3(1-e^{-\lambda t})]/D. \tag{16}$$

By somewhat similar methods it is possible to deal with a detector which has a constant recovery time τ. If $t\gg\tau$, and $f\ll 1/\tau$,

$$P'=\{\exp[-A(t-\tau)-B(1-e^{-\lambda(t-\tau)})/\lambda]\}\times[1-F_2h_2h_3(e^{-\lambda\tau}-e^{-\lambda t})]/D. \tag{17}$$