APRIL 1, 1937 PHYSICAL REVIEW VOLUME 51

A Note on the Search for Element 87

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(Received January 7, 1937)

K. T. BAINBRIDGE, in 1929, reported a "search for element 87 by analysis of positive rays."[1] Although positive ray analysis is undoubtedly the most sensitive method for the detection of the presence of a new element, Bainbridge, studying likely sources (lepidolite and pollucite, minerals rich in the alkali metals) failed to detect in them the presence of element 87.

Since then, others have searched for this new element. F. Allison and E. J. Murphy[2] reported the presence of element 87 in lepidolite and pollucite. Their method is open to question.

J. Papish and E. Wainer[3] reported the finding of several of the stronger x-ray lines of element 87, using a concentrate derived from samarskite. The validity of their findings will be discussed later in this paper.

Recently, Hulubei,[4] using a curved crystal focusing x-ray spectrograph of the transmission type, reported weak lines at 1032 and 1043 X.U., which he assumed were the $L\alpha_{1,2}$ doublet of element 87; from which he concluded that his pollucite extract contained element 87. Fig. 1 shows a Moseley plot ($(\nu/R)^{\frac{1}{2}}$ *vs.* atomic number) of data (indicated by circles) for the $L\alpha_{1,2}$ doublets of various elements as secured from Siegbahn's *Spektroskopie der Roentgenstrahlen*. Also plotted, for atomic number 87, are values of $(\nu/R)^{\frac{1}{2}}$ calculated from Hulubei's data (indicated by crosses).

The Siegbahn data (indicated by circles) follow the Moseley law accurately (to less than 0.01 X.U.), but at $Z=87$ the crosses calculated from Hulubei's data are in error by 6 and 7 X.U. for $L\alpha_1$ and $L\alpha_2$, respectively. This may be due to a consistent error in the wave-length scale used by Hulubei. He remarks in his paper that his x-ray tube target is contaminated by mercury, and it does not seem unreasonable to suppose that these (so designated) lines of element 87 might actually be the mercury lines $L\beta_3$ and $L\beta_1$ ($\lambda=1030$ and 1047 X.U., respectively). Hulubei's measured wave-lengths for 87 differ from these mercury wave-lengths by less than his error in determining the wave-lengths of 87 $L\alpha_{1,2}$.

Recently the present author undertook a search for x-ray lines of element 87, at the request of Dr. T. G. Kennard of Claremont Colleges, California. The x-ray target material, very kindly supplied by Dr. Kennard, was $CsHSO_4$ derived from lepidolite; the x-ray tube was operated at 30 kv d.c. and 8–10 ma (the L excitation potential for element 87 was determined to be 19 kv). A Moseley graph was used to determine the Bragg angle. The rocking device,[5] used to oscillate the crystal in the spectrograph, was set for minimum amplitude of oscillation.

An eight-hour plate was taken at the proper angle to register the $L\alpha$ lines of element 87, the target material being $CsHSO_4$ concentrated for element 87 (as supplied by Dr. Kennard) pounded into a roughened copper wedge. A faint narrow band or "line" appeared at the center of the crystal reflection zone at the position calculated for the $L\alpha$ doublet of element 87. Fig. 2 (a) shows a 10-hr. spectrogram taken with $CsHSO_4$ more strongly concentrated for element 87, used as target; the concentration of element 87 here was estimated by Dr. Kennard at about six times the concentration for the previous sample. The band or "line" which appeared on the first plate seemed to have increased roughly sixfold in intensity. Such an increase in intensity with increase in concentration for element 87 would seem to be suggestive.[6]

However, the band or "line" under discussion here is broader than an L x-ray line such as Th $L\alpha_{1,2}$. Accordingly, an investigation was made to determine whether the Th (90) $L\alpha$ lines (unresolved by this spectrograph, and very nearly in

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[1] K. T. Bainbridge, Phys. Rev. **34**, 752 (1929).

[2] F. Allison and E. J. Murphy, Phys. Rev. **35**, 285 (1930).

[3] J. Papish and E. Wainer, J. Am. Chem. Soc. **53**, 3818 (1931).

[4] H. Hulubei, Comptes rendus **202**, 1927 (1936).

[5] F. K. Richtmyer, Phys. Rev. **37**, 472 (1931).

[6] At this time, the concentrate which was assumed to contain element 87 in strong concentration, was tested for α, β and γ radioactivity with the result that none was detected within experimental error (5 percent of the total number of counts).

the same spectral region) would broaden as the concentration of ThO_2 in the $CsHSO_4$ target material was decreased. No such broadening could be detected in concentrations as low as 0.125 percent of ThO_2 in $CsHSO_4$. A 10-hr. plate was made using a blank copper wedge as target. The band or "line" again appeared at the position where the $L\alpha$ doublet of element 87 should be (see Fig. 2 (b)). It was then suspected that this band or "line" might be due to a crystal imperfection located at the center of the calcite crystal: an abnormally strong reflection of Cu continuous radiation occurs for a narrow vertical zone of the crystal surface. Moreover, with close scrutiny, in Figs. 2 (a) and 2 (b) the band or "line" is observed to have the same peculiarities, *viz.*: it is split (double) at the top and center. This would point to the band or line as undoubtedly due to crystal imperfections which were accidently permitted to register photographically. Play in the rocking device, coupled with small amplitude of oscillation (rocking) would keep the crystal from moving appreciably. It is known that crystal reflection defects can be eliminated by rocking a crystal which is reflecting x-rays since the locus, on the crystal surface, of reflection of a given wave-length, then moves back and forth along the length of the crystal thus eliminating the crystal defects which are "ironed out." With a view to the possibility of eliminating the band or line in this manner, the amplitude of oscillation of the calcite crystal was increased to about 45′; the "line" vanished (see Fig. 2 (c), a 10-hr. plate) showing conclusively that it was due to a crystal surface defect.

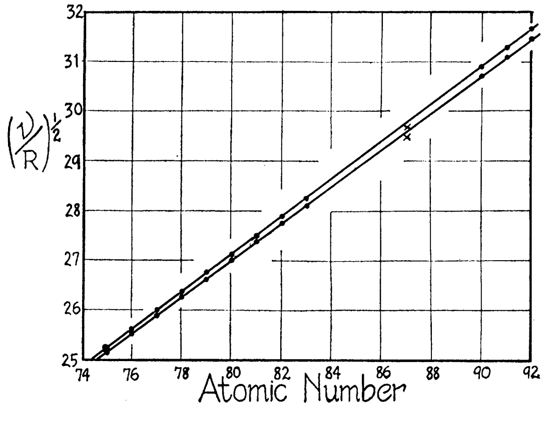

FIG. 1. Moseley graph for the x-ray lines $L\alpha_{1,2}$: circles are from data of Siegbahn's *Spektroskopie der Roentgenstrahlen;* crosses are from data given by Hulubci.[4]

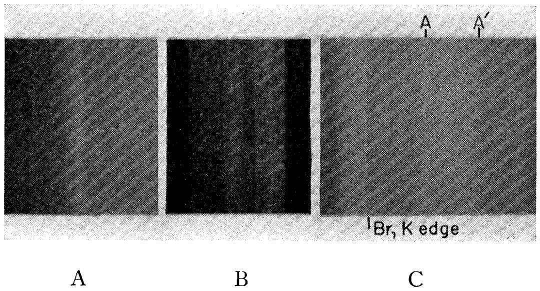

FIG. 2. (a) 10-hr. spectrogram taken using lepidolite-derived $CsHSO_4$, strongly concentrated for element 87. Note "line" at center. (b) 10-hr. spectrogram taken using blank Cu wedge, showing same "line" as in Fig. 2. (a) With identical peculiarities. (c) 10-hr. spectrogram taken with blank Cu wedge as target, with increased amplitude of crystal oscillation. Note "line" has vanished.

An important point of this study is that *the calcite crystal used by the present author is the same calcite crystal which was used by Papish and Wainer.*[3] They make no mention in their article of oscillating the crystal to eliminate the photographic registration of its surface defects;[7] hence the possibility cannot be ruled out that their reported "lines" may be due to the same crystal defect as reported in this note. Without crystal oscillation, faint lines would be masked by the crystal defect "line." Since the zone of intense reflection occurs at the exact center of the crystal (see Fig. 2 (a)), and since the crystal is cut to just match the crystal holder in length, the "line," which is due to this crystal defect, will appear at the position where the element 87 line should appear irrespective of the magnitude of

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[7] If the crystal rocking device had been set for a small amplitude of oscillation, say, for example, 45′, the play (~25′) in the crystal rocking device on the spectrograph which Papish and Wainer (reference 3), used would be sufficient to keep the crystal from oscillating. If the crystal rocker had been set for a large amplitude, for example, 120′, the heavy background (see Fig. 2 (c), zone $A \rightarrow A'$), resulting from the "smearing" out on the plate of the false "line" (see Fig. 2 (a)) would very easily mask faint lines of element 87 as the crystal would only "reflect" the line of element 87 for half of its excursion. This can easily be seen from a simple geometrical construction, as the locus of "reflection" of the line at 9° 45′, the Bragg angle for element $87L\alpha$, passes beyond the ends of the crystal for each final quarter of its travel across the crystal face. (The spectrograph radius is 18.36 cm; the crystal is 2.5 cm long.) In any event, in order to have true lines of element 87 appear on their plates, Papish and Wainer must have rocked their crystal through a large amplitude and must have had element 87 present in considerable concentration. An examination of Papish and Wainer's plates would clear up the whole question; however, they do not seem to be available to me in Ithaca or elsewhere.

the Bragg angle at which the element 87 line is calculated to be found.

This crystal which Papish and Wainer[3] used, was temporarily transferred to the spectrograph which the writer is using,[8] as the other specially cut calcite crystal was not available at the time.

The author feels that the search for element 87 is still open. The writer failed to detect the presence or absence of the $L\alpha_{1,\,2}$ lines of element 87, due mainly to the lack of a sufficient amount of concentrate to permit a conclusive search.

The author wishes to thank Dean F. K. Richtmyer for permission to use certain x-ray apparatus granted to Dean Richtmyer by the Heckscher Research Council of Cornell University. The writer also wishes to thank Dean Richtmyer, Professor Kennard and Dr. Parratt for many helpful discussions during the progress of this study.

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[8] It was not used in research leading to other publications of the present author.

APRIL 1, 1937 PHYSICAL REVIEW VOLUME 51

A Note on the Density of Eigenfunctions for an Electron Obeying Dirac's Equation

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(Received January 14, 1937)

In dealing with the theory of an ideal gas by the method of statistical mechanics, one requires to know the number of independent eigenfunctions in a given energy range. As is well known, in the nonrelativistic mechanics, the required enumeration of the states can be carried out in two ways. We may enclose the electrons in a box, by making the wave functions vanish on a certain boundary, or we may impose periodicity conditions on the wave function, the resulting density of eigenfunctions being the same in each case. For an electron obeying Dirac's equation the number of eigenfunctions is easily found by the latter method.[1] However, such periodic boundary conditions for the wave function can never in practice be realized. In the following note we shall therefore show, that for electrons enclosed in a spherical potential hole, the two methods lead to the same result. This calculation is also of special interest, because it recently has been questioned[2] whether it is justifiable to use the result proved for progressive waves, when the physical problem might require the use of "standing waves." In the special case examined, namely spherical enclosure, "standing waves" does not lead to a new physical situation.

The wave equation $(H-H')\psi=0$, written in terms of the components, gives the equations

$$\left.\begin{aligned}\frac{H'-V}{c}\psi_1-h\frac{\partial\psi_2}{\partial r}-j\frac{h}{r}\psi_2+mc\psi_1=0\\ \frac{H'-V}{c}\psi_2+h\frac{\partial\psi_1}{\partial r}-j\frac{h}{r}\psi_1-mc\psi_2=0\end{aligned}\right\}. \tag{1}$$

Putting

$$\beta_1=\left(mc-\frac{H'-V}{c}\right)/h,\quad \beta_2=\left(mc+\frac{H'-V}{c}\right)/h$$

(1) reduces to

$$\left.\begin{aligned}\beta_2\psi_1-\frac{\partial\psi_2}{\partial r}-\frac{j}{r}\psi_2=0\\ -\beta_1\psi_2+\frac{\partial\psi_1}{\partial r}-\frac{j}{r}\psi_1=0\end{aligned}\right\}. \tag{2}$$

Eqs. (2) are of standard type, the solutions involving Bessel functions in the general sense.

We have to choose solutions which satisfy the given physical conditions. Let us enclose the electron in a spherical box by assuming po-

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* Submitted from Cambridge University, England.

[1] Møller and Chandrasekhar, M. N. Roy. Astr. Soc. **95**, 673 (1935).

[2] Eddington, Proc. Roy. Soc. **A152**, 258 (1935).

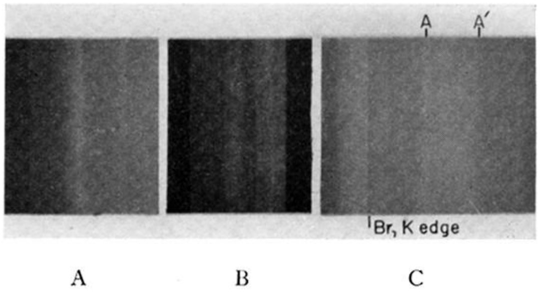

FIG. 2. (a) 10-hr. spectrogram taken using lepidolite-derived $CsHSO_4$, strongly concentrated for element 87. Note "line" at center. (b) 10-hr. spectrogram taken using blank Cu wedge, showing same "line" as in Fig. 2. (a) With identical peculiarities. (c) 10-hr. spectrogram taken with blank Cu wedge as target, with increased amplitude of crystal oscillation. Note "line" has vanished.