we see that the higher orders contribute no correction to the orbital part of the magnetic moment. This rule is quite general, for all nuclei, and is of course independent of the error-curve shape of the interaction function here employed. It follows also from conservation of orbital angular momentum, if one thinks of expanding in the ϕ's.

We have therefore to consider only excited states with $M_{S\nu}=3/2$, $M_{S\pi}=-\frac{1}{2}$ or with $M_{S\nu}=-\frac{1}{2}$, $M_{S\pi}=3/2$. The former are exactly as numerous as the latter, in the special case of Li^6, and have exactly corresponding elements H_{oa}', so their contributions to the projected spin magnetic moment, $\int\psi^*(g_\nu\sigma_{z\nu}+g_\pi\sigma_{z\pi})\psi d\tau$, cancel one another. In Li^6 the zero-order result, which makes the magnetic moment of Li^6 equal to that of the deuteron (as observed),[22] is exact (insofar as V is negligible in H_{oa}', cf. reference 14: *note added in proof*).

In other nuclei correction terms appear due to states analogous to the last four types of Table IV. These are small, of order g^2, for the forms of interaction (9), but may be quite large for (17) with large g_σ. There is a remote possibility that they might furnish an additional means of testing the interaction assumptions.

I am especially grateful to Doctors H. Bethe, E. Feenberg, and L. A. Young for discussion of this and related problems, and for communication of certain of their results before publication.

[22] Manley and Millman, Phys. Rev. 50, 380 (1936).

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On the Magnetic Scattering of Neutrons

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The scattering of slow neutrons by atoms is considered, assuming that, in addition to the ordinary nuclear forces, there is a magnetic interaction between the neutron and the atomic electrons. It is found that the neutrons scattered from an unpolarized beam will be partially polarized in virtue of this magnetic interaction. Since the scattered intensity depends not only upon the intensity, but also upon the spin density of the incident beam, the polarization thus produced can manifest itself by a second scattering. An expression is derived for the neutron intensity after double scattering from magnetized iron plates. Under optimum conditions, it is found that the scattered intensity with parallel orientation of magnetizations is 15 times that with antiparallel orientation. The partial polarization of the scattered neutrons indicates that the undeviated neutron beam will also have a nonvanishing spin density. Expressions are derived for the intensity and spin density of a neutron beam after traversing a certain thickness of magnetized iron. These results are used in the discussion of three types of experiments for producing and detecting a polarized beam of neutrons.

Introduction

The magnetic moment of the neutron has not been measured directly, but has been obtained from the magnetic moments of the proton and the deuteron.[1] The assumption of simple additivity of magnetic moments, involved in this indirect deduction, is, however, open to some objection from the point of view of the β-ray theory of heavy particle interactions and magnetic moments.[2] Since the neutron and proton are symmetrical with respect to interaction with the electron-neutrino field, the magnetic moment of the deuteron should be equal to the "elementary moment" of the proton, $e\hbar/2Mc$. The observed value is 0.85 $e\hbar/2Mc$, which is probably to be explained by the additional moment arising from the process of neutron-proton interaction, and by the fact that the proton is decomposed and does not possess its "elementary moment" during a large fraction of the time.

Recently, Bloch[3] has suggested a direct method of measuring the magnetic moment of the neu-

[1] J. M. B. Kellogg, I. I. Rabi and J. R. Zacharias, Phys. Rev. 50, 472 (1936).
[2] G. C. Wick, Lincei Rend. 22, 170 (1935); H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. 8, 82 (1936).
[3] F. Bloch, Phys. Rev. 50, 259 (1936).

tron which depends upon the fact that an atom may scatter a neutron either in virtue of the nuclear interaction with the neutron, or the magnetic coupling between the atomic electrons and the neutron spin. It is this magnetic scattering of neutrons which we shall investigate. It will appear as a result of this calculation that the expression for the scattering cross section given by Bloch is in error. The difference between the results to be presented and his results arise from the use of the correct Dirac value of the current density and the corresponding magnetic field rather than the "classical interaction" between two magnetic dipoles.

I

Van Vleck[4] has shown that, despite the exchange nature of the forces between nuclear particles, the interaction energy of a neutron and a nucleus may be approximately described by an ordinary potential $V(\mathbf{r}_n)$.[5] Consequently, the Hamiltonian of a neutron and an atomic system of Z electrons may be written:

$$\mathcal{H}=\mathcal{H}_0+\frac{1}{2M}p_n^2+V(\mathbf{r}_n)+e\mu_n\sum_{i=1}^{z}\boldsymbol{\sigma}_n\cdot\frac{\mathbf{r}_i-\mathbf{r}_n}{|\mathbf{r}_i-\mathbf{r}_n|^3}\times\boldsymbol{\alpha}_i, \quad (1)$$

where $\mathcal{H}_0$ is the Hamiltonian of the unperturbed atom, $\mu_n\boldsymbol{\sigma}_n$ is the magnetic moment operator of the neutron, and $\boldsymbol{\sigma}_n$, $\boldsymbol{\alpha}_i$ are, respectively, the Pauli spin-matrix vector of the neutron and the Dirac matrix vector of the i^{th} electron.

If we denote the unperturbed energy of the atom and the energy of the incident neutron by E_0 and E, respectively, the wave equation for the system becomes:

$$\mathcal{H}\Psi=(E_0+E)\Psi, \quad (2)$$

which can be solved approximately by neglecting inelastically scattered waves. We therefore write

$$\Psi=\psi(\mathbf{r}_n)\psi_0(\mathbf{r}_1,\cdots\mathbf{r}_z), \quad (3)$$

where $\psi_0(\mathbf{r}_1,\cdots\mathbf{r}_z)$, a matrix in the spin variables of the electrons, is the wave function of the unperturbed state of the atom, and $\psi(\mathbf{r}_n)$ is to be regarded as a matrix in the spin variable of the neutron.

Inserting (3) into the wave equation (2), we obtain the differential equation:

$$(\hbar^2/2M\nabla^2+E-V(\mathbf{r}))\psi(\mathbf{r})= -e\mu_n\boldsymbol{\sigma}_n\cdot\left\{\sum_{i=1}^{z}\int\psi_0\dagger(\mathbf{r}_1,\cdots\mathbf{r}_z)\frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3}\times\boldsymbol{\alpha}_i\psi_0(\mathbf{r}_1,\cdots\mathbf{r}_z)d\tau_1\cdots d\tau_z\right\}\psi(\mathbf{r}). \quad (4)$$

With the introduction of the abbreviation:

$$\mathbf{H}(\mathbf{r})=e\sum_{i=1}^{z}\int\psi_0\dagger(\mathbf{r}_1,\cdots\mathbf{r}_z)\frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3}\times\boldsymbol{\alpha}_i\psi_0(\mathbf{r}_1,\cdots\mathbf{r}_z)d\tau_1\cdots d\tau_z, \quad (5)$$

Eq. (4) becomes

$$(\hbar^2/2M\nabla^2+E-V(\mathbf{r}))\psi(\mathbf{r})=-\mu_n\boldsymbol{\sigma}_n\cdot\mathbf{H}(\mathbf{r})\psi(\mathbf{r}), \quad (6)$$

which is the wave equation of a neutron interacting with the nuclear field and the static magnetic field of an atom.

In order to solve this equation we shall treat the right side as a small perturbation, that is, we write

$$\psi(\mathbf{r})=\psi^{(0)}(\mathbf{r})+\psi^{(1)}(\mathbf{r}), \quad (7)$$

where

$$(\hbar^2/2M\nabla^2+E-V(\mathbf{r}))\psi^{(0)}(\mathbf{r})=0, \quad (8)$$

and

$$(\hbar^2/2M\nabla^2+E-V(\mathbf{r}))\psi^{(1)}(\mathbf{r}) =-\mu_n\boldsymbol{\sigma}_n\cdot\mathbf{H}(\mathbf{r})\psi^{(0)}(\mathbf{r}). \quad (9)$$

Since $V(\mathbf{r})$ is independent of the neutron spin, a solution of (8) can be found in the form of a spatial wave function $\psi_0(\mathbf{r})$, times a spin wave function which may be expanded in terms of the eigenstates χ_{m_s} of the z component of the spin. Hence,

$$\psi^{(0)}(\mathbf{r})=(C_{\frac{1}{2}}\chi_{\frac{1}{2}}+C_{-\frac{1}{2}}\chi_{-\frac{1}{2}})\psi_0(\mathbf{r}), \quad (10)$$

and

$$(\hbar^2/2M\nabla^2+E-V(\mathbf{r}))\psi_0(\mathbf{r})=0. \quad (11)$$

If we choose the direction of motion of the incident neutron as the axis of our polar coordinate system, the wave function $\psi_0(\mathbf{r})$ can be expanded in a series of Legendre polynomials,

[4] J. H. Van Vleck, Phys. Rev. **48**, 367 (1935); see also C. H. Fay, Phys. Rev. **50**, 560 (1936).

[5] In general, it would be necessary to take into account the effect on scattering of the virtual levels of the system: neutron+nucleus. However, in what follows we shall be primarily concerned with the interaction of iron nuclei and neutrons of thermal energy, for which no such level is known.

viz.:

$$\psi_0(\mathbf{r})=\sum_{l=0}^{\infty}(2l+1)i^l e^{i\delta_l}\frac{u_l(r)}{kr}P_l(\cos\vartheta),\qquad(12)$$

with

$$k=(1/\hbar)(2ME)^{\frac{1}{2}}.\qquad(13)$$

The functions $u_l(r)$ satisfy the equation

$$\frac{\hbar^2}{2M}\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)u_l(r)+(E-V(r))u_l(r)=0\qquad(14)$$

and have the asymptotic form

$$u_l(r)\sim\sin(kr-\tfrac{1}{2}l\pi+\delta_l)\qquad(15)$$

We shall confine our attention to slow neutrons since it is only in this case that the long duration of the collision compensates for the small magnitude of the magnetic forces. Under these circumstances the neutron wave-length $\lambda=2\pi/k$ is very large compared with the range of the nuclear forces, and it is well known that all the "phase shifts," δ_l, will be small except that of the s partial wave, δ_0.

If r be greater than the range of the nuclear forces, r_0, Eq. (14) reduces to:

$$\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)u_l(r)+k^2u_l(r)=0,\qquad(16)$$

whose solution, having the desired asymptotic form (15), is

$$u_l(r)=\left(\frac{\pi kr}{2}\right)^{\frac{1}{2}}(\cos\delta_l J_{l+\frac{1}{2}}(kr)+(-1)^l\sin\delta_l J_{-l-\frac{1}{2}}(kr)).\qquad(17)$$

Hence, if $l\neq 0$, $u_l(r)(r>r_0)$ is approximately $(\frac{1}{2}\pi kr)^{\frac{1}{2}}J_{l+\frac{1}{2}}(kr)$, i.e.; the wave function of a free particle with angular momentum l. δ_0, however, is not small and $u_0(r)$ becomes $\sin(kr+\delta_0)$. The wave function which describes the nuclear scattering is, therefore, for $r>r_0$:

$$\psi_0(\mathbf{r})=F(r,\vartheta)=\sum_{l=0}^{\infty}(2l+1)i^l\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}}\times J_{l+\frac{1}{2}}(kr)P_l(\cos\vartheta)+\frac{e^{ikr}}{2ikr}(e^{2i\delta_0}-1)$$
$$=e^{ikr\cos\vartheta}+\frac{e^{ikr}}{2ikr}(e^{2i\delta_0}-1).\qquad(18)$$

Since $\psi^{(1)}(\mathbf{r})$ is a matrix in the spin variable of the neutron, it can be written

$$\psi^{(1)}(\mathbf{r})=\psi_{\frac{1}{2}}^{(1)}(\mathbf{r})\chi_{\frac{1}{2}}+\psi_{-\frac{1}{2}}^{(1)}(\mathbf{r})\chi_{-\frac{1}{2}}.\qquad(19)$$

Substituting this and the similar expansion of $\psi^{(0)}(\mathbf{r})$ into Eq. (9), we find

$$\left(\nabla^2+k^2-\frac{2M}{\hbar^2}V(\mathbf{r})\right)\psi_{m_s}^{(1)}(\mathbf{r})=-\frac{2M\mu_n}{\hbar^2}\sum_{m_s'}\mathbf{H}(\mathbf{r})\cdot(m_s|\boldsymbol{\sigma}_n|m_s')C_{m_s'}\psi_0(\mathbf{r}).\qquad(20)$$

These equations must be solved subject to the boundary condition that they contain only scattered waves. The asymptotic form of the desired solution is:[6]

$$\psi_{m_s}^{(1)}(\mathbf{r})\sim\frac{M\mu_n}{2\pi\hbar^2}\frac{e^{ikr}}{r}\int F(r',\pi-\Theta)\sum_{m_s'}\mathbf{H}(\mathbf{r}')\cdot(m_s|\boldsymbol{\sigma}_n|m_s')C_{m_s'}F(r',\vartheta')d\tau',\qquad(21)$$

where

$$\cos\Theta=\cos\vartheta\cos\vartheta'+\sin\vartheta\sin\vartheta'\cos(\varphi-\varphi').$$

Here, (r, ϑ, φ) and $(r', \vartheta', \varphi')$ are the polar coordinates corresponding to the vectors $\mathbf{r}$ and $\mathbf{r}'$.

The integral occurring in (21) can be approximately evaluated by using the expression (18) for $F(r,\vartheta)$ since the region in which this formula fails gives a negligible contribution to the integral. Furthermore, we can, by the same reasoning, replace $F(r,\vartheta)$ with $e^{ikr\cos\vartheta}$ since, for slow neutrons, the scattered wave in (18) is comparable in magnitude with the plane wave only if r is of the order of nuclear dimensions. Replacing $e^{ikr'\cos\vartheta'}$ by $\exp(i\boldsymbol{\kappa}_0\cdot\mathbf{r}')$ and $e^{-ikr'\cos\Theta}$ by $\exp-(i\boldsymbol{\kappa}\cdot\mathbf{r}')$, where $\boldsymbol{\kappa}_0$ and $\boldsymbol{\kappa}$ are, respectively, the propagation vectors of the incident wave and the wave scattered in the direction of $\mathbf{r}$, we obtain

$$\psi_{m_s}^{(1)}(\mathbf{r})\sim\frac{M\mu_n}{2\pi\hbar^2}\frac{e^{ikr}}{r}\int\exp(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}')\sum_{m_i}\mathbf{H}(\mathbf{r}')\cdot(m_s|\boldsymbol{\sigma}_n|m_{s'})C_{m_s'}d\tau'.\qquad(22)$$

From the expression (5) for $\mathbf{H}(\mathbf{r})$ we find, after a simple calculation,

[6] Mott and Massey, *The Theory of Atomic Collisions* (Oxford Press, 1933).

$$\int \exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r})\mathbf{H}(\mathbf{r})d\tau=4\pi e i\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|^2}\times\sum_{i=1}^{z}\int \exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i)\psi_0{}^\dagger\boldsymbol{\alpha}_i\psi_0 d\tau_1\cdots d\tau_z. \quad (23)$$

We shall restrict our considerations to the slow neutrons which are strongly absorbed in cadmium, the so-called C neutrons. These neutrons have energies in the thermal region,[7] so that the corresponding de Broglie wave-lengths are of the order of 10^{-8} cm. If the neutron temperature is sufficiently low, the C neutron wave-lengths will be large compared with atomic dimensions.[8] The term $\exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i)$ will, therefore, not vary appreciably over the atom and can be replaced by the first term in its expansion which gives a nonvanishing contribution to the integrals in (23).

With this approximation, Eq. (23) becomes

$$\int \exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r})\mathbf{H}(\mathbf{r})d\tau=-4\pi e\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|^2}\times\sum_{i=1}^{z}\int \psi_0{}^\dagger(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i\boldsymbol{\alpha}_i\psi_0 d\tau_1\cdots d\tau_z. \quad (24)$$

This expression for $\int \exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r})\mathbf{H}(\mathbf{r})d\tau$ may be put in a more convenient form by using the approximate relation:

$$(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i\boldsymbol{\alpha}_i=-\tfrac{1}{2}(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\times(\mathbf{r}_i\times\boldsymbol{\alpha}_i)+\frac{1}{2i\hbar c}((\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i\mathbf{r}_i\mathcal{H}_0-\mathcal{H}_0(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i\mathbf{r}_i). \quad (25)$$

The diagonal matrix element of this equation referring to the ground state gives

$$\int \psi_0{}^\dagger(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r}_i\boldsymbol{\alpha}_i\psi_0 d\tau_1\cdots d\tau_z=-\tfrac{1}{2}(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\times\int \psi_0{}^\dagger(\mathbf{r}_i\times\boldsymbol{\alpha}_i)\psi_0 d\tau_1\cdots d\tau_z). \quad (26)$$

Therefore,

$$\int \exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r})\mathbf{H}(\mathbf{r})d\tau=2\pi e\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|^2}\times\left[(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\times\int \psi_0{}^\dagger(\sum_i\mathbf{r}_i\times\boldsymbol{\alpha}_i)\psi_0 d\tau_1\cdots d\tau_z\right]. \quad (27)$$

Since the average magnetic moment of the atom, $\boldsymbol{\mu}$, is given by the expression

$$\boldsymbol{\mu}=-\frac{e}{2}\int \psi_0{}^\dagger(\mathbf{r}_1,\cdots\mathbf{r}_z)(\sum_i\mathbf{r}_i\times\boldsymbol{\alpha}_i)\times\psi_0(\mathbf{r}_1,\cdots\mathbf{r}_z)d\tau_1\cdots d\tau_z, \quad (28)$$

we obtain, finally:

$$\int \exp\,(i(\boldsymbol{\kappa}_0-\boldsymbol{\kappa})\cdot\mathbf{r})\mathbf{H}(\mathbf{r})d\tau=-4\pi\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}+4\pi\boldsymbol{\mu}. \quad (29)$$

This expression is valid when the neutron wave-length is large compared with atomic dimensions. The value of the integral decreases rapidly with decreasing wave-length.

Collecting our formulae, we may write for the asymptotic solution of Eq. (4):

$$\psi(\mathbf{r})\sim\exp\,(i\boldsymbol{\kappa}_0\cdot\mathbf{r})\sum_{m_s}C_{m_s}\chi_{m_s}+\frac{e^{ikr}}{r}\left\{\frac{e^{2i\delta_0}-1}{2ik}\sum_{m_s}C_{m_s}\chi_{m_s}-\frac{2M\mu_n}{\hbar^2}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\sum_{m_s\;m_s'}\chi_{m_s}\left(m_s\left|\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\sigma}_n\right|m_s'\right)C_{m_s'}+\frac{2M\mu_n}{\hbar^2}\sum_{m_s,\,m_s'}\chi_{m_s}(m_s|\boldsymbol{\mu}\cdot\boldsymbol{\sigma}_n|m_s')C_{m_s'}\right\}. \quad (30)$$

II

The intensity of the neutrons scattered in the direction of the vector $\mathbf{r}$ is given by

$$I=v\psi'^\dagger(\mathbf{r})\psi'(\mathbf{r}). \quad (31)$$

$\psi'(r)$ denotes the scattered wave in Eq. (30) and v is the neutron velocity. It is easily shown that:

$$I=\frac{I_0}{r^2}\left(\frac{\sin^2\delta_0}{k^2}-\frac{4M^2\mu_n{}^2}{\hbar^4}\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2+\frac{4M^2\mu_n{}^2}{\hbar^4}|\boldsymbol{\mu}|^2\right)-\frac{1}{r^2}\frac{2\mu_n}{\hbar}\sin 2\delta_0\times\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\sigma}_0)_a-\boldsymbol{\mu}\cdot(\boldsymbol{\sigma}_0)_a\right), \quad (32)$$

[7] E. Amaldi and E. Fermi, Phys. Rev. **50**, 899 (1936).

[8] The ferromagnetic elements occupy a singular position in this respect, since their magnetic moment arises from an incomplete inner shell.

where $$I_0=v\sum_{m_s}|C_{m_s}|^2,$$

$$(\boldsymbol{\sigma}_0)_a=\sum_{m_s,\,m_s'} C_{m_s}^*(m_s|\boldsymbol{\sigma}_n|m_s')C_{m_s'} \qquad (33)$$

are the intensity and the spin density, respectively, of the incident beam. According to the above formula, the total cross section for scattering from an unpolarized beam ($(\boldsymbol{\sigma}_0)_a=0$) is greater than $(4\pi/k^2)\sin^2\delta_0$. All experimental scattering cross sections are very small compared with $4\pi/k^2$ at C neutron energies, which indicates that $\delta_0\ll 1$. If we write $\delta_0=ka$ and neglect squares and higher powers of δ_0, Eq. (32) simplifies to:

$$I=\frac{I_0}{r^2}\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2+\frac{4M^2\mu_n^2}{\hbar^4}|\boldsymbol{\mu}|^2\right)-\frac{1}{r^2}\frac{4\mu_n ka}{\hbar}\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\sigma}_0)_a-\boldsymbol{\mu}\cdot(\boldsymbol{\sigma}_0)_a\right). \qquad (34)$$

Bloch's formula, however, has

$$-\frac{1}{r^2}\frac{4\mu_n ka}{\hbar}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\sigma}_0)_a$$

for the term in the scattered intensity which depends upon the spin density of the incident beam.

The spin density of the scattered beam at the point $\mathbf{r}$ is:

$$\boldsymbol{\sigma}_a=\psi'^\dagger(\mathbf{r})\boldsymbol{\sigma}_n\psi'(\mathbf{r})=\frac{1}{r^2}\Bigg\{-\frac{4M\mu_n a}{\hbar^2 v}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\times\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}I_0+\frac{4M\mu_n a}{\hbar^2 v}\boldsymbol{\mu}I_0+a^2(\boldsymbol{\sigma}_0)_a$$
$$+\frac{8M^2\mu_n^2}{\hbar^4}\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}-\boldsymbol{\mu}\right)\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\sigma}_0)_a-\boldsymbol{\mu}\cdot(\boldsymbol{\sigma}_0)_a\right)$$
$$-\frac{4M^2\mu_n^2}{\hbar^4}\left(|\boldsymbol{\mu}|^2-\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2\right)(\boldsymbol{\sigma}_0)_a\Bigg\}. \qquad (35)$$

Evidently the neutrons scattered from a beam whose average spin density is zero will be partially polarized. Furthermore, Eq. (34) indicates that this polarization is detectable by a second scattering. Consider, therefore, the following double scattering experiment: A beam of neutrons, whose propagation vector is $\boldsymbol{\kappa}_0$, is incident upon a magnetized plate of iron. The neutrons scattered in the direction of $\boldsymbol{\kappa}_1$ fall upon a second magnetized plate of iron, placed at a distance of r_1 cm from the first, and the intensity of the neutrons which are rescattered in the direction of $\boldsymbol{\kappa}$, say, is measured at a distance of r_2 cm from the second scatterer.

If there are N_1 atoms in the first piece of iron, considered for simplicity of negligible dimensions, the intensity of the singly-scattered neutrons will be:[9]

$$I_1=N_1\frac{I_0}{r_1^2}\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\times\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\cdot\boldsymbol{\mu}\right)_1^2\right]_a+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_1^2]_a\right), \qquad (36)$$

while their spin density will be:

$$(\sigma_1)_a=-N_1\frac{I_0}{r_1^2}\frac{4M\mu_n a}{\hbar^2 v}\times\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\cdot(\boldsymbol{\mu}_1)_a-(\boldsymbol{\mu}_1)_a\right). \qquad (37)$$

We have denoted averages over the atoms by the subscript a, so that $(\boldsymbol{\mu}_1)_a$, for example, is the average magnetic moment per atom.

The expression for the intensity of the doubly-scattered neutrons is:

$$I=N_2\frac{I_1}{r_2^2}\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\left[\left(\frac{\boldsymbol{\kappa}_1-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_1-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)_2^2\right]_a+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_2^2]_a\right)-\frac{N_2}{r_2^2}\frac{4\mu_n ka}{\hbar} \qquad (38)$$
$$\times\left(\frac{\boldsymbol{\kappa}_1-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_1-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\mu}_2)_a\frac{\boldsymbol{\kappa}_1-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_1-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\sigma}_1)_a-(\boldsymbol{\mu}_2)_a\cdot(\boldsymbol{\sigma}_1)_a\right).$$

Substituting the expressions (36) and (37) for I_1

[9] The following formulae, although derived for amorphous solids, are approximately applicable to crystalline substances.

and $(\boldsymbol{\sigma}_1)_a$, we obtain

$$I=N_1N_2\frac{I_0}{r_1^2r_2^2}\left\{\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\cdot\boldsymbol{\mu}\right)^2\right]_1+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_1^2]_a\right)\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\times\left[\left(\frac{\boldsymbol{\kappa}_1-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_1-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2\right]_{2a}+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_2^2]_a\right)+\left(\frac{4M\mu_na}{\hbar^2}\right)^2\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\cdot(\boldsymbol{\mu}_1)_a-(\boldsymbol{\mu}_1)_a\right)\cdot\left(\frac{\boldsymbol{\kappa}_1-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_1-\boldsymbol{\kappa}|}\frac{\boldsymbol{\kappa}_1-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_1-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\mu}_2)-(\boldsymbol{\mu}_2)_a\right)\right\}. \quad (39)$$

A convenient set of experimental conditions is described in part by the equations:

$$\boldsymbol{\kappa}_0=\boldsymbol{\kappa},$$
$$(\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1)\cdot(\boldsymbol{\mu}_1)_a=0,$$
$$(\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1)\cdot(\boldsymbol{\mu}_2)_a=0.$$

Under these circumstances, the resultant intensity depends upon the angle between the directions of magnetization of the two scatterers. The asymmetry ϵ, defined as the difference in intensity between parallel and antiparallel orientation divided by the average intensity, is then given by

$$\epsilon=\frac{2\left(\frac{4M\mu_na}{\hbar^2}\right)^2|(\boldsymbol{\mu}_1)_a||(\boldsymbol{\mu}_2)_a|}{\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\cdot\boldsymbol{\mu}\right)^2\right]_{1a}+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_1^2]_a\right)\times\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|}\cdot\boldsymbol{\mu}\right)^2\right]_{2a}+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_2^2]_a\right)}. \quad (40)$$

If the magnetizing fields are sufficiently strong to produce saturation, it is permissible to neglect $[(\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1/|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}_1|\cdot\boldsymbol{\mu})^2]_a$ and replace $(|\mu|^2)_a$ with $|\boldsymbol{\mu}_a|^2$. The expression for the asymmetry then becomes

$$\epsilon=\frac{2\gamma_1\gamma_2\gamma_n^2(e^2/amc^2)^2}{(1+\frac{1}{4}\gamma_1^2\gamma_n^2(e^2/amc^2)^2)\times(1+\frac{1}{4}\gamma_2^2\gamma_n^2(e^2/amc^2)^2)}, \quad (41)$$

where $\mu_n=\gamma_n(e\hbar/2Mc)$ and $|\mu_a|_{1,2}=\gamma_{1,2}(e\hbar/2mc)$.

The saturation value of the intensity of magnetization is about 1700 gauss for iron at ordinary temperatures, which corresponds to a value of $\gamma_{1,2}\sim2.2$. Adopting provisionally the atomic beam value of $\gamma_n=-2$, and utilizing the experimental result $4\pi a^2\sim10^{-23}$,[10] we obtain $\epsilon=1.75$. From the definition of ϵ we see that the intensity of the double scattering with parallel orientation of magnetizations is 15 times that with antiparallel orientation. However, despite the large magnitude of the asymmetry, this effect will be difficult to detect with present methods because of the small intensity of the doubly-scattered neutrons if conditions of single scattering obtain in both iron plates.

III

We have seen that the neutrons scattered from an unpolarized beam will be partially polarized. It is apparent that, as a result of this effect, the undeviated beam will also become polarized, the amount of polarization per neutron increasing with the penetration of the beam. In order to investigate this effect, suppose that an unpolarized neutron beam of intensity I_0 is incident upon a magnetized sheet of iron. The neutron beam will be described by the solution of the wave equation:

$$i\hbar\frac{\partial}{\partial t}\psi=\left\{-\frac{\hbar^2}{2M}\nabla^2+\sum_i V(\mathbf{r}-\mathbf{r}_i)-\mu_n\boldsymbol{\sigma}_n\cdot\sum_i\mathbf{H}(\mathbf{r}-\mathbf{r}_i)\right\}\psi. \quad (42)$$

$\mathbf{r}_i$ is the position vector of the i^{th} iron nucleus,

[10] A. C. G. Mitchell, C. J. Murphy and M. D. Whitaker, Phys. Rev. 50, 133 (1936).

and the symbols $V(\mathbf{r})$, $\mathbf{H}(\mathbf{r})$ represent the same quantities as in Eq. (6). The resultant conservation theorem, applied to a section of the iron of area A and of thickness dx, such that dx is large compared with atomic dimensions but yet so small that I and $\boldsymbol{\sigma}_a$ do not vary appreciably over it, is

$$\frac{\partial}{\partial t}\int \psi^\dagger\psi d\tau=\frac{i\hbar}{2M}\int\{\psi^\dagger\nabla\psi-\nabla\psi^\dagger\psi\}\cdot d\mathbf{S}. \quad (43)$$

We may at this point take into account the neutron absorption in the iron caused by nuclear capture by replacing the left side of Eq. (43) with:

$$\frac{\partial}{\partial t}\int \psi^\dagger\psi d\tau+n\sigma_c IAdx,$$

where n is the number of atoms per cm and σ_c is the capture cross section. When the stationary state is reached,

$$n\sigma_c IAdx=\frac{i\hbar}{2M}\int\{\psi^\dagger\nabla\psi-\nabla\psi^\dagger\psi\}\cdot d\mathbf{S}. \quad (44)$$

If we neglect the waves scattered by other atoms in the calculation of $\psi'(\mathbf{r}-\mathbf{r}_i)$ (this is equivalent to neglecting multiple scattering), we obtain an approximate solution of Eq. (42), *viz.:*

$$\psi(\mathbf{r})=\exp(i\boldsymbol{\kappa}_0\cdot\mathbf{r})\sum C_{m_s}\chi_{m_s}+\sum_i\psi'(\mathbf{r}-\mathbf{r}_i). \quad (45)$$

The C_{m_s} are to be regarded as slowly varying functions of x in order to describe the decreasing intensity of the undeviated beam with increasing penetration. Inserting (45) into Eq. (44) and evaluating the surface integrals, we obtain:

$$-\frac{dI}{dx}=n\bigg(\sigma_c+4\pi a^2+\frac{4\pi M^2\mu_n{}^2}{\hbar^4}\bigg(3[|\boldsymbol{\mu}|^2]_a-\bigg[\bigg(\boldsymbol{\mu}\cdot\frac{\boldsymbol{\kappa}_0}{k}\bigg)^2\bigg]_a\bigg)\bigg)I-n\frac{4\pi\mu_n ka}{\hbar}\bigg(\boldsymbol{\mu}_a\cdot\frac{\boldsymbol{\kappa}_0}{k}\frac{\boldsymbol{\kappa}_0}{k}-3\boldsymbol{\mu}_a\bigg)\cdot\boldsymbol{\sigma}_a. \quad (46)$$

If we denote the quantity in brackets multiplying I by σ and neglect the third term therein, Eq. (46) becomes

$$-\frac{dI}{dx}=n\sigma I-\frac{4\pi n\mu_n ka}{\hbar}\bigg(\boldsymbol{\mu}_a\cdot\frac{\boldsymbol{\kappa}_0}{k}\frac{\boldsymbol{\kappa}_0}{k}-3\boldsymbol{\mu}_a\bigg)\cdot\boldsymbol{\sigma}_a. \quad (47)$$

Returning to Eq. (42), we easily derive the relation:

$$\frac{\partial}{\partial t}\int\psi^\dagger\boldsymbol{\sigma}_n\psi d\tau+n\sigma_c vAdx\boldsymbol{\sigma}_a=\frac{i\hbar}{2M}\int\{\psi^\dagger\boldsymbol{\sigma}_n\nabla\psi-\nabla\psi^\dagger\boldsymbol{\sigma}_n\psi\}\cdot d\mathbf{S}+\frac{2\mu_n}{\hbar}\boldsymbol{\sigma}_a\times\sum_i\int\mathbf{H}(\mathbf{r}-\mathbf{r}_i)d\tau, \quad (48)$$

which, in the stationary state, gives approximately

$$\frac{d\boldsymbol{\sigma}_a}{dx}=n\sigma\boldsymbol{\sigma}_a-\frac{4\pi nM\mu_n a}{\hbar^2 v}\bigg(\boldsymbol{\mu}_a\cdot\frac{\boldsymbol{\kappa}_0}{k}\frac{\boldsymbol{\kappa}_0}{k}-3\boldsymbol{\mu}_a\bigg)I-\frac{16\pi n\mu_n}{3\hbar v}\boldsymbol{\sigma}_a\times\boldsymbol{\mu}_a. \quad (49)$$

We shall solve Eqs. (47) and (49) subject to the arbitrary initial values I^0 and $(\boldsymbol{\sigma}^0)_a$. The abbreviations:

$$\frac{\boldsymbol{\mu}_a}{|\boldsymbol{\mu}_a|}=\mathbf{e}_3,\quad \frac{\boldsymbol{\kappa}_0}{k}=\mathbf{n},\quad \mathbf{e}_3\cdot\mathbf{n}=\cos\Theta,\ \frac{4\hbar}{3Mva}=\delta,\quad \frac{4\pi nMa\mu_n|\boldsymbol{\mu}_a|}{\hbar^2}x=\xi \quad (50)$$

simplify these equations to

$$\frac{d}{d\xi}\frac{I'}{v}=(\cos\Theta\mathbf{n}-3\mathbf{e}_3)\cdot(\boldsymbol{\sigma}')_a,$$
$$\frac{d}{d\xi}(\boldsymbol{\sigma}')_a=(\cos\Theta\mathbf{n}-3\mathbf{e}_3)\frac{I'}{v}+\delta(\boldsymbol{\sigma}')_a\times\mathbf{e}_3, \quad (51)$$

where

$$I=I'e^{-n\sigma x},\quad \boldsymbol{\sigma}_a=(\boldsymbol{\sigma}')_a e^{-n\sigma x}. \quad (52)$$

These equations can be solved exactly. However, the complexity of the resultant solution is such that it is convenient to make certain approximations which involve the fact that $\delta\sim 3400$ at thermal energies. We thus obtain:

$$\frac{I'}{v}=\tfrac{1}{2}\left(\frac{I^0}{v}-(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_2\right)\times e^{(3-\cos^2\Theta)\xi}+\frac{1}{2}\left(\frac{I^0}{v}+(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_2\right)e^{-(3-\cos^2\Theta)\xi}-\frac{1}{\delta}\sin\Theta\cos\Theta(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_2\cos\delta\xi+\frac{1}{\delta}\sin\Theta\cos\Theta(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_1\sin\delta\xi,$$

$$(\boldsymbol{\sigma}')_a=-\frac{1}{2}\left(\left(\frac{I^0}{v}-(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_2\right)\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta\left(\frac{I^0}{v}-(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3\right)\mathbf{e}_2\right)e^{(3-\cos^2\Theta)\xi}$$
$$+\frac{1}{2}\left(\left(\frac{I^0}{v}+(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_2\right)\mathbf{e}_3-\frac{1}{\delta}\sin\Theta\cos\Theta\left(\frac{I^0}{v}+(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3\right)\mathbf{e}_2\right)e^{-(3-\cos^2\Theta)\xi}$$
$$+\left((\boldsymbol{\sigma}^0)_a-\mathbf{e}_3(\boldsymbol{\sigma}^0)_a\cdot\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta\frac{I^0}{v}\mathbf{e}_2\right)\cos\delta\xi+\left((\boldsymbol{\sigma}^0)_a\times\mathbf{e}_3+\frac{1}{\delta}\sin\Theta\cos\Theta\frac{I^0}{v}\mathbf{e}_1\right)\sin\delta\xi,\quad(53)$$

where

$$\mathbf{e}_1=\frac{1}{\sin\Theta}(\mathbf{n}-\cos\Theta\mathbf{e}_3),\quad \mathbf{e}_2=\frac{1}{\sin\Theta}\mathbf{e}_3\times\mathbf{n}.\quad(54)$$

If we apply these results to the case under consideration, i.e.; $I^0=I_0$, $(\boldsymbol{\sigma}^0)_a=0$, we find that the intensity and spin density of the beam after traversing a thickness of x cm of iron are given by

$$I=I_0e^{-n\sigma x}\cosh\times\left((3-\cos^2\Theta)\frac{4\pi nMa\mu_n|\boldsymbol{\mu}_a|}{\hbar^2}x\right),$$
$$\boldsymbol{\sigma}_a=-\frac{\boldsymbol{\mu}_a}{|\boldsymbol{\mu}_a|}\frac{I_0}{v}e^{-n\sigma x}\sinh\times\left((3-\cos^2\Theta)\frac{4\pi nMa\mu_n|\boldsymbol{\mu}_a|}{\hbar^2}x\right).\quad(55)$$

The polarization thus produced can manifest itself either by an absorption or a scattering experiment. The first procedure, which we shall call the double transmission method, gives the following expression for the intensity of the undeviated beam:

$$I=I_0e^{-n\sigma(x_1+x_1)}\cosh\beta_1\cosh\beta_2\times\left(1+\frac{(\boldsymbol{\mu}_1)_a}{|(\boldsymbol{\mu}_1)_a|}\cdot\frac{(\boldsymbol{\mu}_2)_a}{|(\boldsymbol{\mu}_2)_a|}\tanh\beta_1\tanh\beta_2\right),\quad(56)$$

where

$$\beta_{1,\,2}=(3-\cos^2\Theta_{1,\,2})\frac{4\pi nMa\mu_n|(\boldsymbol{\mu}_{1,\,2})_a|}{\hbar^2}x_{1,\,2}.\quad(57)$$

The subscripts 1, 2 refer, of course, to the first and second sheets of iron, respectively. The asymmetry ϵ, defined as the difference in intensity between parallel and antiparallel orientations divided by the average intensity, is then given by

$$\epsilon=2\tanh\beta_1\tanh\beta_2.\quad(58)$$

The magnitude of the asymmetry which may be obtained experimentally is limited by the undesirability of reducing the intensity to such a large extent that the effect is hidden by the high speed neutron background. Thus, if we do not wish to diminish the intensity by more than 75 percent, we can take $\Theta_1=\Theta_2=30°$ and use thicknesses of 0.35 cm ($x_1=x_2=0.7$ cm), since $\sigma=12\cdot10^{-24}$.[11] If both plates are saturated, the asymmetry, as calculated from Eq. (58), is 37 percent.

The second procedure, which we shall term the transmission scattering method, leads to the following expression for the intensity:

$$I=N_2\frac{I_0}{r^2}e^{-n\sigma x_1}\cosh\beta_1\left(a^2-\frac{4M^2\mu_n^2}{\hbar^4}\times\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2_2\right]_a+\frac{4M^2\mu_n^2}{\hbar^4}[|\boldsymbol{\mu}|_2{}^2]_a\right)$$
$$+N_2\frac{I_0}{r^2}e^{-n\sigma x_1}\frac{4\mu_nMa}{\hbar^2|(\boldsymbol{\mu}_1)_a|}\sinh\beta_1\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\mu}_2)_a\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\mu}_1)_a-(\boldsymbol{\mu}_2)_a\cdot(\boldsymbol{\mu}_1)_a\right).\quad(59)$$

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[11] J. R. Dunning, G. B. Pegram, G. A. Fink and D. P. Mitchell, Phys. Rev. **48**, 265 (1935).

The subscripts 1, 2 refer to the polarizing plate and scatterer, respectively; $\boldsymbol{\kappa}_0$ and $\boldsymbol{\kappa}$ are the propagation vectors of the incident and scattered wave; and r is the distance from the scatterer to the point of observation.

The best experimental conditions are obtained when

$$\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\mu}_1)_a=\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot(\boldsymbol{\mu}_2)_a=0.$$

Under these circumstances, the intensity is given by

$$I=N_2\frac{I_0}{r^2}e^{-n\sigma x_1}\cosh\beta_1\left(a^2-\frac{4M^2\mu_n{}^2}{\hbar^4}\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2_2\right]_a+\frac{4M^2\mu_n{}^2}{\hbar^4}[|\boldsymbol{\mu}|_2{}^2]_a\right)$$
$$-N_2\frac{I_0}{r^2}e^{-n\sigma x_1}\frac{4\mu_n Ma}{\hbar^2|(\boldsymbol{\mu}_1)_a|}\sinh\beta_1(\boldsymbol{\mu}_1)_a\cdot(\boldsymbol{\mu}_2)_a. \quad (60)$$

The asymmetry, defined in this case as the difference in intensity between antiparallel and parallel orientation of magnetizations divided by the average intensity, is:

$$\frac{8\mu_n Ma|(\boldsymbol{\mu}_2)_a|}{\hbar^2}\frac{\tanh\beta_1}{\left(a^2-\frac{4M^2\mu_n{}^2}{\hbar^4}\left[\left(\frac{\boldsymbol{\kappa}_0-\boldsymbol{\kappa}}{|\boldsymbol{\kappa}_0-\boldsymbol{\kappa}|}\cdot\boldsymbol{\mu}\right)^2_2\right]_a+\frac{4M^2\mu_n{}^2}{\hbar^4}[|\boldsymbol{\mu}|_2{}^2]_a\right)}. \quad (61)$$

It is interesting to note that the maximum intensity occurs with antiparallel orientation of magnetizations, in agreement with what one would expect by elementary considerations.

If both polarizer and scatterer are saturated, $x_1=0.7$ cm, and $\Theta_1=30°$, the asymmetry is 81 percent. With given values of $|(\mu_1)_a|$, $|(\mu_2)_a|$ and x_1, the maximum asymmetry is obtained at $\Theta_1=90°$. For example, under the above conditions, but with $\Theta_1=90°$, the asymmetry, as calculated from Eq. (61), is 92 percent.

There is still a fourth possible type of experiment in which a neutron beam is polarized by scattering, and then allowed to pass through a magnetized iron plate. If the iron plate is of such dimensions that it is permissible to neglect the fact that the scattered waves are spherical and not plane waves, the intensity is given by a formula identical with Eq. (59).

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Photoelectric Cross Section of the Deuteron

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Photoelectric cross section curves for a Majorana-Heisenberg potential of the type $V=-V_0e^{-r^2/a^2}$ and a velocity dependent potential determined by $J_0=-(2B/a)e^{-(r+\rho)/a}$ are compared with a cross section curve for a square hole Majorana force calculated by Breit, Condon and Stehn. In each case the values of the constants used are those which have been determined as the best for accounting for the binding energies of H^2, H^3, and He^4. Results show that the cross section values for the first two potentials differ considerably from the third but very little from each other. A general formula for the area under the cross section curve, which holds for exchange as well as for ordinary forces is derived. For exchange forces $\int\sigma(\nu)d(h\nu) \cong (\pi e^2h/2Mc)(1+a\alpha)$ and this depends only on a, the range of interaction, α being defined by $\alpha^2\hbar^2/M=\epsilon$, the binding of energy the deuteron. The addition of a long range repulsive force to the velocity dependent interaction is found to decrease the cross section for this potential type considerably. The classical equivalent of the velocity dependent potential operator is determined.