The Sign of the Magnetic Moment of the K^{39} Nucleus

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The assignment of quantum numbers to the magnetic levels of alkali atoms depends on the sign of the nuclear magnetic moment. If a beam of normal alkali atoms is split in a weak inhomogeneous magnetic field and transitions are then induced among the component magnetic levels, a subsequent strong field analysis of a selected state suffices to identify its quantum numbers and hence to determine the sign of the moment. This method applied to K^{39} determines its nuclear magnetic moment to be positive in contradiction to the findings of Jackson and Kuhn from h.f.s. data. A similar experiment on Na^{23} shows its nuclear moment to be positive in agreement with the h.f.s. result of Granath and Van Atta.

A METHOD for the determination of the signs of nuclear magnetic moments based on the use of nonadiabatic transitions[1] in an atomic beam was suggested by Rabi,[2] and has been used by Kellogg, Rabi and Zacharias[3] to determine this property of the proton and the deuteron. This method has now been extended to include the alkalis and, specifically, to determine the sign of the nuclear magnetic moment of K^{39}.

General Considerations

If a beam of neutral alkali atoms passes through a weak inhomogeneous magnetic field, it is separated into $2(2I+1)$ component magnetic states where I is the nuclear spin. The atoms in $2(I+\frac{1}{2})+1$ of these states have a total angular momentum $F=I+\frac{1}{2}$, while those in the remaining $2(I-\frac{1}{2})+1$ states have $F=I-\frac{1}{2}$. In the case of K^{39}, $I=3/2$[4] and there are eight states in all, for five of which, $F=2$ and for the remaining three, $F=1$. The effective magnetic moments of the atoms in these eight levels as a function of the magnetic field intensity are given by[5]

$$\mu_m=(\mp\delta)\frac{x+2\delta m/(2I+1)}{\{1+[4\delta m/(2I+1)]x+x^2\}^{\frac{1}{2}}}\mu_0, \qquad x=2\mu_0 H/\Delta W, \tag{1}$$

where $\delta=+1$ if the nuclear moment is positive and -1 if negative; μ_0 is the Bohr magneton, H is the external magnetic field intensity, m is the magnetic quantum number and ΔW is the energy separation of the two groups of levels in the absence of magnetic field. These magnetic moments are plotted against x in Figs. 1a and 1b

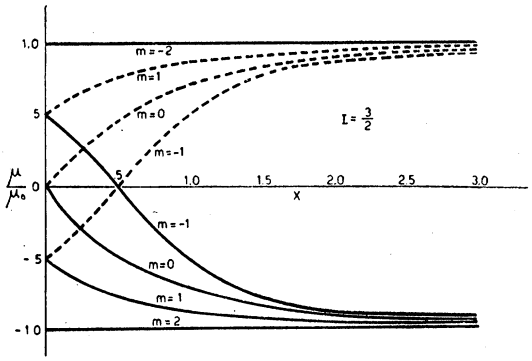

FIG. 1*a*. Variation of moment with magnetic field. The dotted lines are the moments of the magnetic levels arising from the $F=I-1/2$ state. Nuclear moment assumed positive.

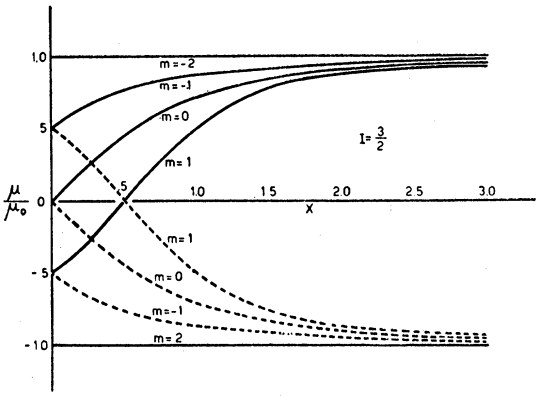

FIG. 1*b*. Same as Fig. 1*a*, but with nuclear moment assumed negative.

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[1] T. E. Phipps and O. Stern, Zeits. f. Physik **73**, 183 (1931); R. Frisch and E. Segrè, Zeits. f. Physik **80**, 610 (1933).

[2] I. I. Rabi, Phys. Rev. **49**, 324 (1936).

[3] J. M. B. Kellogg, I. I. Rabi and J. R. Zacharias, Phys. Rev. **50**, 472 (1936).

[4] S. Millman, Phys. Rev. **47**, 739 (1935).

[5] G. Breit and I. I. Rabi, Phys. Rev. **38**, 2082 (1931).

for nuclear moment positive and negative, respectively. In the case of K^{39} one x unit is equivalent to 159 gauss. As is evident from these curves, the only asymmetries due to the sign of the nuclear magnetic moment consist in the assignment of the quantum numbers m and F to the individual states. Since the magnetic moments are the important quantities in an atomic beam experiment, the sign can be determined only by having recourse to these distinctions.

As was shown by Rabi,[2] it is possible to utilize these differences by causing the atoms of the beam to make transitions among the component magnetic levels. These transitions are produced by passing the beam of atoms through the immediate neighborhood of a point of zero field intensity. Such a region is characterized by a rapid change in direction of the magnetic field intensity along the path of the beam. The spacial quantization of an atom passing through this region is thus subject to perturbation which, in certain cases, is sufficient to induce a nonadiabatic transition to another magnetic state. The probabilities of such transitions for this type of perturbation have been calculated by Majorana[6] in terms of a parameter α, which is, physically, the angle between the two directions, before and after transition, with respect to which the atom is space-quantized with the same quantum number m. It can be shown that α is the same for all m and is given by a certain function of the distance of the beam from the point of zero field and the field intensity at the position of closest approach of the beam to the zero point. Majorana's equations are reproduced in Rabi's paper. In Table I, the transition probabilities for $F=2$ are given in terms of $\cos \alpha/2$ and $\sin \alpha/2$. The transition which is significant in this experiment is the one in which m changes from $-\delta$ to -2δ. Its probability is $4 \cos^6 \alpha/2 \sin^2 \alpha/2$ which has the maximum value 0.422 at $\alpha=60°$. The success of the method of nonadiabatic transitions depends largely on the following circumstance: Since the perturbation takes place in very weak fields (less than one gauss), it is not sufficiently powerful to induce transition unless the energy separation of the two states concerned is quite small. This condition obtains among the Zeeman components of either of the two h.f.s. levels, but is not satisfied by the comparatively large interval ΔW. The transitions are therefore subject to the selection rule $\Delta F=0$.[7]

It will be shown later that if the beam is first split in a weak inhomogeneous magnetic field and transitions are then caused to take place, a subsequent analysis of the beam in a second inhomogeneous magnetic field will suffice to identify a given state and hence to determine the sign of the nuclear magnetic moment.

Apparatus

The apparatus is represented diagrammatically in Fig. 2. The essential parts are housed in a brass tube 5 in. inside diameter and $4\frac{1}{2}$ ft. long. This tube is divided by a partition into two chambers. The source chamber contains the oven and is made short so that a pressure of 10^{-5} mm can be tolerated without detriment to the beam intensity. To permit the passage of the beam, the partition separating the two chambers is fitted with a pair of slit jaws which have a separation of about 0.5 mm and form a channel 2 mm long which offers a high resistance to gas flow. Each chamber is evacuated with its own high speed oil diffusion pump of the Zabel[8] type. These two pumps feed into an interstage vacuum provided by a small oil diffusion pump which is backed by a Cenco Megavac.

[6] E. Majorana, Nuovo Cimento **9**, 43 (1932).

[7] L. Motz and M. E. Rose, Phys. Rev. **50**, 348 (1936).

[8] R. M. Zabel, Rev. Sci. Inst. **6**, 54 (1935).

Table I. *Transition probabilities for $F=2$ in terms of $\cos \alpha/2$ and $\sin \alpha/2$.*

m	-2	-1	0	$+1$	$+2$
-2	c^8	$4c^6s^2$	$6c^4s^4$	$4c^2s^6$	s^8
-1	$4c^6s^2$	$c^4(c^2-3s^2)^2$	$6s^2c^2(c^2-s^2)^2$	$s^4(3c^2-s^2)^2$	$4c^2s^6$
0	$6c^4s^4$	$6s^2c^2(c^2-s^2)^2$	$(c^4-4s^2c^2+s^4)^2$	$6s^2c^2(c^2-s^2)^2$	$6c^4s^4$
$+1$	$4c^2s^6$	$s^4(3c^2-s^2)^2$	$6s^2c^2(c^2-s^2)^2$	$c^4(c^2-3s^2)^2$	$4c^6s^2$
$+2$	s^8	$4c^2s^6$	$6c^4s^4$	$4c^6s^2$	c^8

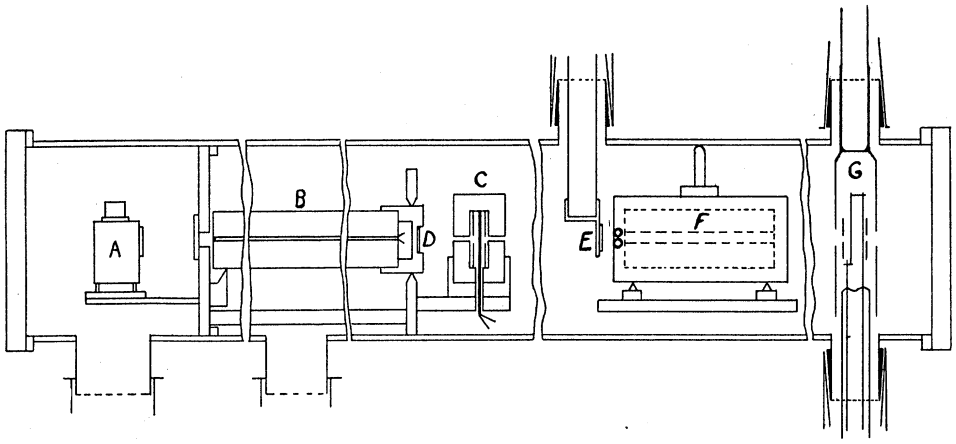

FIG. 2. Schematic diagram of the apparatus. *A*, oven; *B*, *A* field block; *C*, transition field assembly; *D*, collimator slit; *E*, selector slit; *F*, *B* field magnet; *G*, detector.

The beam is produced in a nickel oven (*A*) of conventional design. Its slit has a width of about 0.02 mm.

The first deflecting field or "*A*" field is produced in the manner described by Rabi, Kellogg and Zacharias,[9] by two horizontal straight copper tubes carrying currents in opposite directions. These are held in a duralumin block, 2 ft. long, (shown at *B*) which is supported at the source end by a knife edge and at the other end by two screw points. It is movable for purposes of lineup about a vertical axis through these points by a screw at the source end. The collimator slit, *D*, is rigidly fixed to the end of the field block and is collinear with the axis of the screw points so that a motion of the block about this axis does not shift the beam. The width of this slit is 0.02 mm.

The transitions are produced by sending the beam between two vertical straight copper wires 3.5 mm apart which carry currents of the order of 0.1 amp. in the same direction. These currents produce a magnetic field which changes in direction through 180° along the path of the beam and has a zero point on a line perpendicular to the two wires and joining their centers. The distance of this zero point from the path of the beam as well as the magnetic field intensity at the distance of closest approach to the zero point can be controlled by the currents in the wires. Fig. 3 shows a plot of the lines of force in the neighborhood of the two wires in the special case of equal currents. The region in the neighborhood of these wires is shielded magnetically by a permalloy cylinder. To avoid magnetizing the permalloy the current having passed through the two wires is returned by a cylindrical brass sheath inside the permalloy. This assembly (shown at *C*) immediately follows the collimator slit. The oven, partition, *A* field block, collimator slit, transition field assembly and their supports form a complete unit which can be lined up outside before installation.

The function of the slit shown at *E*, is the selection of part of the beam for future analysis. It is suspended from a double eccentric ground joint. Immediately following the selector slit is the electromagnet (*F*) which produces the analyzing, or "*B*," field. It is placed on the opposite side of the beam to the *A* field block in order to give deflections in the opposite direction. The magnet, a cross section of which is shown in Fig. 4, is a cylindrical shell 4 in. long of Armco iron which is slotted to a width of 3/16 in. along its length. It is activated by a current in eight turns of hollow water-cooled copper tubing. The author is indebted to Professor I. I. Rabi for suggesting this type of magnet which has considerable utility in molecular beam experiments. Its construction is easy and its small size permits inclusion in the vacuum system. Activation by high currents with a low voltage power source is most convenient, as the few turns necessary

[9] I. I. Rabi, J. M. B. Kellogg, J. R. Zacharias, Phys. Rev. **46**, 157 (1934).

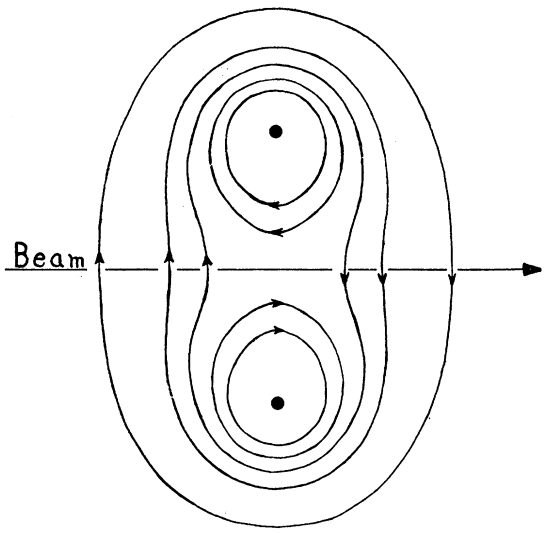

FIG. 3. Lines of force in the neighborhood of the transition field wires in the special case of equal currents.

under these conditions are easily cooled and insulated. The magnet in its present form was developed by Dr. J. R. Zacharias and the author. The magnetic field intensity in the region of the slot opening was measured by a point to point plot of the magneto-resistance of a straight fine bismuth wire. The gradient was then computed from these measurements by numerical differentiation. The direct measurements were preferred to calculation by a Schwarz transformation since the latter is very tedious. The beam is sent along parallel to the axis of the magnet just outside the pole faces, as in this region the gradient is approximately constant over the height of the beam. The power source is a large two volt lead cell with a capacity of 3000 amp. hr. Two volts across the field coil yields a current of 250 amp. and a field of 5000 gauss inside the gap and 4000 gauss at the region traversed by the beam. The ratio of gradient to field at the latter position is about 2 cm^{-1}.

Detection is effected by a surface ionization gauge with a one mil tungsten filament (shown at G). The filament motion is furnished by a double eccentric ground joint.

The distances from the oven slit to the collimator slit, transition field wires, selector slit and detector are respectively 65, 70, 95 and 125 cm.

THE EXPERIMENTAL METHOD

The oven is loaded with a piece of potassium metal and the apparatus is immediately sealed and evacuated. Upon attainment of a sufficiently low pressure, the oven is heated slowly until a beam of desired intensity is obtained. The current in the A field wires is turned on to produce a field of 120 gauss at the region traversed by the beam. The corresponding value of x (cf. Figs. 1a and 1b) is about 3/4. At this field the eight magnetic levels have magnetic moments separated by approximately equal intervals. Since the force on an atom in the y direction in a magnetic field is $\mu_m(\partial H/\partial y)$ when μ_m is the effective magnetic moment of the atom, the deflections produced by the A field are about equally spaced. Four of the eight states are deflected to the right and four to the left. The selector slit, which is so wide that only one jaw is used at a time, is introduced to stop one of the two groups of four states. The remaining atoms all of which have magnetic moments of the same sign are allowed to enter the analyzing field. The magnetic field intensity in this region is so large (over 1000 gauss) that the coupling between the nuclear and electronic spins is destroyed and the atoms of all these four states have the same magnetic moment equal in absolute magnitude to one Bohr magneton. Consequently this field exerts the same force on each atom. By a suitable choice of the current activating this field, it is thus possible to

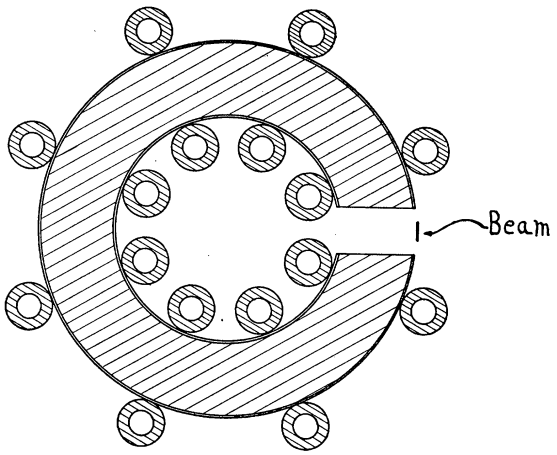

FIG. 4. Cross section of the electromagnet producing the B field.

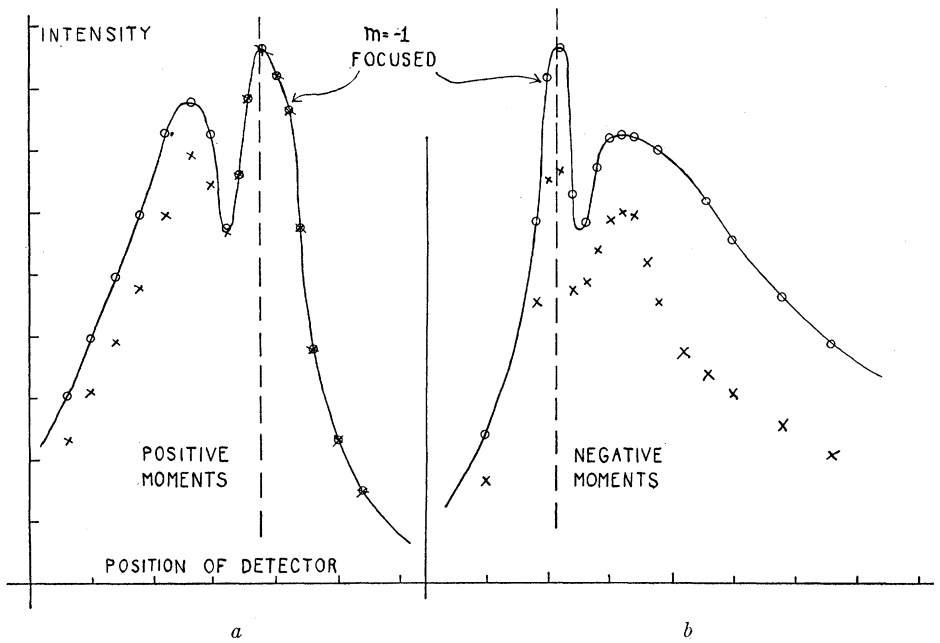

FIG. 5. Effect of nonadiabatic transitions on intensity distribution at detector. Broken line shows position of net zero deflection.

neutralize the deflections received by the atoms of one of the four states in the A field. The atoms of this state are now focused at the detector which has not been moved from its original position at the maximum of the undeflected beam. The atoms of the remaining three states, passed by the selector slit, still have net deflections and miss the detector. Thus the atoms now being focused belong entirely to one of the eight original magnetic levels.

Of the four selected states, the one having $m=-\delta$ is best resolved and, for this reason, the current in the B field is adjusted until the atoms belonging to this state are focused. With no current in the transition field wires, the filament is moved across the beam and the intensity is taken at each position. This yields an intensity distribution curve which shows two peaks: a narrow one at the position of net zero deflection due to the focused atoms and a broad one to one side due to the atoms of the three unfocused states. At each position of the filament, the transition field currents are varied in an effort to alter the intensity. Whenever this is possible, the intensity of maximum change is recorded. Let us inquire as to what effect, if any, transitions may be expected to have on the intensity. If the atoms received by the filament make transitions to states with atomic moments of the same sign[10] in the B field, no change will be noticed in the intensity since the magnetic moment and hence the deflection in the strong B field is unaltered. If, however, the transitions result in change of sign of the net atomic moment, the deflection in the B field will be opposite in direction and these atoms will no longer strike the detector. In general it can be said that transitions are unobservable unless the sign of the atomic moment in the B field is changed, and that observable transitions are manifested by reductions in the intensity at the detector.

Since the value of x in the A field is greater than $\frac{1}{2}$, the three states for which $F=1$ have in this field atomic moments of the same sign. This may be seen by reference to Fig. 1 in which the magnetic moments of these states are represented by the broken line curves. This circumstance, together with the rule that F does not change in a transition, means that an atom,

[10] Atomic moments are not to be confused with nuclear moments.

originally in one of these states, is confined to atomic moments of one sign in the B field and is therefore incapable of making an observable transition. Such is not the case, however, for an atom in one of the five states for which $F=2$. Such an atom always has open to it the possibility of an observable transition consistent with the selection rule.

In view of these considerations, we are led to the following conclusions:

(a) If it is found possible to alter the intensity at a given position of the filament by manipulation of the transition field currents, then at least some of the atoms received at this position have $F=2$.

(b) If no combination of currents in the transition field wires succeeds in altering the intensity at a given position of the filament, then all of the atoms striking the detector at this position have $F=1$.

The latter statement is, of course, only true within the limits of experimental error which may be estimated to be less than 2 percent. It is further qualified by the fact that the maximum probability of an observable transition is usually less than 1. However, these probabilities, given by the matrix elements in Table I, all have maxima in excess of 0.37.

Results

Typical results are shown in Figs. 5a and b which are plots of the intensity at the filament against filament position taken in the manner described above. Fig. 5a shows the results obtained when atoms with positive atomic moments were selected; Fig. 5b shows the corresponding curve for negative atomic moments. The broken line indicates in each case the position of the filament when it is receiving atoms with net zero deflections. The peaks at this position in the two curves, a and b, are due to the focused atoms of the two states for which $m=-\delta$. The crosses give the intensity at each point after the original intensity has been altered as much as possible by manipulation of the transition field currents.

Since the atoms of the state m with positive strong field moments do not show observable transitions, they must belong to the $F=1$ level by the result (b) of the last section. The intensity of the $m=-\delta$ state with negative strong field atomic moments shows a maximum change of 25 percent. This state must, then, be one of the group for which $F=2$ by the result (a) of the last section. It is seen that the atoms of the three unfocused states show observable transitions in both Figs. 5a and 5b. This result should be expected in any case, since at least one of the states composing each selected group has $F=2$.

By an inspection of Fig. 1, it is clear that these results are consistent only with the hypothesis that the hyperfine structure of the $^2S_{\frac{1}{2}}$ state of K^{39} is regular and the nuclear magnetic moment is positive.

Discussion

During the progress of this experiment Jackson and Kuhn[11] published results from the intensities of the h.f.s. of K^{39} to show that the nuclear moment is negative. This result, if valid, would place K^{39} in a unique position since the h.f.s. of the other alkalis so far examined (Li^7, Na^{23}, Rb^{85}, Rb^{87} and Cs^{133}) has been found to be regular. In view of the disagreement of Jackson and Kuhn's result with the findings of the method of nonadiabatic transitions, it should be emphasized that the determination of the sign of the nuclear moment by the latter method depends only on the detection of qualitative differences in the behavior of two states with the same m and different F. The final result is independent of the manner in which the transitions are brought about, provided the selection rule, $\Delta F=0$, is obeyed.

Since the first instance in which it has been possible to compare results of the two methods has led to contradiction, it was thought advisable to make a further comparison on another alkali nucleus. Na^{23} was chosen for this purpose. It should be noted that all above considerations will still apply to sodium since its nuclear spin is also 3/2.

Further Experiments

The apparatus used for K^{39} did not have sufficient resolving power for Na^{23} so that another very similar apparatus was modified for these measurements. The author wishes to thank

[11] D. A. Jackson and H. Kuhn, Nature **137**, 107 (1936).

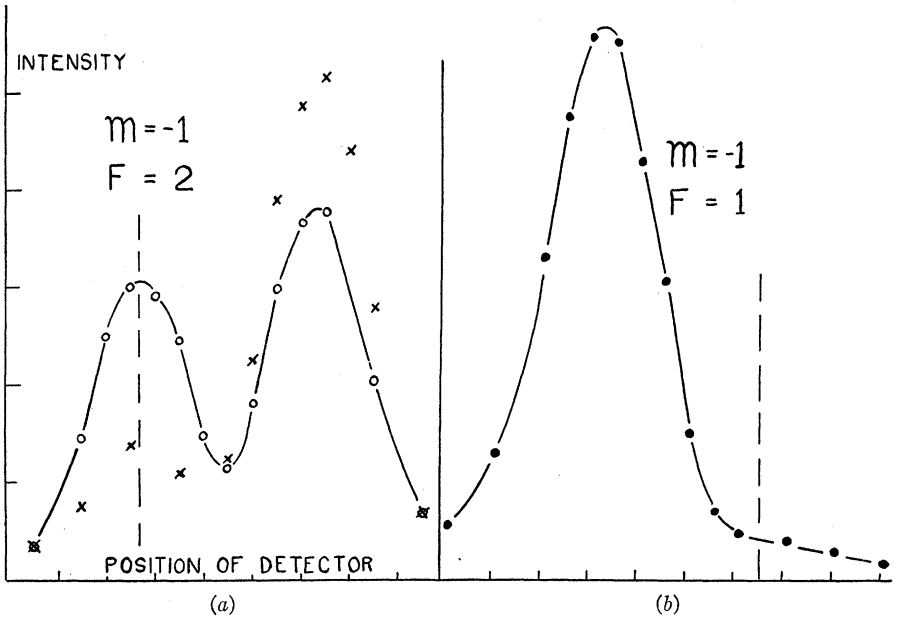

FIG. 6. Asymmetries in deflection patterns of (*a*) $m=-1$ atoms with positive strong field moments, (*b*) $m=-1$ atoms with negative strong field moments.

Dr. Millman and Dr. Zacharias for the loan of this apparatus and for their cooperation in securing the results. After modification to meet the demands of the above method, the only major difference between the two apparatus, aside from resolving power, was in the location of the analyzing field which is now on the same side of the beam as the A field block. As a result of this change an atom with a magnetic moment of the same sign in both fields will not be focused. However, selection is now effected not by focusing but by narrowing the selector slit to 0.035 mm so that only atoms of the state in question are passed. The states examined are again the two for which $m=-\delta$. The procedure is now as follows. With all fields off, the filament is placed at the center of the beam. The A field is then set at a value of x slightly less than 1/2. The filament is moved slightly to one side so that the atoms which strike it belong only to one of the $m=-\delta$ states. The atoms of the other states have relatively large deflections and miss the detector. The selector slit is introduced to pass the atoms directed towards the filament. The B field is now turned on, deflecting these atoms to one side, and the filament is moved the necessary amount to pick up the deflected peak. These preliminary adjustments made, the transition field is turned on and varied in an effort to change the intensity at the final position of the filament. As in the former experiment, transitions, to be observable, must take place to $m=-2\delta$ and this is not possible if the original state has $F=1$. When the state $m=-\delta$ with positive strong field atomic moments was selected in this manner, it was found that no change in intensity to within 1 percent could be produced by the transition field, whereas the peak intensity of the corresponding state with negative moments in the B field was reduced from a maximum of 14.2 cm of galvanometer deflection to a minimum of 9.5 cm, a net change of 30 percent. Hence by reference to Figs. 1*a* and 1*b* it is seen that the sign of the nuclear magnetic moment of Na^{23} is positive in agreement with the results of Granath and Van Atta.[12]

In order to effect a direct comparison of the two alkalis, the apparatus was also used with K^{39}. The A field was set at a value of x slightly larger than $\frac{1}{2}$; otherwise the preliminary experimental procedure is the same as for Na^{23}. In this case, however, the intensity distribution of the

[12] L. P. Granath and C. M. Van Atta, Phys. Rev. **48**, 725 (1935).

selected atoms was found by moving the filament across the beam (*a*) when the transition field current was set at the value for which the peak intensity was a maximum and (*b*) when this current was set to minimize the peak intensity. The results when the state $m=-\delta$ with negative strong field moments was selected is shown in Fig. 6*a*. The circles are the experimental points for procedure (*a*); the crosses are those for procedure (*b*). The peak in the solid line curve at the position of the dotted line is due to atoms which have made observable transitions, in consequence of which they have received smaller deflections in the B field. The other peak in this curve is due to atoms which have not made observable transitions. Since this state shows observable transitions, it must have $F=2$. Fig. 6*b* shows the corresponding situation when atoms of the state $m=-\delta$ with positive strong field atomic moments were selected. Atoms making observable transitions would give rise to a peak at the position of the dotted line. In this case procedures (*a*) and (*b*) are equivalent, since it was found impossible to alter the intensity of the observed peak by varying the transition field currents. It was also impossible to produce a displaced peak in this manner. The absence of observable transitions identifies this state as $F=1$. These results determine the sign of the magnetic moment of the K^{39} nucleus to be positive in agreement with the results of the first experiment.

In conclusion the author wishes to express his gratitude to the members of the molecular beam laboratory for generously contributed assistance and especially to Professor I. I. Rabi who suggested the problem and whose advice and resourcefulness insured its successful solution.

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Effect of CCl_4 Vapor on the Dielectric Strength of Air

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The effect of CCl_4 vapor on the dielectric strength of air has been studied at a number of different air pressures up to a maximum of 75 lb./in.^{2}. With air at 25°C and 15 lb. pressure the dielectric strengths of the mixtures relative to pure air ranged from a value of 1.25 for a CCl_4 vapor pressure of 5 percent of saturation to a value of 1.80 for a saturated vapor pressure. At higher air pressures the addition of CCl_4 vapor gives a greater increase in the absolute value of the dielectric strength, but the percentage increase, relative to the pure air, is less than at 15 lb. pressure.

INTRODUCTION

THE effect of CCl_4 vapor on the dielectric strength of air was first noticed three years ago by one of the authors (R.G.H.) while working with an electrostatic generator of the Van de Graaff type which operates in a steel tank under an air pressure up to 45 lb. With air at atmospheric pressure containing a high concentration of CCl_4 vapor the maximum generator potential was found to be about 1.7 times the maximum potential obtainable with pure air.

Qualitative measurements were then made on the breakdown potential of the generator as a function of CCl_4 vapor pressure, with air at atmospheric pressure and also at higher pressures. The electrostatic generator proved to be unsatisfactory, however, for a thorough study of the CCl_4 effect because of sparking along a textolite support which determined an upper voltage limit that could not be improved by higher air pressure or higher concentrations of CCl_4 vapor. A small pressure chamber, equipped with an adjustable sphere gap was therefore constructed and with this apparatus sparking potentials were determined as a function of CCl_4 vapor pressure at a number of different air pressures. Publication of these results was de-