where T_a is the average kinetic energy of the system.

If ψ is an eigenfunction for any $S_{\alpha\beta}$ the corresponding term of the last member of Eq. (2) vanishes. Thus if the ψ function is made antisymmetrical in the protons and neutrons separately, and the S's are Heisenberg operators, the last member will contain only cross terms between protons and neutrons. Again, this term vanishes for the separate states of the deuteron.[3]

Writing $J_{\alpha\beta}$ as a function of $r_{\alpha\beta}$ only, we get

$$\sum_i x_i \frac{\partial J_{\alpha\beta}}{\partial x_i} = (\mathbf{r}_\alpha - \mathbf{r}_\beta)\cdot\nabla_\alpha J_{\alpha\beta} = r_{\alpha\beta}(dJ_{\alpha\beta}/dr_{\alpha\beta}),$$

so that for the total energy we find

$$E = \sum_\alpha \sum_{\beta<\alpha} \int \psi^* \left\{ J_{\alpha\beta} + \frac{1}{2} r_{\alpha\beta} \frac{dJ_{\alpha\beta}}{dr_{\alpha\beta}} \right\} (S_{\alpha\beta}\psi) d\tau + \frac{1}{2} \sum_\alpha \sum_{\beta<\alpha} \sum_i \int \left[J_{\alpha\beta} x_i \left\{ \psi^* \frac{\partial}{\partial x_i}(S_{\alpha\beta}\psi) - \frac{\partial \psi}{\partial x_i}(S_{\alpha\beta}\psi^*) \right\} \right] d\tau.$$

In some simple cases this formula permits a relatively quick method of estimating the energy from approximate wave functions.

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Minneapolis, Minnesota,
February 8, 1937.

[1] W. H. Wells and E. L. Hill, Phys. Rev. **49**, 858 (1936).
[2] J. C. Slater, J. Chem. Phys. **1**, 687 (1933).
[3] Cf. H. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936), especially pages 105–119, for a discussion of the properties of the deuteron wave functions.

Neutron Optics

The remarkable results already obtained from the study of neutrons show what an extraordinarily fertile field of theoretical and experimental research would be opened if a beam of neutrons, like a beam of light, could be collimated, focused, or broken into a spectrum. Results have been obtained in two experiments[1, 2] which have hitherto been attributed to the diffraction of slow neutrons by a crystal lattice, but it seems unlikely that diffraction by crystals can carry us far toward the goal of a complete mastery of the neutron beam.

Indeed the achievement of such a goal seems at first a little hopeless. The de Broglie wave-length, even of the slower neutrons with which we have to deal, does not much exceed 1A, compared with which any ordinary optical surface is extremely rough. Nevertheless, if some form of radiation is capable of penetrating deeply into a given medium, it may not be greatly affected by conditions at the surface.

In the theory of light there is another distance which is almost as important in the interpretation of wave phenomena as the wave-length. This distance is the length of the wave train, or, for short, the coherence distance, which may be roughly measured as the maximum difference in path, between two parts of a divided beam, that still permits interference. This coherence distance, according to modern theory, is determined in part by the history of the emitting atom, and chiefly by the life of that excited state of the atom which precedes the emission of a photon. Some time ago, however, I pointed out[3] that, according to the law of the symmetry of time in physics, which is one of the most universal of physical laws, "If a transition depends upon the properties of the state preceding the transition, it must in equal measure depend upon the properties of the state following." We may therefore state that the coherence distance depends symmetrically upon the lifetime of the emitting atom preceding the transition and the lifetime of the absorbing atom after the transition. This dependence of optical coherence upon the nature of the absorbing as well as of the emitting substance has never been sought for experimentally, but will undoubtedly be found.

It has seemed to me that a search for ordinary optical effects in a neutron beam would be profitable if, and only if, the neutrons can be expected to possess a high degree of coherence. Such coherence is indeed to be predicted from the view of capture in neutron orbits that I have recently set forth.[4] It seems to me that there can hardly be any other explanation of the remarkable effect of paraffin[5] in slowing neutrons down even to 20°K, except that neutrons fall into and are held by paraffin molecules, and are later reejected through thermal agitation. At least this view suggested a large degree of coherence of the slow neutrons from paraffin and led to the experiments described in the accompanying communication.

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Berkeley, California,
February 14, 1937.

[1] von Halban and Preiswerk, Comptes rendus **203**, 73 (1936).
[2] Mitchell and Powers, Phys. Rev. **50**, 486 (1936).
[3] Lewis, Science **71**, 569 (1930).
[4] Lewis, Phys. Rev. **50**, 857 (1936).
[5] Libby and Long, Phys. Rev. **50**, 577 (1936).

Experiments on the Magnetic Moment of the Neutron

Further experiments[1] have been performed to observe directly the effect of the magnetic moment of the neutron, and magnetic scattering. In one experiment, the neutrons were directed normally through two iron plates, each 0.5 cm thick, 24 cm apart, and magnetized to saturation by separate electromagnets. Counts were taken with the "polarizing" plate successively magnetized in opposite directions and then demagnetized. The "analyzing" plate was continuously magnetized in one direction. Table I shows the results of this experiment.

From these results it is seen that while there is no significant difference between the parallel and antiparallel cases, the differences between these cases and the zero case is significant. By considering only the slow (absorbed in Cd) neutrons it is seen that these differences represent 3.3 percent ±1.2 percent and 2.7 percent ±1.3 percent effects. These figures are in good agreement with a number of experiments which we have performed, including the results given in Table I of the previous letter,[1] in which the differences between the parallel and antiparallel cases are small, but in which much larger differences occur

TABLE I.

	Total No. Counts	No./min.	Differences from Demagnetized Case
Fields parallel	16,256	208.9 ± 1.7	5.5 ± 2.1
Fields antiparallel	16,220	207.8 ± 1.7	4.4 ± 2.1
"Polarizer" demagnetized	27,686	203.4 ± 1.2	0
Background with Cd	787	26.7 ± 1.0	176.7 ± 1.5

TABLE II.

	Total No. Counts	No./min.	Differences
Magnetized	99,880	149.85 ± 0.47	4.00 ± 0.66
Demagnetized	98,542	145.85 ± 0.47	
Background with Cd	6,310	28.30 ± 0.36	117.55 ± 0.59

between these and the demagnetized cases. These results in the parallel-antiparallel case are also consistent with those obtained by Hoffman, Livingston and Bethe.[2] (They give no data for the demagnetized case.)

The difference between the parallel and demagnetized cases suggested that an experiment should be performed using a single magnet. In order to reduce multiple scattering the experiment was performed using three 0.65 cm plates of Armco steel, spaced 5.7 cm apart. The plates were magnetized and then demagnetized. The results obtained in this experiment are shown in Table II. This difference represents a 3.4 percent$\pm$0.6 percent effect which is approximately six times the probable error.

In all experiments of this type, the effect of the magnetic field on the operation of the ionization chamber and the first tube of the amplifier must be considered. Spurious effects may be caused if they are not sufficiently shielded. In this experiment test runs were made which showed no such effects larger than 0.5 percent.

All of these results are at least in qualitative agreement with the theory which has been developed.[3] According to this theory, the scattering cross section for neutrons whose spin is parallel to the magnetic field in the iron is different from those whose spin is antiparallel. The transmission in the parallel case is proportional to

$$e^{-n\sigma(x_1+x_2)} \cosh n\sigma p(x_1+x_2)$$

and in the antiparallel case to

$$e^{-n\sigma(x_1+x_2)} \cosh n\sigma p(x_1-x_2),$$ [2-4]

where n is the number of atoms per cc, $\sigma(1+p)$ and $\sigma(1-p)$ are the cross sections for the two directions of polarization, and x_1 and x_2 are the thicknesses of the plates. The expression for the antiparallel case becomes the same as that for the demagnetized case, namely, $e^{-n\sigma(x_1+x_2)}$ provided that $x_1=x_2$.

In general, therefore, the transmission should be greatest in the parallel case, and the same in the antiparallel case as in the demagnetized case. In the first experiment, only one plate was demagnetized; hence this transmission should be slightly greater than the antiparallel case. Evidence for the reorientation of the neutrons in the antiparallel case is found, since the number transmitted was nearly as great as in the parallel case; i.e., the plates were so far apart that the neutrons passed through a region in which the gradient of the field was of the same order as the Larmor precession frequency of the neutrons, and hence a large fraction of them reoriented themselves parallel to the field of the second plate. Thus it approximates the parallel case and the difference would be small as all of the experiments indicate.[1, 2]

From these and the earlier experiments[1, 2] the existence of magnetic interaction with magnetized materials arising from the neutron magnetic moment is definitely proven; but the results must be interpreted with caution.

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February 17, 1937.

[1] Dunning, Powers and Beyer, Phys. Rev. **51**, 51 (1937).
[2] Hoffman, Livingston and Bethe, Phys. Rev. **51**, 214 (1937).
[3] Schwinger, in process of publication.
[4] Bloch, Phys. Rev. **50**, 259 (1936).