## On the Absorption of Cosmic-Ray Electrons in the Atmosphere

The satisfactory description of shower phenomena[1, 2] obtained under the assumption of the validity of radiation theory up to very high energies reopens the question of the effects due to primary electrons in the atmosphere itself. Qualitatively this problem can be treated very simply by the method of Carlson and Oppenheimer.[1] Their diffusion Eq. (12) admits for the distribution $P(E, t)$ as function of energy $E$ and traveled distance $t$ (unit length for air 0.4 m water equivalent) the solution[3]

$$P(E, t)dE = e^{-k_n t}E^{-n}dE, \tag{1}$$

$$k_n = 4/3 - 1/n + \{(2/3 - 1/n)^2 + 4/3(n-1)n\}^{1/2}. \tag{2}$$

An initial distribution of form $E^{-n}$ is therefore preserved as such and absorbed exponentially. At the limit of divergence of the incident energy, i.e., $n=2$, $k$ would be 0 (no apparent absorption) and any apparent coefficient of absorption can thus be accounted for by a suitable exponent $n$.

In case the primary distribution can be approximated by a sum of falling powers in $E$ its change in the atmosphere can be worked out easily. At large $t$ the terms with smaller $n$ will be the most important. To the $k=0.2$ (0.5/m water equivalent) for the soft component alone of the cosmic radiation near sea level there corresponds an $n=2.3$. Since the maximum near the top of the atmosphere observed by Regener and Pfotzer[4] and Millikan[5] can be well explained by the preponderance of primaries just above the minimum energy for penetration of the earth magnetic field at our latitudes (about $3\times10^9$ ev) and since the change in the distribution (1) due to the absence of primaries below this energy can be estimated to be negligible for $t>12$ (about half the atmosphere) it is evident that any absorption curve of the type found by Pfotzer can be represented as due to a suitable primary distribution.[6]

A distribution $P(E, 0) = E^{-n(E)}$ with $n(E)$ decreasing smoothly from 2.8 at $3\times10^9$ ev to 2.3 at about $10^{12}$ ev has been found to give the entire Pfotzer curve within an error smaller than 20 percent at every point.

The consequences of such a primary distribution seem to be quite compatible with our other knowledge. The distribution in energy at a definite depth would be nearly independent of $t$ from sea level ($t=25$) to $t=15$ and would approximate an $E^{-2.3}$ law. This means that the probability of showers should be nearly proportional to the intensity of the soft component in latitude as well as in altitude. The same should hold for the relative probability of showers of different size. Also the optimal thickness of shower generating lead screens should be independent of altitude for moderate altitudes. All this seems to be approximately correct except possibly in the case of large showers or bursts which show a more rapid increase with altitude than the soft component alone. This could be interpreted as due to a stronger falling off than $E^{-2.3}$ of $P(E, 0)$ at extremely high energies (over $10^{12}$ ev). For the geomagnetic effect it would follow that the latitude effect at sea level must be entirely due to the hard component. At pressures of about 50 cm Hg a considerable effect (about 25 percent) already would exist for the soft component and near the top of the atmosphere (8 cm Hg) the intensity at the equator should be only a few percent of the intensity at 50° latitude.

It seems therefore to be quite possible to retain the assumption of the unlimited validity of the radiation theory for electrons and photons provided a rather slow falling off of the primary distribution and its extension to at least $10^{12}$ ev is admitted.[7] A small change in the radiative probabilities at these energies would, however, affect this analysis appreciably.

L. W. NORDHEIM

Purdue University,
Lafayette, Indiana,
May 18, 1937.

[1] J. F. Carlson and J. R. Oppenheimer, Phys. Rev. **51**, 220 (1937).
[2] H. J. Bhabha and W. Heitler, Proc. Roy. Soc. **159**, 432 (1937).
[3] This solution holds only for electrons of energies above $1.5\times10^8$ ev where the ionization losses become preponderant over the radiation losses. The effect of the ionization and the number of low energy electrons can be estimated by a method analogous to Eqs. (27) to (30) (reference 1) and their inclusion will not modify the absorption coefficient (2).
[4] G. Pfotzer, Zeits. f. Physik **102**, 23, 41 (1936).
[5] R. A. Millikan, H. V. Neher and S. K. Haynes, Phys. Rev. **50**, 992 (1936).
[6] Prior to an analysis in terms of an electron distribution an allowance for the hard component has to be made. We assumed its contribution at sea level to be 70 percent of the total and extrapolated to other altitudes with an absorption coefficient of 0.16/m water. The analysis is not very sensitive against a change in these assumptions.
[7] A somewhat similar analysis of the absorption curve has been given by W. F. G. Swann, Phys. Rev. **50**, 1103 (1936), and C. G. Montgomery and D. D. Montgomery, Phys. Rev. **51**, 217 (1937). The difference in our treatment is that we use the actual theoretical formulae for the radiative effects and identify the electrons with the soft component of the cosmic radiation.

---

## Theory of Recombination of Ions Over an Extended Pressure Range

Recent work by M. E. Gardner[1] in this department has strongly indicated the correctness of the J. J. Thomson[2] theory of ion recombination below one atmosphere pressure. Recent work of Mächler[3] and earlier results indicate that above 5 to 15 atmospheres the recombination occurs according to the classical Langevin[4] theory. As suspected by Thomson[2] these two results are not inconsistent. If a sphere of radius $d$ defined by $e^2/d = 3kT/2$ is drawn about an ion in a gas, ions of opposite sign will undergo random diffusion in general away from the ion if outside $d$ and will be actively attracted[2] to it if inside $d$. Here $e$ is the electron, $k$ the Boltzmann constant and $T$ the absolute temperature. If the electron attaches outside of $d$ to form an ion the recombination will follow Thomson's theory giving a coefficient $\alpha_T = \pi(\bar{c}_+^2 + \bar{c}_-^2)^{1/2}(2\omega - \omega^2)$.[5] $c_+$ and $c_-$ are velocities of agitation of the ions and $\omega$ is the chance of energy loss by one ion by molecular impact in $d$. If it attaches inside $d$ it will recombine according to the classical Langevin equation $\alpha_L = 4\pi e(k_+ + k_-)$ (preferential recombination) where $k_+$ and $k_-$ are the mobilities of the two ions.[4] The fraction $f$ of the electrons diffusing to a distance $d$ or more before attaching can be determined. The fraction $1-f$ attach within $d$. Hence the true value of the coefficient $\alpha$ is $\alpha = \alpha_T f + (1-f)\alpha_L$. In the absence of the ionic field an electron diffuses a distance $d$ in a time $t$ given by $d = (12Dt/\pi)^{1/2}$.[6] Here $D$ is the coefficient of diffusion roughly given by $D = \frac{1}{3}\bar{c}\lambda$, where $\lambda$ is the electron free path. The field reduces diffusion *away* from the parent molecule[7] and thus $D$ should be diminished by a numerical factor $\eta$, whose

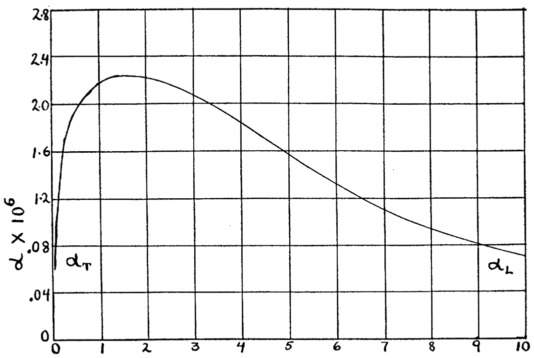


FIG. 1. Coefficient of recombination of ions in oxygen, the function of pressure. The abscissae give the pressure in atmospheres. At 1 atmos. $\alpha_T=2\times10^{-6}$, $\alpha_L=6.8\times10^{-6}$, $\eta=2\times10^6$.

theoretical evaluation is very difficult and which may be quite large, so that $d=(12Dt/\pi\eta)^{\frac{1}{2}}$. Let $n$ be the number of electrons so that in a time $dt$ the number of electrons attaching will be given by $dn=nh\bar{c}dt/\lambda$, where $h$ is the probability of attachment and $\bar{c}/\lambda$ is the collision frequency. If $n$ is the number of electrons after a time $t$ and $n_0$ is the number at $t=0$, then $f=(n/n_0)e^{-h\bar{c}t/\lambda}$.[8] Hence $f=e^{-\pi hd^2\eta/4\lambda^2}$, which neglecting the variation of $\lambda$ and $h$ with electron energy can be written $f=e^{-a(p/760)^2}$. Since again $4\pi e(k_++k_-)=b(760/p)$ the equation for $\alpha$ becomes

$$\alpha=\alpha_T e^{-a(p/760)^2}+[1-e^{-a(p/760)^2}]b(760/p),$$

where $p$ is the pressure in mm of Hg. There are at present no reliable extensive data to check this theory and our experience indicates that they will be very difficult to obtain. Insertion of the values of the constants into the equation indicates from the unsatisfactory data on hand that $\eta$ will have to be of the order of $10^6$. The shape of the curve for $O_2$ is indicated below. The 1902 data of Langevin[9] strongly indicate a behavior of this sort and in the limits of high and low $p$ it is naturally in agreement with the character of the results of Gardner and Mächler.

LEONARD B. LOEB

University of California,
Berkeley, California,
May 8, 1937.

[1] M. E. Gardner, Phys. Rev. **51**, 144 (1937).
[2] J. J. Thomson, *Cond. Elect. Through Gases*, Vol. 1, third edition p. 44. L. B. Loeb, *Kinetic Theory of Gases*, second edition p. 586.
[3] W. Mächler, Zeits. f. Physik **104**, 1 (1936).
[4] P. Langevin, Ann. Chim. Phys. **28**, 287, 433 (1903). L. B. Loeb, *Kinetic Theory*, first edition p. 480.
[5] L. B. Loeb, *Kinetic Theory*, second edition p. 595.
[6] A. Einstein, Ann. d. Physik **17**, 558 (1905).
[7] Loeb and Marshall, J. Frank. Inst. **208**, 371 (1929). Loeb, *Kinetic Theory*, second edition p. 590.
[8] Loeb, *Kinetic Theory*, second edition p. 617. N. E. Bradbury, Phys. Rev. **44**, 883 (1933). J. Chem. Phys. **2**, 827 (1934).
[9] P. Langevin, Comptes rendus **137**, 177 (1902).

## Thin Film Field Emission

Recently, Malter[1] reported the existence of anomalous secondary electron emission from specially treated electrolytic aluminum oxide. The same type of phenomenon has been produced by evaporating $BaOB_2O_3$ or quartz on to a metal plate and treating the film with caesium and oxygen; the treatment is similar to that used in the case of aluminum oxide. Films showing first and second order

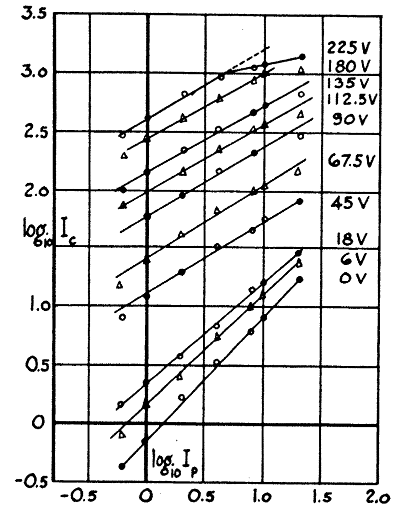


FIG. 1. $\text{Log}_{10} I_c$ plotted against $\log_{10} I_p$ for various collector voltages. Currents are expressed in $\mu$a.

interference colors, roughly from 600A to 6000A thick, are best in the production of this effect.

A summary of measurements made on treated barium borate films in a tube similar in structure to that used by Malter[1] is presented in the accompanying figures. They

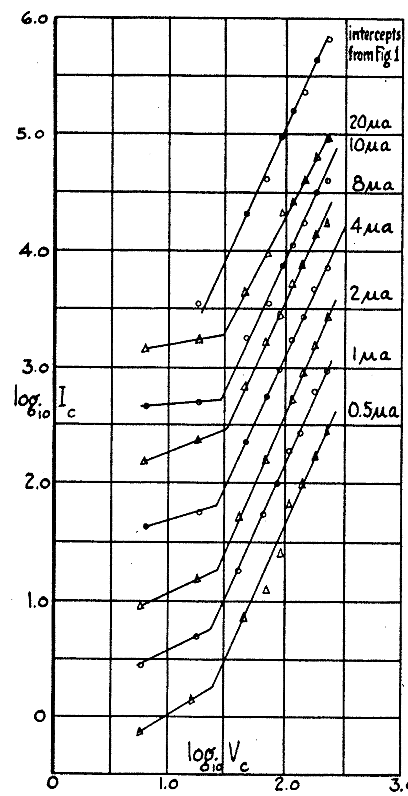


FIG. 2. $\text{Log}_{10} I_c$ plotted against $\log_{10} V_c$ for various primary currents. $I_c$ is in $\mu$a, $V_c$ in volts. The zero of the ordinate for each successive line is moved up 0.3 units.