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The Beta-Ray Spectra of Radium E and Radioactive Phosphorus

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The beta-ray spectra of Ra E and P^{32} were obtained by means of a magnetic spectrometer of high resolution. The observed end points of the spectra of Ra E and P^{32} are at 5280 ± 20 $H\rho$ (1.15 Mev) and 7150 ± 100 $H\rho$ (1.69 Mev), respectively. The end points obtained by extrapolating the Konopinski-Uhlenbeck plots are in each case about 17 percent greater than the observed ones, indicating that for some spectra these plots should not be used to determine the upper limits.

INTRODUCTION

SEVERAL attempts have been made to formulate a theory of beta-decay which would predict closely the observed form of the continuous momentum distribution spectrum of electrons emitted from a radioactive substance. One of the recent and most successful ones is due to Konopinski and Uhlenbeck[1] and is a modification of Fermi's theory.[2] Many observers have found that the shape of the distribution obtained from the K–U theory was in fair accord with the experimentally determined shape over the major portion of the spectrum. By assuming that the K–U theory was correct over the whole spectrum, it was then possible to extrapolate data taken over the middle portion of the spectrum to give the end point of the spectrum. In order to test the validity of the K–U prediction of the end point, a study of the spectra of radium E and radioactive phosphorus, P^{32}, both emitting electrons only, was undertaken by means of a spectrometer of high resolution.[3]

[1] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **48**, 7 (1935).
[2] E. Fermi, Zeits. f. Physik **88**, 161 (1934).
[3] E. M. Lyman, Phys. Rev. **50**, 385 (1936).

APPARATUS AND EXPERIMENTAL METHOD

A diagram of the magnetic spectrometer is shown in Fig. 1. The radioactive material, deposited on a plate or foil, was placed inside an aluminum box at O, and tipped at an angle of 45° to the plane of AD. Beta-particles passing through the incident slit, AB (2 mm wide, 8 mm long) into the evacuated chamber, were acted upon by a magnetic field, H, perpendicular to the chamber, so that their paths became circular. Particles of the proper momentum (proportional to $H\rho$) arrived at the counter slit, CD (4 mm wide, 8 mm long). The distance BD was 30.00 cm, making the minimum radius of curvature, ρ, of particles admitted to the counter slit 14.80 cm. The maximum radius of curvature, defined by the stops, E, F, was 15.20 cm. The separation of the aluminum-lined flat sides of the chamber was 14 mm. Auxiliary aluminum stops were located on the sides and on the top, BEC, and bottom, AFD, to prevent particles outside the proper $\delta\rho$ interval from being scattered from the walls into the counter slit. The stops were constructed so as to present a minimum of scattering material to the beam, coupled with a thickness sufficient to absorb completely 2 Mev electrons. The depth of

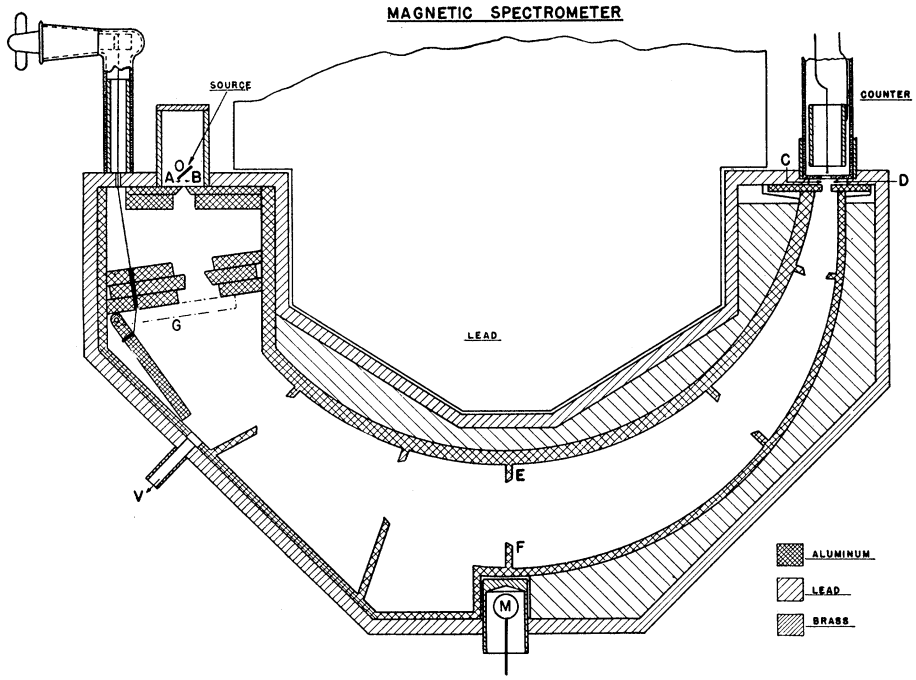

FIG. 1. Magnetic spectrometer for obtaining the beta-ray spectrum.

the beam was 8 mm. The particles passing through *CD* entered a Geiger counter which was connected to a three stage amplifier and a recording circuit capable of recording 60 counts per second. An aluminum gate, *G*, could be raised to cut off the beam in order to obtain the background count. Since the spectrometer was designed for use with a source emitting γ-rays as well as electrons, lead shielding was placed as shown to protect the counter from direct and scattered radiation. The chamber was evacuated through the outlet, *V*, by a Cenco Hyvac pump kept running continuously throughout the experiments.

In order to measure the magnetic field, a small flip coil, connected to a ballistic galvanometer, was inserted into the cavity, *M*. The system was calibrated by connecting the secondary of a standard Leeds and Northrup 50 millihenry mutual inductance in series and obtaining galvanometer deflections due to reversing measured currents in the primary. Then, from the dimensions of the flip coil, the value of H corresponding to any deflection produced by flipping the coil through 180° in a field could be computed. As a check upon this calibration, deflections of the galvanometer were compared with the expected deflections when the coil was flipped in the field at the center of a long solenoid through which a measured current flowed. For an absolute calibration, both methods required a knowledge of the absolute values of the currents. In this case it was considered sufficiently reliable to measure the currents by passing them through a standard resistance and comparing the potential differences produced with the e.m.f.'s of several Weston standard cells. A further check was made by comparing the expected deflections with those produced in strong fields previously calibrated by a bismuth spiral. All three methods agreed to within 0.3 percent. The magnetic field was measured for each point on the following curves, and the probable error is estimated to be well within ± 0.25 percent of H.

The field, produced by a half-ton electromagnet, was mapped over all the region traversed by the particles and found to be homogeneous to within 0.5 percent up to a distance of 1 cm below the incident and exodus slits. At the slit position it was 4 percent lower than at *M*. By applying

Hartree's[4] correction to the nonhomogeneous field, it was found that the effective correction, DH_0, to be applied to H, the value of the field measured at M, was -0.23 percent. Furthermore, it was observed that the relative inhomogeneities in the field-map were constant for all values of H in the range employed; therefore, this correction applied throughout.

The counter above CD was in a weak part of the magnetic field due to the fringing flux at the edges of the poles. By varying the magnetic field and observing that the counting rate due to a nearby γ-ray source remained constant, it was concluded that the variations in the field had no effect upon the counter. The field-map showed that the beta-particles always traversed the same region of the counter regardless of the value of the field. The counter and counting system were operated in a "plateau" region and were tested for variation in counting efficiency at different observed counting rates. For some counters used this variation was as high as 15 percent of the observed rate, while for others it was negligible at the high counting rates. The ordinates of the curves have been reduced to a standard efficiency.

Correction for the absorption and scattering of the mica window (0.002 cm thick) of the counter was made by adding, successively, two extra sheets of mica to it, observing a spectrum in each case, and then extrapolating to a spectrum corresponding to zero thickness of mica. This was done by plotting the three ordinates at various values of $H\rho$ against the thickness of the mica, and extrapolating linearly to an ordinate corresponding to zero thickness. The ratios between ordinates of the extrapolated spectrum and the spectrum corresponding to the window alone presented a method whereby the ordinates of any observed spectrum could be corrected for the effect of the window. In this case, the probable errors of the extrapolation were rather large, resulting in large probable errors in the ordinates of the following curves for values of $H\rho$ less than 2800 $H\rho$. The window had no effect upon the spectrum above 2800 $H\rho$. There was no material between the source and the initial slit; hence the initial distribution was undistorted when it reached the counter slit, and the above correction eliminated the distortion due to the mica window.

[4] D. R. Hartree, Proc. Camb. Phil. Soc. **21**, 746 (1923).

Each point of the following spectra is the result of about 1000 counts. When the background is very small compared to the counting rate, the probable error in the points is about ± 3 percent of the value of the ordinates: ± 2 percent due to statistical fluctuations alone, and ± 1 percent additional, estimated from the other sources of error involved (chiefly the variation of the counting efficiency due to fluctuations in the high voltage supply). However, as the counting rate approaches the background level, the relative probable errors in the ordinates become very large because the ordinates are small differences between two very nearly equal counting rates. This difficulty, and the fact that there is no *a priori* knowledge of the shape of the distribution near the end point obviously make it impossible to assign a definite upper limit to a spectrum.

There was no appreciable scatterng from the stops or walls of the spectrometer chamber. This was shown, first, by increasing $H\rho$ to a point slightly beyond the upper limit of the spectrum and observing that there was no noticeable count above background. If there had been scattering of the high velocity particles from the walls or stops, some of them would have been scattered into the counter slit, resulting in a long tail on the end of the spectrum. Next, the field was decreased to a value so low that the particles arriving at CD were unable to penetrate the mica window. Here again, no count above background was observed, although the multitude of particles with slightly greater penetrating power would certainly have been scattered into the counter, had such scattering existed. It was concluded therefore, that the scattering from the walls and stops of the chamber was negligibly small. As a further indication that this was true, an entirely different slit system, involving more auxiliary stops and defining slits at G, gave within the limits of error the same distribution for the P^{32} spectrum.

The radio-phosphorus sample was prepared by bombarding, in vacuum, a red phosphorus target backed by a copper plate, with 30 microampere-hours of 5 Mev deuterons produced in the cyclotron.[5] The copper plate was cooled by a

[5] E. O. Lawrence and M. S. Livingston, Phys. Rev. **45**, 608 (1934).

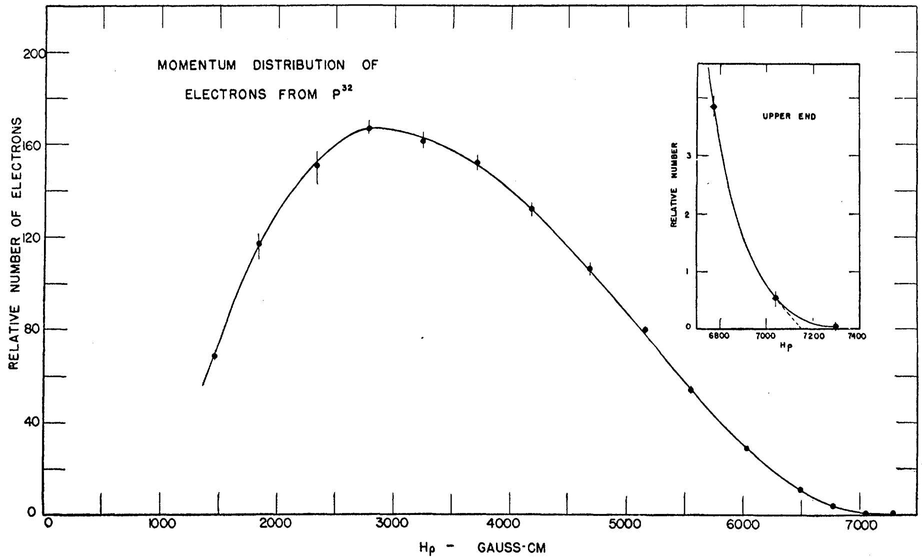

FIG. 2. The momentum distribution of P^{32} electrons. Probable errors in the points due to statistical fluctuations and experimental sources are indicated by the crosses through the points. The $H\rho$ of the last point is computed using the minimum radius of curvature.

liquid air flask in order to minimize evaporation of the phosphorus.[6] The sample obtained was approximately 2 millicuries in strength. It was placed about 1 cm above the incident slit and tipped at an angle of 45° to the plane of *AD*.

The Ra E sample was prepared from old radon tubes by deposition of the Ra E and F from 0.1 *N* HCl solution at 80°C onto a nickel plate. The strength was about 2.5 millicuries.

Results

To obtain the momentum distribution (or distribution with respect to $H\rho$) from the observed data, namely, the number of counts per minute for various values of H, the values of H were multiplied by the mean radius of curvature of the particles entering the counter slit, *CD*, and the numbers of counts per minute were divided by the corresponding values of H. To find the mean radius of curvature, ρ, it was observed that particles of radii ranging from 14.80 cm to 15.20 cm could enter the counter slit, and if the source were uniformly homogeneous in beta-particles of all radii, a maximum number would enter for $\rho = 15.00$ cm. By applying the geometrical method due to Wooster[7] to obtain the graph of distribution with radius of curvature of beta-particles entering *CD* from such a source, it was shown that the mean radius of curvature was 14.99 cm, and that the distribution was more nearly symmetrical about this radius than about that of the maximum ($\rho = 15.00$ cm). Thus, $\rho = 14.99$ cm was regarded as the center of the $\delta\rho$ interval and $H \times 14.99$ gauss-cm was therefore the mean value of $H\rho$ of the particles throughout a $\delta(H\rho)$ interval.

In computing the $H\rho$ value of the last point of each spectrum, the minimum radius of curvature (14.80 cm) was used. This procedure implies that the last observed positive count (above background) was due to particles which had radii just great enough to allow them to enter the counter slit, and this, in turn, involves the assumption that the distribution actually ended at this last observed point, or very close to it.

In any magnetic spectrometer, distortion of the true momentum distribution always occurs due to the finite dimensions of the slits. The amount of distortion depends upon the resolution of the spectrometer: in general, the better the resolution, the less the observed spectrum will be distorted. In this case, the fractional resolution, that is, the difference in $H\rho$ of two resolvable monochromatic lines divided by their mean $H\rho$, was 0.014. The amount of distortion introduced

[6] I am indebted to Mr. H. C. Paxton for the use of this target.

[7] W. A. Wooster, Proc. Roy. Soc. **A114**, 729 (1927).

by this spectrometer was investigated by the method suggested by Henderson,[8] in which one constructs, by the trial and error process, a distribution curve which, when scanned by the finite-sized counter slit, reproduces the observed spectrum. The distribution curve thus invented is not unique, but at least it is one step closer to the true distribution curve than the observed curve. In the cases of Ra E and P^{32}, the distribution curves thus obtained were identical with those observed, well within the limits of the probable errors of the ordinates, showing that the distortion present in the observed spectra (*after* having corrected the end points by using the minimum radius of curvature) was negligible.

The evaluation of the end points and estimation of the probable errors in them was accomplished by considering the probable errors in the last two points of the spectrum. For example, the last point of the Ra E spectrum, at $H\rho=5290\pm 13$ (using the minimum radius of curvature) was obtained by subtracting the background counting rate, 14.84 ± 0.46, obtained with the gate closed, from 15.37 ± 0.64, the rate with it open. Thus the counting rate at $H\rho=5290\pm 13$ was 0.53 ± 1.10. At $H\rho=5274\pm 13$ (using the mean ρ), the rate was 1.61 ± 1.30. These observations indicate that the end point probably lies between 5290 ± 13 and 5274 ± 13, or, roughly, it is at 5280 ± 20 $H\rho$.

The momentum distribution curves for P^{32} and Ra E electrons are shown in Figs. 2 and 3. The lower ends of the spectra have not been investigated. The data from the two distributions have been applied to the Fermi and K–U theories in order to test their validity near the upper ends. As has already been observed,[9, 10] the attempt to fit the Fermi theory to data of this type has been unsuccessful, while the K–U modification has fit somewhat better. According to the two theories, the momentum distribution of electrons may be expressed as

$$(N/(\eta_e)^2)^{1/\alpha}=K[(1+(\eta_e)_{\max.}{}^2)^{\frac{1}{2}}-(1+(\eta_e)^2)^{\frac{1}{2}}] \quad (1)$$

for light elements, and

$$(N/\eta_e(1+0.355\eta_e))^{1/\alpha}=K[(1+(\eta_e)_{\max.}{}^2)^{\frac{1}{2}}-(1+(\eta_e)^2)^{\frac{1}{2}}] \quad (2)$$

for heavy elements, where N is the number of electrons in small, equal momentum intervals, $\eta_e=H\rho/1700$ is the mean momentum of the electrons in the interval, $(\eta_e)_{\max.}$ is the maximum momentum of the electrons, i.e., the value at the end point, K is a constant of proportionality, and $\alpha=2$ for the Fermi theory and 4 for the K–U theory.

Thus plots of

$$(N/(\eta_e)^2)^{1/\alpha} \quad \text{or} \quad (N/\eta_e(1+0.355\eta_e))^{1/\alpha}$$

against $(1+(\eta_e)^2)^{\frac{1}{2}}$ should be linear and strike the $(1+(\eta_e)^2)^{\frac{1}{2}}$ axis at $(1+(\eta_e)_{\max.}{}^2)^{\frac{1}{2}}$ when $\alpha=2$ if the Fermi theory is correct and when $\alpha=4$ if the K–U theory is correct.

The data have been plotted according to the two theories in Fig. 4 for P^{32} and Fig. 5 for Ra E. As may be seen, neither of the two theories is in agreement with the data. A straight line may be drawn through the central portion of the K–U plot, or the final portion of the Fermi plot in each case, but in neither instance are the data fitted throughout. It thus appears that neither the Fermi nor the K–U curves of Eqs. (1) and (2) can serve to determine the end points of spectra when

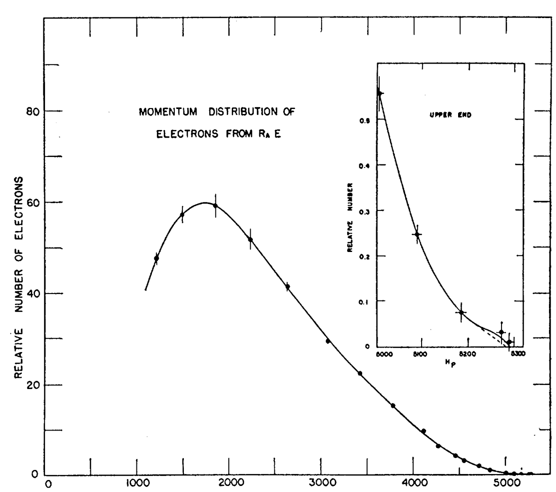

FIG. 3. The momentum distribution of Ra E electrons. The $H\rho$ of the final point is computed using the minimum radius of curvature.

[8] W. J. Henderson, Proc. Camb. Phil. Soc. **31**, 285 (1935).
[9] F. N. D. Kurie, J. R. Richardson and H. C. Paxton, Phys. Rev. **49**, 368 (1936).
[10] W. A. Fowler, L. A. Delsasso and C. C. Lauritsen, Phys. Rev. **49**, 561 (1936).

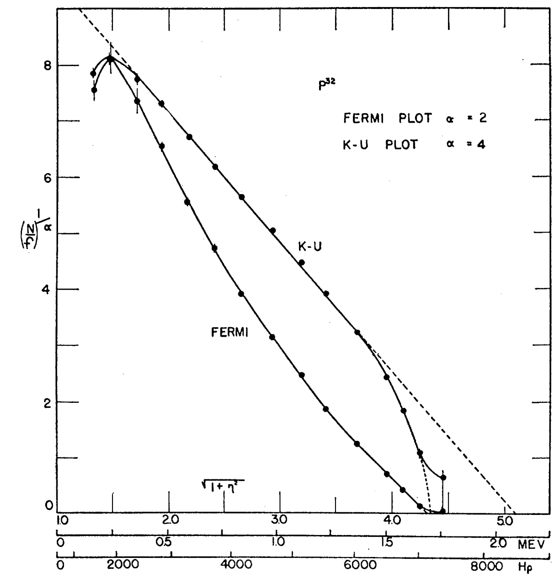

FIG. 4. The Fermi and K–U treatment of the data for P^{32} (Eq. (1)). The solid lines are drawn through the data, assuming that the distributions continue on beyond the observed end points. The curved, broken line at the end of the K–U plot is drawn, assuming that the distribution ends at the observed end point. The extrapolated Fermi end point is at $(1+(\eta_e)^2)^{\frac{1}{2}}=4.32$, the K–U, at 5.10, and the observed, at 4.33.

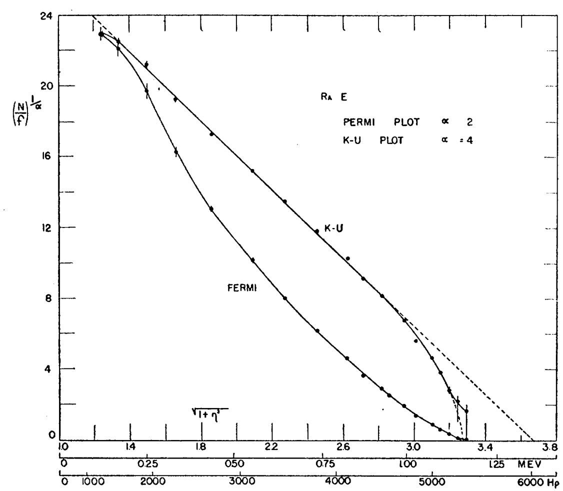

FIG. 5. The Fermi and K–U treatment of the data for Ra E (Eq. (2)). The lines are drawn as in Fig. 4. The extrapolated Fermi end point is at $(1+(\eta_e)^2)^{\frac{1}{2}}=3.26$, the K–U, at 3.69 and the observed at 3.25.

data are taken throughout the central portions alone. However, for purposes of comparison, straight lines have been drawn through the points of the central portions of the two K–U plots and the end portions of the Fermi plots. The "extrapolated end points" corresponding to the intercepts of these straight lines on the $(1+(\eta_e)^2)^{\frac{1}{2}}$ axis are shown in Table I, along with the observed end points obtained from the data given in Figs. 2 and 3:

In plotting the final points in Figs. 4 and 5, the mean radius of curvature was used to compute the values of $(1+(\eta_e)^2)^{\frac{1}{2}}$ because the central portions of the K–U plots indicate that the distributions should not end at the observed end points, but rather, should continue on out considerably further than was experimentally observed. In addition, however, the trend of the K–U points, assuming that the distributions do end at the observed end points, have been indicated by the broken lines in each case. The Fermi points have been plotted in the same manner.

It may be observed that the deviations from the K–U straight lines, as drawn, are far greater than the experimental errors involved. For example, in the case of P^{32} at $H\rho=7000$, the number of particles that should be observed according to the K–U straight line is $(2/1.1)^4$ or 11 times that actually obtained, while at $H\rho=7200$, the number is 26 times that observed.

The deviations at the lower ends of the K–U plots are probably due in part to the inability of the extrapolation method to compensate completely for the absorption and scattering of the mica window.

The ratios of the extrapolated K–U to the observed end points are the same for the two substances, within the limits of error: the K–U limits are about 17 percent higher than the observed ones.

The shape and value for the end point of the Ra E spectrum are in essential agreement with what has been found by some other experimenters. Among these, Alichanow, Alichanian, and Dzelepow,[11] Madgwick[12] and Champion[13] found end points at 5200, 5000 and 5500 $H\rho$, respectively. However, neither the shape nor the end point agrees with Scott's results.[14] On the other hand, many observers working with cloud

[11] Alichanow, Alichanian and Dzelepow, Nature **137**, 314 (1936).
[12] Madgwick, Proc. Camb. Phil. Soc. **23**, 982 (1927).
[13] Champion, Proc. Roy. Soc. **A134**, 672 (1932).
[14] F. A. Scott, Phys. Rev. **48**, 391 (1935).

chambers find agreement with the K–U formula near the upper limit for P^{32} or other radioactive substances, but, as shown by Fowler, Delsasso and Lauritsen[10] and by Paxton,[15] this agreement is only apparent, due to the large probable errors inherent in cloud chamber technique.

Discussion

In the light of the results, it must be concluded that the K–U distribution as expressed in (1) and (2) does not fit the data throughout the spectrum. The fact that the K–U plot drops to the axis long before the extrapolated end point has suggested the possibility that a neutrino mass different from zero should be considered in applying formulas (1) and (2). The correction to the formulas for this case was given to me by Mr. W. E. Lamb, Jr., of this department. Application of the correction to the data showed that no positive neutrino mass, of any magnitude whatever, was capable of bringing about agreement.

Table I. *Extrapolated end points given by theories and experiment.*

	Observed $H\rho$	Observed Mev	Fermi $H\rho$	K–U $H\rho$	Obs/K–U	Maximum $H\rho$
P^{32}	7150 ± 100	1.69	7140	8500	0.84	2800
Ra E	5280 ± 20	1.15	5300	6950	0.87	1750

[15] H. C. Paxton, Phys. Rev. (in press).

Furthermore, it may be pointed out that the formulas (1) and (2) apply to the so-called "allowed" beta-transitions, whereas, Ra E is a "singly forbidden" transition and P^{32}, judging from its position on the Sargent curves, is probably "doubly forbidden." There are a number of forms that can be chosen for the coupling of heavy and light particles, all of which lead essentially to the K–U plots for the allowed transitions, but give substantially different curves for the forbidden ones. The simplest forms of the coupling, including the one actually suggested by Konopinski and Uhlenbeck, have been investigated by Mr. Lamb and only partially remove the discrepancy at the upper limit, giving, however, a much better fit for Ra E than for radiophosphorus. It is not yet clear how these experimental curves can be fitted by a suitable and more complicated choice of the coupling, but here it must be emphasized that the failure of the K–U plots to fit these data need not be regarded as an argument against their applicability to allowed beta-spectra. A formula of the form (1) or (2), but with α taken equal to 3, represents the data very closely, although it has no theoretical foundation.

It is a pleasure to express my appreciation to Professor E. O. Lawrence and Professor J. R. Oppenheimer and to other members of the department for their helpful suggestions and advice during the course of the experiments, and to Mr. W. E. Lamb, Jr., who so kindly made the theoretical calculations.