Neutron-Proton Interaction: The Scattering of Neutrons by Protons

WILLIAM D. HARKINS, MARTIN D. KAMEN, HENRY W. NEWSON AND DAVID M. GANS, *University of Chicago*

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The inadequacy of present theories of proton-neutron interaction is shown in work in which 730 proton tracks produced by collisions of fast neutrons with hydrogen nuclei have been studied and measured to determine the distribution-in-angle. The recoils have been observed in three gaseous media; hydrogen, ethylene and hydrogen sulphide, by the use of a sylphon-type Wilson chamber and two cameras mounted in such a way as to give two views of each recoil, thus enabling a stereoscopic projection and a direct measurement of the angle of scattering to be made for each track. In this way it is found that the intensity distribution of recoil protons shows a maximum in the neighborhood of 25°. The distribution referred to unit solid angle in each angle interval exhibits a sharp maximum at 0° in close agreement with the work of Kurie, but in contradiction to those who have believed the scattering to be of the classical type. Various factors, both geometrical and statistical, which may lead to distortion of the angular distribution are cited, and it is shown that their effect is taken into account in these experiments, but not necessarily in those of the other workers. The contradictions in the results reported at present in the literature are assumed to arise from these factors. The bearing of the distribution found in these experiments on the neutron-proton interaction is considered in the light of the modern nuclear theories and evidence is deduced for an exchange or some other type of interaction at energies lower than predicted on the basis of these theories. The mechanism favored by both the classical and quantum theories, namely elastic collision of like spheres, is inadequate to explain the distribution. The present distribution is found to be consistent with Fermi's explanation for the efficiency of hydrogen nuclei in slowing fast neutrons, but indicates an even higher efficiency.

1. INTRODUCTION

THE prominence of the neutron-proton model in contemporary nuclear theory invests with importance all types of measurement which lead to an elucidation of the neutron-proton interaction. Among these, the measurement of the angular distribution of recoil protons set in motion by fast neutrons is of immediate interest as it affords a direct test of the presence of the "exchange" force which is assumed to play a fundamental role in the energetics of neutron-proton combination. As suggested by Wick, and as will be shown later, the "exchange" force should manifest itself by the presence of an asymmetry in the angular distribution such that there is a preferential scattering of the protons into the smaller angles. Ordinary collisions, as in the case of like elastic spheres, should lead to a distribution following the $\sin\theta\cos\theta$ law (θ being the angle variable); that is, a chart of the intensity of scattering as a function of the scattering angle should reveal a maximum at 45°.

The unique properties of the neutron impose limitations on the flexibility of scattering experiments carried out with them. With the experimental conditions thus far used, the neutron is always the projectile, the proton the target. In addition, there exists no way at the present time whereby a beam of extremely fast neutrons, homogeneous in velocity, may be obtained. The consequent heterogeneity of the velocities used in the experiments renders difficult an explanation of any deviations from the expected distribution which are found. Thus, at present all experiments on scattering with fast neutrons must be considered as somewhat negative in their bearing on the problem, since they serve to show deviations from theoretical prediction, but give no unambiguous clue as to the manner in which the theory should be changed.

Since the inception of the work described in the paper, rather meager data have been published, most of which are subject to large error because of the small number of recoils measured. The results are highly contradictory. Thus, while Monod-Herzen[1] and Meitner and Philipp[2] present data which they consider agree with the distribution to be expected from the collision of like elastic spheres, Kurie[3] exhibits data which show a marked asymmetry in the small angles, a result also found by Harkins and co-workers.[4, 5] Nor does it appear possible to reconcile such results

[1] Monod-Herzen, J. de phys. et rad. Feb. (1934).
[2] Meitner and Philipp, Zeits. f. Physik **87**, 484 (1934).
[3] Kurie, Phys. Rev. **44**, 461 (1933).
[4] Harkins, Gans, Kamen and Newson, Phys. Rev. **47**, 511 (1935).
[5] Harkins and Gans, Report to Washington meeting, American Physical Society, April, 1935; Abstract in Phys. Rev. **47**, 795 (1935).

as those of Kurie and of Harkins with any of the models proposed so far for the neutron.[6] It is hoped that the presentation of the somewhat more extensive data collected in this laboratory along with an exposition of some statistical and geometrical factors which affect these data, as well as those already reported in the literature, will indicate how such discrepancies may arise. This should help to clarify the present status of the measurements of angular distribution.

2. Experimental Details

The photographs of the recoil protons were obtained by means of a Wilson chamber of the type described by Harkins, Gans and Newson.[7] A closely fitting steel piston is sealed gas tight to the cylinder wall by means of a brass sylphon (cf. Fig. 1). Gas may be withdrawn or introduced into both sylphon and chamber proper by means of capillary stopcock valves (not shown in Fig. 1) and the operation of the chamber is usually carried out at pressures slightly higher than atmospheric. In this way, changes in the composition of the gas due to chance leakage through the glass walls of the chamber or through the piston are minimized. Actually, no sensible leakage of this sort was found to occur when the chamber was properly assembled. The chamber was lighted by a discharge from a condenser at 20,000 volts through two Pyrex capillaries filled with air at about 4 cm pressure. These were placed at the focal point of a lens-parabolic reflector arrangement so that most of the light emitted during the discharge was effective in photographing the chamber. The general design of the chamber as well as the arrangement for lighting is shown in Fig. 1. A B-eliminator was used to supply a steady 400-volt field across the chamber. In the earlier work of this laboratory the gamma-ray background was not intense, so the field could be removed for the short time of the expansion, but with the more powerful source used in the later work it was found that the field was required during the expansion; otherwise, the gamma-ray background became so intense as to render indistinguishable a large number of the proton recoils. With gases like ethylene, propane, etc., which were tried in an attempt to increase the yield, the background became too intense even with the field on continually. It was feared that some distortion of the tracks by the presence of the field would occur, but visual and photographic tests showed no appreciable distortion for the vast majority of recoils studied. Such tracks as appeared to be split or bent by the field, especially near the periphery of the chamber, were not measured.

Two views of the chamber at approximately 73° to each other were obtained by means of two cameras mounted rigidly on runways. Eastman Supersensitive Panchromatic film was used for the later work and Gevaert Safety Motion Picture film in the earlier work. The cameras could be shifted by means of a double track of steel so

[6] Massey and Mohr, Proc. Roy. Soc. **A148**, 206 (1935).
[7] Harkins, Gans and Newson, Phys. Rev. **47**, 52 (1933).

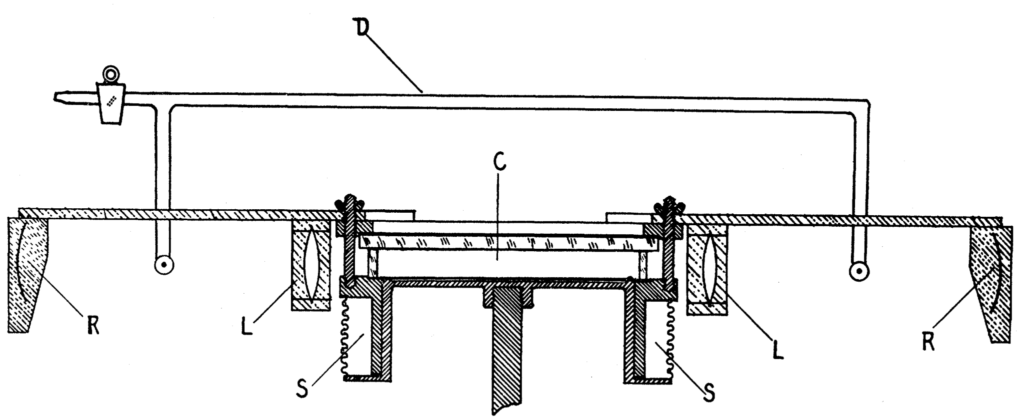


Fig. 1. Diagram of assembled Wilson chamber with optical system. C, Wilson chamber; S, sylphon compartment; L, cylindrical lens; R, metallic parabolic reflector; D, capillary discharge lights.

it was possible to project the two views of the recoil track stereoscopically on a semi-transparent onion skin screen in the ordinary way. Thus, an exact reproduction of the recoil as it occurred in the chamber was obtained and the scattering angle, and range of the proton and neutron were measured directly. The center of the source was indicated by a pointer free to move along any one of the three space axes. In all the work, the neutrons were assumed to emanate isotropically from the source and the neutron path was taken to be a straight line from the center of the source to the point of incidence of the recoil track. Protons were easily distinguishable from adventitious marks such as might arise from scratches on the cover glass, irregularities in the background and chance disturbances in the gas. All true recoil tracks *showed increase in density of ionization along the recoil path*, whereas no such effect was evident for the scratches. All tracks were examined first under a magnification of 16 diameters before being projected, so that the possibility of a scratch being mistaken for a recoil was quite remote. For those tracks which ended in the field of the photograph, straggling was also in evidence. The effect of stray contamination was checked by the use of a source known to be contaminated; this served as a calibration of the experimental method, for the contamination background was found to yield a completely random distribution. In the actual experiments with hydrogen, ethylene and hydrogen sulphide, the contamination was found to be negligible, about 1 track in 50 expansions being observed with no source of neutrons present as compared with a yield of 1 in 5 with the source in the center of the chamber. Moreover, most of the contamination was recognizable since the tracks extended along the whole length of the field and made improbable angles with the assumed neutron path. In measuring the scattering angle of the recoils, it was assumed that the end of the track nearest the source was the point of incidence. This assumption was borne out by the fact that the assumed beginning of each track was found to be thinner than its end.

The measurements to be discussed shortly have been separated into two sets, depending on the type of source used. In set A which represents the earlier work 505 recoils were observed in hydrogen, ethylene, and hydrogen sulphide.[8] For this set the source was a mixed one and consisted of salts of mesothorium and thorium X in intimate contact with beryllium powder. The powder so made was placed in silver capsules which in turn were embedded in a platinum vessel 9 mm in internal diameter and 3 mm thick to minimize the gamma-radiation as much as possible. In set B, there are 225 recoils found in hydrogen by the use of a source of radiothorium salt (28 mc equiv. of Ra) mixed with beryllium powder and placed in a spherical soft glass bulb, housed in a stainless steel pellet about 1 mm thick so shaped as to reduce surge in the gas. These details are schematically shown in Fig. 2. Some polonium and radon sources were available but proved to be too weak. Placement of the source at one side of the chamber reduced the yield too drastically to make it advisable to take advantage of the extra accuracy gained by the longer neutron path. It was felt that the center of the chamber, offering as it did the most symmetrical disposition of the source, was the best position for the source, since it renders less probable multiple scattering by the glass walls, cover glass and steel bottom of the chamber. In all, about 9000 photographs were examined.

Details of the synchronization of the lights with the expansion, mechanics of the expansion, adjustment of the expansion ratio and shutters, circuit for the high voltage necessary to discharge through the lights, etc., have been omitted since they are fully described in the report by Harkins, Gans and Newson.[7]

3. Data on Proton-Neutron Scattering

A complete discussion of the data obtained as described in the previous section must contain a thorough treatment of all factors in the experiments which tend to introduce distortion of the true distribution sought. Tables I and II exhibit the complete numerical results of the measurements of the angle of scattering. The numbers shown give the percentages of proton recoils found in the various angle intervals which throughout this work have been taken in steps of 10°. In Table I, the recoils have been classified

[8] Harkins, Gans, Kamen and Newson, Phys. Rev. **47**, 511 (1935).

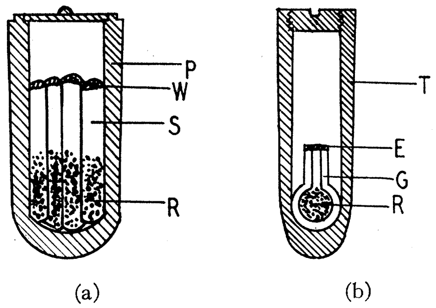


FIG. 2. (a) Source for recoils of set A. P, platinum wall; S, silver vessel for radioactive mixture R; W, seal of Wood metal. (b) Source for recoils of set B. G, soft glass bulb for radioactive mixture R; E, sealing wax; T, stainless steel wall.

according to the medium in which they were studied. The set A recoils include 287 in ethylene, 139 in hydrogen, and 79 in hydrogen sulphide, 505 in all. The set B recoils were all obtained in hydrogen and number 225. Thus 730 recoils have been observed in order to obtain the distribution shown in Fig. 3. It will be seen that a marked asymmetry exists in the small angles. The agreement between the results as found for the various media studied in the case of the set A recoils as well as the agreement of the summed results of set A with those of set B is strong evidence that the asymmetry observed is intrinsic in the angular distribution of the recoil protons. In order to facilitate comparison with the work of Kurie, with which these data are in striking agreement, the percentage scattering intensity as given in Table I, has been referred to unit solid angle subtended by each angle interval and the numbers thus determined are exhibited in Table II together with the results of Kurie. A plot of the data of Table II is given in Fig. 4. The results check well within experimental error, with the exception of the ordinates at 45° and 55°; at these angles our work gives a curve which is the smoother.

At these points the normal deviation of Kurie's results is in the neighborhood of 30 percent so that the discrepancy cannot be regarded as serious. The set B recoils are to be regarded as the data most worthy of confidence since they were picked in such a way as to render nonexistent certain geometrical factors which might have distorted the distribution found for the recoils of set A. Moreover, the source used in the set B measurements had only half the diameter of the source for the set A recoils. Hence, the accuracy with which the angle could be measured was approximately twice as high in the case of set B as compared with set A. The agreement in the data obtained in both cases shows that no great distortion took place even when no great care was taken to insure against it. This must be regarded as good fortune and should not be taken to mean that neglect of the factors to be discussed shortly is permissible.

Some criticism may be leveled against the results obtained for ethylene because of the presence of carbon nuclei which could superimpose a false distribution. To check this source of error, all recoils which might be carbon (less than 9 mm long) were segregated and plotted separately. The resulting curve for carbon showed no serious deviation from the distribution found for the main body of data so that the effect of carbon recoils was thus shown to be negligible. In the case of the hydrogen recoils of set A, some fear was occasioned by the possibility that distortion of the tracks which originated near the source was being brought about, partly by surge of the

TABLE I. *Proton scattering in various gases. Percent scattering into interval $\Delta\theta$.*

Angle interval	Recoils in hydrogen	Recoils in ethylene	Recoils in hydrogen sulphide	Average
		Set A		
0°– 10°	12.0	13.2	14.6	13.2
10°– 20°	19.7	19.0	17.7	18.9
20°– 30°	19.2	19.1	18.5	19.0
30°– 40°	14.6	15.4	13.0	14.6
40°– 50°	12.0	11.7	12.4	11.9
50°– 60°	7.3	9.1	9.9	8.8
60°– 70°	7.0	5.6	6.4	6.2
70°– 80°	4.0	3.6	4.4	3.9
80°– 90°	2.8	1.8	3.0	2.4
90°–100°	1.4	0.8	0	0.8
100°–110°	0	0.4	0	0.3
		Set B. Proton recoils in hydrogen		
0°– 10°		8.4		
10°– 20°		16.9		
20°– 30°		19.8		
30°– 40°		17.8		
40°– 50°		14.0		
50°– 60°		9.6		
60°– 70°		6.5		
70°– 80°		4.0		
80°– 90°		2.0		
90°–100°		1.0		

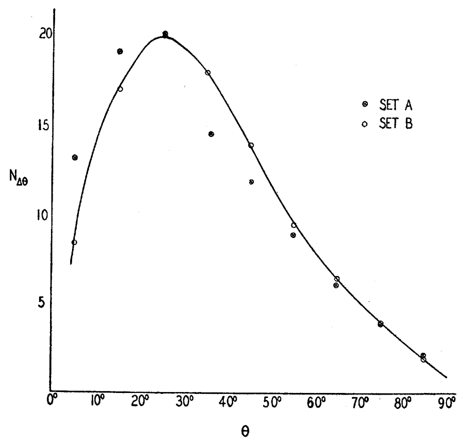


FIG. 3. Angular distribution of recoil protons. The ordinates $N_{\Delta\theta}$ are the average intensities in the angle intervals $\Delta\theta$ which throughout the work have been taken in steps of 10°.

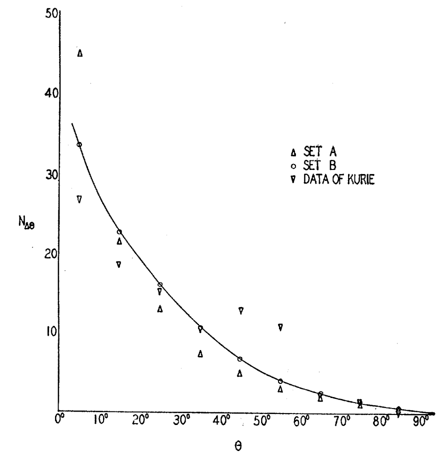


FIG. 4. Distribution in angle interval $\Delta\theta$ per solid angle subtended in each angle interval.

gaseous medium and partly by condensation of water on the source. A similar treatment of such tracks as that described in the case of the short tracks in ethylene showed no sensible departure from the general nature of the results as a whole. Fig. 5 shows the data of ethylene and hydrogen in set *A* plotted in this way. The values of the ordinates were arrived at in a manner to be described in the discussion of the data that follows.

4. Discussion of Data

To be comparable with theory, the determination of variation of intensity of scattering with angle must be carried out in an experimental system free from distortion, and the number of recoils studied should be sufficient to constitute a satisfactory statistical sample. In actuality, geometrical factors are always present which may lead to fallacious results and no investigation can be considered complete without a careful analysis of the way in which they affect the experimental results. Neglect of this fact together with the study of too few recoils can lead to apparently highly contradictory results. It would seem that such is the situation with regard to measurements of the angular distribution of proton recoils from fast neutrons.

TABLE II. *Percent scattering into angle interval per unit solid angle.* The data for set *B* are supposed by us to be more accurate than those of set *A*.

Angle interval	Set *A* recoils	Set *B* recoils	Kurie's data
0°– 10°	44.9	33.6	27.0
10°– 20°	21.7	22.8	18.6
20°– 30°	13.3	16.3	15.5
30°– 40°	7.5	10.8	10.7
40°– 50°	5.0	6.9	13.2
50°– 60°	3.2	4.1	11.3
60°– 70°	2.0	2.5	2.1
70°– 80°	1.2	1.5	1.6
80°– 90°	0.7	0.7	0
90°–100°	0.3	0.2	0

In the ideal case, a system may be contemplated in which a central point source emanates fast neutrons isotropically into a homogeneous medium at any point of which a proton recoil may arise. All points in such a space will be such that the recoil may proceed in any direction with regard to the incident neutron. By reference to Fig. 6, it is readily ascertained that in the cloud chamber as actually assembled such points in the gas are of this type except those near the source, top and bottom of the chamber. These should be excluded on account of the fact that at such points a recoil will track be interfered with, since it can be observed only in certain preferred directions. Hence, only recoils which originate reasonably far from the source and in the gas proper can be taken for measurement if a true distribution is to be attained. In the study of the set *B* recoils, such a procedure has been followed, since no recoils were measured which originated very near the top or bottom. Nor were any chosen which started less than 20 mm from the source. The results must then be considered free from most of the geometric distortion which may arise from the presence of the source and the boundary regions of the chamber. To further investigate the nature of such an effect, attention is again called to Fig. 6. As in the conditions of the experiment, the center of the source is in the geometric center of the chamber at the moment of expansion. Consider the case in which a neutron which proceeds from the source gives rise to a proton recoil at the point P. The recoil will be free to move off at an angle *in or out of the horizontal plane of the chamber*. Such a recoil will then satisfy the conditions stated above. However, suppose the neutron proceeds to the point P' at the top (or bottom) of the chamber. It is obvious that the recoil will be allowed to go off only toward the surface of the hemisphere which is *below* the top of the chamber. At the bottom this will be *above* the bottom of the chamber. For identical spheres according to the classical theory, all proton recoils will be included in the hemisphere whose base is a plane perpendicular to the initial path of the neutron and which passes through the point of impact. Obviously, the hemisphere lies beyond this plane in the direction of the initial velocity of the neutron.

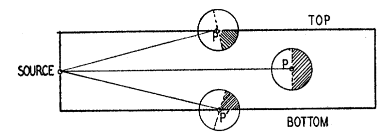


FIG. 6. The shaded portions of the semicircles represent the regions in the space of the chamber which are open to proton recoils at the points P and P'. It is seen that at any point P removed from the top or bottom of the chamber, there is no discrimination against any angle of scattering whereas at the point P' the short angles are discriminated against because of the presence of the top and bottom.

Thus, the top (or bottom) of the chamber will prevent the occurrence of all proton recoils from neutrons which have velocities directly upward (or downward) when the impact occurs first at the surface of the top (or bottom). At any other point on the surface, the recoils observed can lie only in the shaded section of the hemispheres. This means that only the *larger* angles of proton recoils can be observed, since all of the smallest angles are excluded.

Since the data of set A were calculated without attention to this fact, and since this error was avoided in the observations of set B, it is important to compare the distribution given by the two sets. If the error in the distribution given by set A due to this cause is of significance then the angle of maximum scattering found for set A should be shifted toward a *larger* value than that for set B. The data for Table I show conclusively that this is not true. Therefore it may be concluded that this error as introduced into the scattering exhibited by the data of set A is not appreciable.

Another error is introduced by the inclusion of impacts which lie too close to the top or bottom. A proton in such an impact may be scattered at a small angle, strike a nucleus in the solid material of the boundary region and be deflected back into the chamber. Obviously, measurement of such a recoil will give an angle larger than the true angle of scattering.

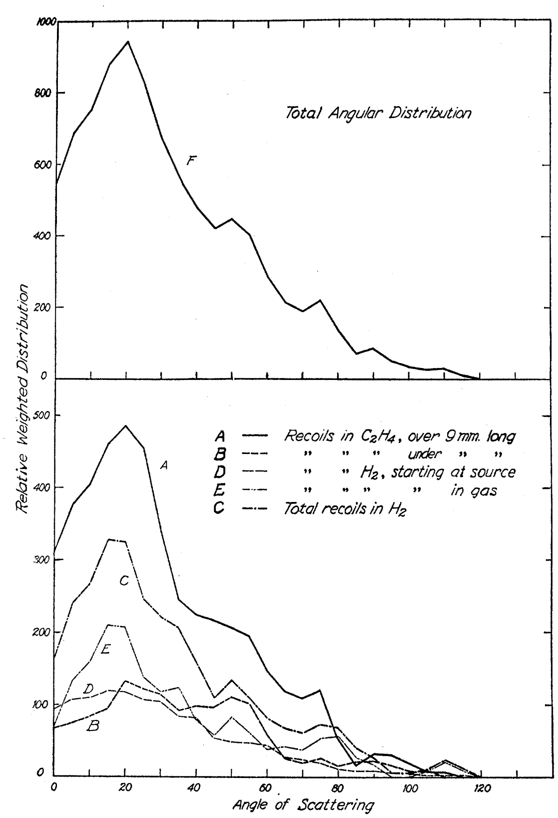


FIG. 5. Angular distribution of proton recoils in ethylene and hydrogen of set A. The ordinates in this figure have been chosen for convenience to be about fifty times as large as those of Figs. 3 and 4. They are not converted to parts per hundred since only the shapes of the various curves are to be compared.

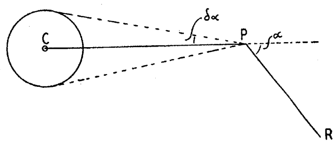


FIG. 7. Representation of the error $\delta\alpha$ due to finite width of the source. CP, neutron path; PR, track of recoil proton, α measured angle of scattering; C, center of source.

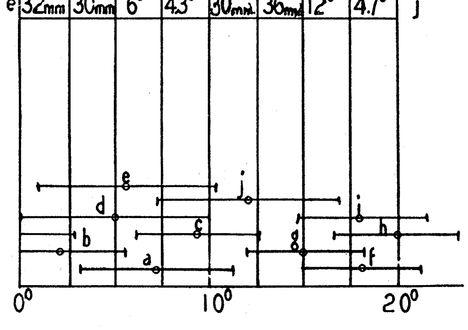
It should also be noted that the presence of an error of this type would be such as to give an apparently *better* agreement with modern theories than would be justified.

The finite size of the source constitutes a second factor which may lead to distortion of the data. In the present set of experiments the source was a small sphere placed so that its center corresponded very closely with the geometric center of the cloud chamber. From Fig. 7 it will be seen that every measurement of angle involves a systematic error due to the finite width of the source and that this error becomes progressively smaller as the neutron path becomes longer. Thus, if a recoil is found to make an angle of α degrees with the incident neutron, the associated error being $\delta\alpha$, the recoil must be considered to have arisen in the *interval* (line) $\alpha \pm \delta\alpha$, and not at the point α, in a determination of the number of recoils scattered into any angle interval $\Delta\theta$. The procedure adopted in obtaining the ordinates of Figs. 3 and 4 is illustrated in Table III. Ten representative tracks taken from the recoils of set B are tabulated in the data at the head of the diagram. These recoils are then plotted as chords on the $N-\theta$ chart, the length of the chord being given by the error $\delta\alpha$ calculated from the known radius of the source and length of the neutron path. Each such line is given the value of *one* track. If a line occurs with 3/10 of its length in the interval between say 0° and 10° it is counted as 3/10 of a track. This is dependent on the assumption that the neutron has a constant probability of origin at any point inside the spherical region which contains the source.[9]

Some chords will lie entirely in the interval $\Delta\theta$, others will contribute only a fraction. Whatever is found in any interval is added to give the ordinate $N_{\Delta\theta}$. These ordinates are then summed and each $N_{\Delta\theta}$ expressed as a fraction of the total $N(=\sum_\theta N_{\Delta\theta})$. These are the numbers exhibited in the tables given at the beginning of this report. It must now be ascertained whether this method of weighting the data has introduced extraneous peaks in the distribution curve. It is evident that an accidental piling up of "good" tracks (i.e., short chords) in any one interval $\Delta\theta$ will cause too high a value of $N_{\Delta\theta}$. If the choice of tracks has been made perfectly at random with respect to the length of neutron path no such disproportionation will occur. Hence, a method of checking the weighting method will also give information as to the randomness of the occurrence of tracks throughout the chamber. The simplest test which has been found consists in reweighting the data, assigning the value of unity to each chord regardless of whether the chord occurs entirely in the interval $\Delta\theta$ under consideration or not. In this way, one recoil may count as many, one for each interval into which its representative chord may extend. The numbers thus obtained by this method of addition are denoted by $N_{\Delta\theta}'$ (cf. Table III). If no disproportionation has oc-

TABLE III. *Method of obtaining intensity numbers $N_{\Delta\theta}$ and checking weighting and randomness in data by means of k.*

	Sn	Sp	α	±δα	Sn	Sp	α	±δα	
a	35mm	15mm	7°	4°	36mm	>33mm	18°	3.9°	f
b	30mm	>32mm	2	4.7	48mm	12mm	15	3.1°	g
c	42mm	>25mm	9°	3.3°	43mm	26mm	20°	3.3°	h
d	32mm	27mm	5°	4.3°	40mm	10mm	18°	3.5°	i
e	32mm	30mm	6°	4.3°	30mm	36mm	12°	4.7°	j



A chord entirely in one interval $\Delta\theta$ (as d in 0°–10°) counts as 1. Thus,

$$N_{0^\circ-10^\circ} = 7/8 \quad 1 \quad 43/66 \quad 1 \quad 1 \quad 27/94 = 4.8,$$
$$N'_{0^\circ-10^\circ} = 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 = 6,$$

and

$$N_{10^\circ-20^\circ} = 59/78 \quad 1 \quad 1/8 \quad 23/66 \quad 1/2 \quad 67/94 \quad 55/70 = 4.2,$$
$$N'_{10^\circ-20^\circ} = 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 = 7.$$

Hence,

$$k_{0^\circ-10^\circ} = N_{0^\circ-10^\circ}/N'_{0^\circ-10^\circ} = 4.8/6 = 0.80.$$

Likewise,

$$k_{10^\circ-20^\circ} \equiv 4.2/7 = 0.60.$$

As more tracks accumulate in the various intervals, the k's approach constancy *provided the tracks occur at random throughout the chamber.*

[9] Other assumptions such as Maxwellian distribution around the source center have not been found to give appreciably different values.

curred, the numbers $N_{\Delta\theta}'$ must be linearly related to the numbers $N_{\Delta\theta}$ by a factor $k=N/N'$ constant for all intervals. Deviations of k from constancy indicate deviations from complete randomness. The values of k should be more constant the greater the number of tracks measured. In this manner, some knowledge of the statistical nature of the data may be obtained. When k shows no deviations greater than experimental error (*ca.* 5–10 percent) the randomness of the tracks as regards the length of the neutron path is established and the number of tracks for which this is true constitutes a "sufficient" number to give a satisfactory statistical sampling, assuming that the azimuthal distribution is also random. The values of k determined for the data of sets A and B are shown in Table IV. The mean values of k have been calculated and the numbers $N_{\Delta\theta}$ redetermined multiplying the various numbers $N_{\Delta\theta}'$ by the average k. The ordinates $N_{\Delta\theta}$ obtained in this way represent the most probable values within experimental error and are to be considered as the final values for the intensity of scattering in the various angle intervals $\Delta\theta$. It will be noted that for all data of set A, k is high for the interval 0°–10°. This shows that this particular value is somewhat too high, as is borne out by the more accurate value given by the recoils of set B. For purposes of comparison, the set B results should be considered the norm, as the accuracy is greater than in set A by approximately a factor of 2. This may be seen from the relative values of k for the two classes of data.

TABLE IV. *Values of the constant k.*

Angle interval	Set A Hydrogen	Set A Ethylene	Set A Hydrogen sulphide	Set A	Set B Hydrogen
0°– 10°	0.42	0.44	0.24	0.38	0.54
10°– 20°	.36	.39	.19	.34	.50
20°– 30°	.33	.38	.18	.33	.55
30°– 40°	.31	.30	.18	.28	.45
40°– 50°	.27	.34	.15	.28	.50
50°– 60°	.36	.41	.16	.32	.47
60°– 70°	.26	.32	.17	.26	.48
70°– 80° *	.39	.46	.14	.35	.47
80°– 90° *	.30	.35	.10	.26	.53
90°–100° *	.18	.65	—	.42	.25
100°–110° *	—	.53	—	.53	—
Average	0.33	0.38	0.16	0.32	0.50

* Large deviation of k in these intervals are not to be regarded as serious since so few recoils occur in them.

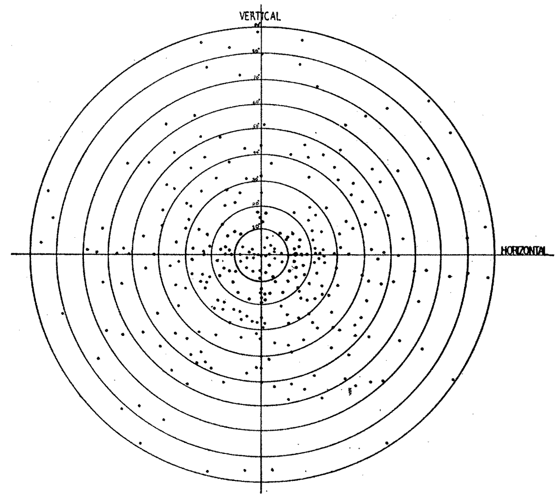


FIG. 8. Azimuthal distribution found for recoils of set B. Each concentric division reading outward from the center represents an increase of 10° in the azimuthal angle. The plot is a projection of the actual azimuthal distribution which would be observed at the surface of a sphere with the neutron source as center (as discussed in the text).

Figs. 3 and 4 have been drawn so that the points of set B determine the curve. It must be emphasized that distortion of the measurements of angle occurs if the finite size of the source is neglected. It does not arise from any method of weighting the data as long as a sufficiently random sampling of recoils is studied.

It was found in the course of the work that the distribution varied with the number of recoils studied *until about 150 had been measured. After this number had been attained the distribution remained practically invariant.* It is obvious that such an effect is due to statistical fluctuations which may be quite large for small numbers of recoils studied. Results based on a small number of recoils are meaningless unless they represent a satisfactory statistical sample. This must be demonstrated both by careful scrutiny of the effects so far described as well as those to be discussed now.

Among these, the determination of the azimuthal distribution is extremely important. If the tracks are not distributed uniformly in azimuth for all angle intervals $\Delta\theta$, effects analogous to projection of a sphere on a plane will become operative and lead to a false distribution. If the plane of the chamber is taken for measurement and all others excluded a correction for the loss in azimuth thus occasioned must be introduced. Such a correction is dependent on the experimental conditions. Kurie has adopted such a procedure in order to avoid such effects as might arise from contamination, the boundary region distortion, etc., and has made such a correction; a perusal of his report shows that this particular correction is contained implicitly in his conversion of data to unit solid angle. The workers on the continent do not mention any effort to check either the effect due to the source size or asymmetric azimuthal distribution, both of which may be important in their case since their results are based on such a small number of recoils. The azimuth of all recoils in set B has been measured and Fig. 8 gives a schematic presentation of the results. It will be seen that the distribution shows no appreciable preference for any plane, even the horizontal plane of the chamber. It is thus shown that the data as presented in Figs. 3 and 4 are satisfactorily random both in azimuth and with respect to the length of the radius vector (neutron path) from the source center.

A further check on the satisfactory statistical nature of the data is afforded by the occurrence of more than one track in a photograph. Occasionally two are found to occur, and very rarely, three. The number of such double events should bear the same relation to the number of single events as the latter does to the total number of photographs taken. Thus, in the case of the set B recoils 4160 photographs were examined. Two hundred and twenty-five recoils occurring singly were found, that is, one such recoil occurred every 18 photographs. On this basis, there should be 225/18 or 13 double events; 15 were found. To extend the same argument to triple events would not be justifiable in view of the extremely small number to be expected in 4160 photographs. Nevertheless agreement has also been found for the case of the triple events. There should be $4160/(18\times 18\times 18)$ or about 1; 1 such triple event was observed. The three criteria (constancy of k, azimuth, occurrence of multiple events) outlined thus far together with the observation that, after measurement of 150–200 recoils, the distribution became invariant to the number of recoils studied make it appear quite certain that the distribution presented in Figs. 3 and 4 is based on a sampling of data which is satisfactory statistically. This coupled with the demonstration that no distortion from geometric factors is present argues strongly that the intrinsic nature of the angular distribution of proton recoils from neutrons in the energy range determined by the sources used (*ca.* 1–8 Mev) is as given in this report and in the report by Kurie.

The one factor which is still unaccounted for is the presence of multiple scattering. An appreciable amount of this would nullify all the results given above, since it has been assumed that the neutron proceeds in a straight line from the source in order that the measurement of the angle of recoil be made possible. It is doubtful whether much multiple scattering occurs appreciably in this cloud chamber. If such were the case, there should be a large number of recoils at angles larger than 90°. Actually only 11 such recoils have been discovered throughout the whole duration of this investigation. This effect of multiple scattering was made smaller as compared with our earlier work, by making the glass top of the chamber as thin as possible. The general effect of multiple scattering would be to increase the number of recoils at large angles, which would tend to make our curve of scattering conform more to the theoretical than it would otherwise. It seems therefore that the presence of some multiple scattering cannot offer any explanation of the small angle of proton scattering which we observe.

Conversion of data to unit solid angle

Before proceeding to the final section in which the theoretical background of the present research is discussed, it is appropriate to insert some comment concerning the plot of unit solid angle given in Fig. 4. Some confusion seems to exist in the literature in regard to the way in which the conversion of data to a form suitable for a plot referred to unit solid angle is affected. Thus, Kurie remarks that the work of Monod-Herzen cannot be compared directly with his own because it has been obtained from a study of recoil protons distributed at random throughout the chamber, and in such a case the meaning of the solid angle subtended in any angle interval $\Delta\theta$ is hazy. To clarify this point, it is necessary only to consider a sphere with the point of impact between the neutron and proton as the center (as at point P in Fig. 6). From this point, a proton recoil may proceed in any direction to the incident neutron. Imagine a plane passing through the point of impact and perpendicular to the direction of the neutron. To obtain the azimuthal angle of the recoil, it is required merely to project the point at which the recoil passes through the surface of the sphere on to this plane. This procedure may be repeated for all the protons, yielding a plot of the kind shown in Fig. 8. Likewise, the solid angle Ω subtended in any interval $\Delta\theta$ will be given by the usual formula, i.e.,

$$\Omega=2\pi(\cos\theta_1-\cos\theta_2);\quad \theta_2>\theta_1 \quad \text{and} \quad \Delta\theta=\theta_2-\theta_1,$$

since all the spheres may be translated to the origin and the point of impact imagined to have been at the origin. This procedure is permissible since the angle of scattering is changed in no way by such a translation. In obtaining the azimuthal plot of Fig. 8, the recoils and their associate spheres were all referred to a plane perpendicular to the line joining the two cameras and passing through the source, since the choice of such a plane is arbitrary and this particular plane was most convenient for reference. Such a choice necessarily involved a rotation as well as a translation of most of the spheres to the particular axis of neutron path determined by the line perpendicular to the reference plane at the source, a procedure which in no wise changed the measured angle of scattering and so was permissible also.

Of course, these considerations are true only if all the conditions described in the previous section are satisfied. Since this has been shown to be the case, the data as collected in these experiments are identical with the ideal case in which the protons all originate at the source and constitute a bundle of tracks around the neutron beam. Hence, the conversion of the observed data as in Fig. 3 to those of Fig. 4 is accomplished merely by dividing the $N_{\Delta\theta}$'s of Fig. 3 by the respective fractions of solid angle subtended in each $\Delta\theta$.

Thus, the disagreement between the data of Kurie and those of Monod-Herzen is real. It is not the result of any indeterminacy in the solid angle but arises in all probability from the presence of some of the factors given in the discussion of the data.

5. Theoretical Aspects

A proper evaluation of angular distribution data is not attainable without a brief review of the present theories concerning the mechanism whereby neutrons are scattered by protons. The production of slow neutrons which is experimentally controllable to an extent not at present possible with fast neutrons lends this particular phase of the nuclear problem special interest.

It is universally assumed that the mechanism to which the slowing and scattering of neutrons by hydrogenic media is attributable is for the most part a process similar to that operative in the elastic collisions of identical spheres. Thus, only a very small effect on the scattering of the "exchange" type for the energy range in which the experiments are performed is postulated by present day theorists. At very high energies, interaction becomes all-important and in the present theory is considered analogous to the forces associated with the presence of the so-called "exchange" integral that arises in the quantum-mechanical treatment of the hydrogen molecule ion. Before sketching the recent treatment of the scattering problem as developed by Bethe and Bacher[10] the picture of the collision process according to the simpler view of classical mechanics will be considered. It may be supposed that the center of gravity (B) between the neutron and proton is fixed. Thus, to an observer at the center of gravity the two particles, identical in every respect, approach from diametrically opposite directions in a line fixed by the points A, B and C (Fig. 9). The velocity for both particles will be one-half that observed for the neutron moving toward a stationary proton, as in actual experiment. The relative velocities for incident and scattered particles are shown vectorially in Fig. 9. The condition that the center of gravity be stationary requires that the sum of the scattering angles be 180°. To determine the law of intensity distribution at various angles for the scattered protons as desired for comparison with experiment, advantage is taken of the fact that the distribution is isotropic in solid angle with respect to the coordinate system with center of gravity fixed. Thus, the number of neutrons dN_n scattered into the inter-

[10] Bethe and Bacher, Rev. Mod. Phys. 8, 82 (1936).

val $d\theta_n$ at an angle θ_n is given by

$$dN_n = \text{const.} \times \sin \theta_n d\theta_n.$$

Likewise, the number of protons dN_p which appear in the interval $d\theta_p$ is given by

$$dN_p = \text{const.} \times \sin \theta_p d\theta_p$$

since for every neutron in $d\theta_n$, there is a proton in $d\theta_p$.

The photographs of the hydrogen recoils refer to a system in which the center of gravity of the two particles has a velocity in the forward direction which is one-half the initial velocity of the neutron. The treatment given above refers to the center of gravity at rest. To transform this into the system with the center of gravity in motion, it is only necessary to add the vector which gives the velocity of the center of gravity (Fig. 9). If φ denotes the angles of recoil in the experimental coordinate system, it is found that

$$\varphi_p = \tfrac{1}{2}\theta_p, \quad \text{and} \quad \varphi_p + \varphi_n = \pi/2.$$

Hence, the desired result is

$$dN_p = \text{const.} \times \sin 2\varphi_p d\varphi_p = \text{const.} \times \sin \varphi_p \cos \varphi_p d\varphi_p.$$

Thus, Fig. 3 should show a somewhat flat maximum at 45° with a symmetrical dropping off toward 0° and 90°. In Fig. 4, the curve should give the appearance of a cosine function since the distribution in unit solid angle for each angle interval $d\varphi_p$ is found by dividing the expression derived for $dN_p/d\varphi_p$ by sin φ_p in each interval $d\varphi_p$. No such behavior is exhibited, however, since both curves show a marked asymmetry in the small angles, an effect which may be attributed to the presence of the exchange type of interaction.

The way in which an interaction between nehe tron and proton of the type postulated in tu- recent theories may affect the angular distribution has been studied in some detail by Bethe and Bacher.[10] These authors have shown that if Wigner's explanation for the anomalously binding energy of the alpha-particle as compared to the deuteron (i.e., a deep narrow potential hole for the interaction function of the neutron and proton) is accepted, then the differential cross section for the scattering of neutrons by protons should be given mainly by the calculation of the

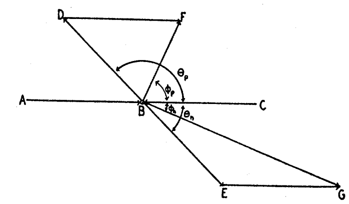


FIG. 9. Vector diagram showing relationship between coordinates in system with center of gravity B fixed and system in which recoils are observed (i.e., with center of gravity in motion). AB, path and velocity of neutron in system with center of gravity fixed; CB, path and velocity of proton in same system; BE and BD, corresponding vectors for scattered neutron and proton, respectively; θ_n and θ_p, scattering angles of neutron and proton with center of gravity at rest. By adding the motion of the center of gravity ($EG = DF$) to the vectors for the neutron and proton, the observed paths and velocities are obtained. Thus, BG and ϕ_n are the observed velocity and scattering angle for the neutron, BF and ϕ_p, like quantities for the proton. (Note that if V be the incident velocity of the neutron in the observer's system, then AB, CB, BD, BE, EG, DF, are all equal to $\frac{1}{2}V$.)

zero-order phase constant δ_0 in the expression for the cross section $d\sigma$ as derived by Mott and Massey[11]

$$d\sigma = \frac{\pi}{2k^2} \left| \sum_l (2l+1) P_l(\theta)(e^{2i\delta_l} - 1) \right|^2 \sin \theta d\theta,$$

where $P_l(\theta)$ are the familiar Legendre polynomials, l is the running index which may take all integral values, and the other symbols represent constants. This conclusion which leads to a result no different from that derived from classical theory is supposed by Bethe and Bacher to be true when the energy of the neutrons does not exceed twenty million electron volts. Hence, according to the present quantum theory, no marked asymmetry of the kind found in these experiments should be evident. The expression for the differential cross section $d\sigma$ when δ_1 is taken into account as well as δ_0 is found by Bethe and Bacher to be

$$d\sigma = \left(\frac{2\pi}{k^2}\right) \sin^2 \delta_0 (1 + 6\delta_1 \cot \delta_0 \cos \theta) \sin \theta d\theta,$$

where $\delta_1 = -(1/18)\mu(\kappa a)^3$ (hence is always negative) and $\cot \delta_0 = -\alpha/k + (\alpha^2 + k^2)a/2k$ (θ is the angle of recoil in the system with fixed center of gravity).

[11] Mott and Massey, *Theory of Atomic Collisions* (Oxford Press, 1935), p. 24.

Here, μ is a constant of order unity, a the "radius" of the deuteron, κ the reciprocal of the "wave-length" of the neutron $\lambda\!\!\!^{-}$, and α and k are functions of the deuteron binding energy, ϵ, and kinetic energy of the neutron, E. Specifically, $\alpha=(M\epsilon)^{\frac{1}{2}}/\hbar$ and $k=(ME)^{\frac{1}{2}}/\hbar$, M being the reduced mass of the neutron-proton system and $\hbar$ Planck's constant. (It is to be noted that the bar indicates division by 2π. Thus $\hbar=h/2\pi$.)

For high neutron energies (k small) the formula shows there will be preferential scattering of the neutrons backward into large angles which will be observed as a forward scattering of protons. The data of this paper as well as those of Kurie seem to indicate that such an effect is appreciable at energies much lower than predicted by the present theory. It must therefore be concluded that the data are at variance with the theory.

It is of interest to examine the way in which present data affect the mechanism given for the production of slow neutrons. A direct calculation shows that the most probable energy lost by a neutron in collision with a proton (assuming elastic collision and the observed distribution) is seven-tenths of the initial energy of the neutron as contrasted with the estimate of five-tenths if the collision is of the identical sphere type. Thus, these experiments indicate that protons are more effective than has been supposed in slowing down neutrons; i.e., a two-million-volt neutron will require about 8 collisions to lose all but two hundred electron volts on the basis of the present data, whereas 13 would be required if the billiard ball type of collision is assumed. Such a discrepancy is not sufficient to affect greatly the arguments of Fermi in regard to the effects observed in the irradiation of various substances by slow neutrons.

The lengths and angles of the proton tracks in the Wilson chamber indicate that about 25 percent of the neutrons used in this work had velocities represented by energies between 10^5 and 10^6 e-volts. A practically identical neutron source gave energies up to 14 or 15 million e-volts. A curve of distribution of velocities of those neutrons which disintegrate nitrogen nuclei is given in an earlier paper from this laboratory[12] and shows a peak in the number distribution at 6×10^6 ev. However, the peak for scattering is at a somewhat lower energy. Many more neutrons of very high energy are given by our radiothorium source than are emitted from the polonium source of the other investigators.

Note added in proof: The lowest range of the protons found in hydrogen at normal temperature and pressure in this work is one centimeter. The smallest proton range which is observable is 2 mm, which according to Blackett and Lees[13] corresponds to about 23,000 volts. To produce a proton recoil at 45° which is 2 mm long requires a neutron and energy $23{,}000/\cos^2 45°$, or about 50,000 volts, while at 65°, 130,000 volts is required. Since no neutrons of energy less than 37,000 volts were found in this work it seemed probable that no distortion of the distribution curve would occur, except at angles greater than 65°. Thus a peak of 45° should not be obscured. However, since the results of this work are in such discord with the present ideas of theorists the data will be recalculated and the results presented in a later note. In recalculations all protons of forward range less than 150 mm, and at any other angle a proton range which is less than that given by a neutron of the energy required to give a forward range of the proton equal to 15 mm in hydrogen, will be discarded. If there is any appreciable error in the results on scattering as presented in this paper it seems probable that it is due to the heterogeneity of the velocities of the neutrons.

The writers wish to thank Professors Lunn, Hoyt and Eckart of the University of Chicago, and Professor Bethe of Cornell for suggestions concerning the treatment of the data. We are indebted to the Penrose Fund of the American Philosophical Society, and to grants from funds of the National Academy of Sciences for a part of the apparatus and the radioactive source used in connection with this and other nuclear problems.

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[12] Harkins, Gans and Newson, Phys. Rev. **44**, 534 (1933).

[13] Blackett and Lees, Proc. Roy. Soc. **134**, 684 (1932).