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Nuclear Radius and Many-Body Problem

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The radioactive α-decay is discussed as a many-body problem. The nuclear radius is found to be $13\cdot 10^{-13}$ cm$\pm$10 percent, as compared to $9\cdot 10^{-13}$ in the older theory. The increased radius makes the theoretical cross section for the disintegration of heavy nuclei by deuterons agree with experiment.

I

THE problem of the natural radioactive decay has thus far been treated from the standpoint of the one-body problem: It has been assumed that the α-particle exists at all times inside the radioactive nucleus and has only to penetrate through the potential barrier in order to be emitted. In reality, the ideas of Bohr[1] and of Breit and Wigner[2] must be applied to this problem. Ordinarily, the particles in a complicated nucleus make extremely complicated motions, and only very rarely will the situation be describable as an α-particle plus a residual nucleus. The probability of α-emission consists therefore of two parts, *viz.*, the probability that the nuclear configuration is such that an α-particle may be emitted times the probability of penetration through the potential barrier. Since the former is certainly quite small compared to unity, the probability of penetration must be much larger than the observed resultant probability of α-decay, i.e., the penetration must be much more probable than has been assumed thus far. This means that the potential barriers of radioactive nuclei must be lower, and the radii larger than has been assumed.

We may write the decay constant of a radioactive nucleus

$$\lambda=\Gamma_\alpha P/\hbar, \tag{1}$$

where P is the penetrability of the potential barrier and $\Gamma_\alpha/\hbar$ the value which the decay constant *would* have if the potential barrier were absent. Γ_α would be the width of the nuclear energy level (in ergs) for no barrier.

There is, of course, no direct evidence about Γ_α. The only evidence we have for the widths of nuclear levels is from the experiments on slow neutrons. The "neutron width" of the first resonance level of Ag (short period) has been found to be about 0.0007 volt. From this value, we may estimate the width of resonance levels for fast neutrons. It is known[2, 3] that, *cet. par.*, the neutron width of a nuclear level is proportional to the velocity of the neutron, or inversely proportional to its wave-length. Furthermore it is plausible[3] that this rule holds until the neutron wave-length[4] $\lambdabar$ becomes of the order of the range of the nuclear forces, i.e., about $2\cdot 10^{-13}$ cm. Since the wave-length of a neutron of 3 volts energy is $\lambdabar=2.5\cdot 10^{-10}$ cm, we find for the neutron width of a nuclear level from which fast neutrons may be emitted,

$$\Gamma_n=0.0007\cdot 2.5\cdot 10^{-10}/2\cdot 10^{-13}\sim 1 \text{ volt}. \tag{2}$$

[1] Bohr, Nature **137**, 344 (1936).
[2] Breit and Wigner, Phys. Rev. **49**, 519 (1936).
[3] Bethe and Placzek, Phys. Rev., to appear shortly.
[4] $\lambdabar$ denotes the ordinary wave-length divided by 2π.

It is not easy to see in what way the α-width will differ from the neutron width. On one hand, the α-width may be expected to be smaller, because it seems less probable that an α-particle is formed than that the nucleus splits into a neutron plus a residual nucleus. Furthermore, it may be expected that the widths of the nuclear levels decrease with increasing atomic weight, along with the decrease of the spacing between the levels,[5] and are therefore smaller for radioactive nuclei than for Ag. On the other hand, it might be argued that it is more difficult to concentrate all the energy of a highly excited nucleus on one neutron than to emit an α-particle from a nucleus in the ground state. Arguments could further be given to invalidate the last two points.

In the absence of any conclusive evidence, we assume the α-width $\Gamma_\alpha = 1$ volt, i.e., of the same order as the neutron width calculated above. We must admit that this value may be wrong by about a factor 100 either way. With this value for Γ_α and the observed decay constants, we may calculate the radii of the radioactive nuclei in the usual straightforward way. We obtain in the average

$$R = 13 \cdot 10^{-13} \text{ cm} \tag{3}$$

as compared to the value $R = 9 \cdot 10^{-13}$ cm derived from the one-particle model of the nucleus.

The values of R derived from various α-radioactive nuclei are very nearly the same,[6] just as in the old model of Gamow and of Condon and Gurney. In other words, the Geiger-Nuttall relation is not affected by the change of the nuclear radius. This can be seen from the well-known approximate formula for the penetrability[7]

$$\log P = -\frac{2\pi z Z e^2}{\hbar v} + \frac{4e}{\hbar}(2MzZR)^{\frac{1}{2}} \tag{4}$$

in which Ze is the charge of the nucleus, and ze, M and v are the charge, the mass and the velocity of the α-particle. P consists, therefore, of one factor depending on the velocity and not on the radius, and another depending on the radius and not on the velocity. The first is the basis of the Geiger-Nuttall relation. The second factor is changed due to the change in the nuclear radius, but the change has the same amount for all radioactive nuclei and just balances the factor introduced in Γ_α by using the many-body rather than the one-body approximation.

The uncertainty in R due to the uncertainty of Γ_α may be estimated from (4). Since the decay constant λ is given experimentally, the penetrability P is just as uncertain as Γ_α, i.e., by a factor 100. Now, according to (4), small variations of R and P are connected by the relation

$$\frac{\delta P}{P} = \frac{\delta R}{R}\frac{2e}{\hbar}\left(\frac{2MzZ}{R}\right)^{\frac{1}{2}}. \tag{5}$$

Inserting the numerical values, this gives

$$\delta P/P \approx 50\delta R/R. \tag{5a}$$

An uncertainty of $\pm \log 100 = \pm 4.6$ in $\log P$, corresponds therefore to an uncertainty of about ± 10 percent in the nuclear radius.

II

The increase in the nuclear radius will, of course, change all quantities in nuclear physics whose estimates are based on the value of R. We mention only the calculations of the density of nuclear energy levels,[5] of the Coulomb energy in heavy nuclei, the nuclear surface tension, and the semiempirical formula for nuclear binding energies.[8] The latter has to be replaced by

$$E = -4.60A + 0.4I + 9.9A^{\frac{2}{3}} + 13.7I^2/A + 0.43Z^2A^{-\frac{1}{3}}, \tag{6}$$

where A is the mass number, Z the charge and, $I = A - 2Z$ the isotopic number of a nucleus. E is the "mass excess" in thousandths of a mass unit.

The change in the nuclear radius has also an important consequence for the probability of transmutations. It is well known that heavy nuclei (Pt, Bi) have been disintegrated by deuterons of 4 to 5 MV. The cross sections for these disintegrations are quite large,[9] about

[5] Bethe, Phys. Rev. **50**, 332 (1936).

[6] A table will be given in the report on Nuclear Physics, part B, by Bethe and Livingston, to appear in Reviews of Modern Physics, 1937.

[7] Strictly speaking, a more complicated formula must be used because the energy of the α-particle is not small compared to the height of the potential barrier. However, the correction term is not very different for different radioactive nuclei.

[8] Cf., e.g., Bethe and Bacher, Rev. Mod. Phys. **8**, 166 (1936), Eq. (185a).

[9] I am indebted to Professor Lawrence for this communication.

10^{-28} cm². This is much more than could be expected with the old radii but fits in quite well with the new ones.

According to the Breit-Wigner formula,[2, 3] the probability of a disintegration by a deuteron of energy E is

$$\sigma = \pi \lambda_d^2 \frac{\gamma_d \gamma_p}{(E-E_0)^2+\frac{1}{4}\gamma^2}, \tag{7}$$

where E_0 is the energy corresponding to the nearest resonance level of the compound nucleus, γ the total width of this level, γ_d the part of the width corresponding to the emission of a deuteron by the compound nucleus, γ_p the width corresponding to the emission of the outgoing particle, and λ_d the wave-length of the deuteron. It has been assumed that, for each deuteron energy, only one resonance level is important (cf. reference 3). If we average (7) over the deuteron energy we obtain

$$\sigma_v = 2\pi^2 \lambda_d^2 \gamma_d \gamma_p / \gamma\Delta, \tag{8}$$

where Δ is the spacing between adjacent levels of the compound nucleus. This formula must be corrected for the possibility that deuterons of various angular momenta up to $l \approx R/\lambda$ may enter the nucleus while (7) was derived for zero angular momentum of the incident particle; as to order of magnitude, this will correspond to an additional factor of about $(R/\lambda)^2$ in the cross section, yielding

$$\sigma_0 = 2\pi^2 R^2 \gamma_d \gamma_p / \gamma\Delta. \tag{9}$$

Δ is, then, the spacing between nuclear levels of given angular momentum.

If the emission of particle p is a probable process, we may put approximately $\gamma_p = \gamma$ thereby slightly overestimating σ_0. Furthermore, we may split γ_d into a factor representing the penetration through the potential barrier, P_d, and a second factor Γ_d which would be the "deuteron width without barrier" (reduced deuteron width). It is plausible to assume that Γ_d is somewhat smaller than the spacing Δ of the nuclear levels. We have now

$$\sigma_0 \approx 2\pi^2 R^2 P_d \Gamma_d / \Delta. \tag{10}$$

Now the penetrability of the potential barrier for deuterons, P_d, may be calculated in a straightforward way. Assuming the nuclear volume to be proportional to the number A of the particles in the nucleus, we find for a deuteron energy of 4.5 MV if the nuclear radius is $R = 9 \cdot 10^{-13}$ cm for $A = 220$:

$$P_d = 4 \cdot 10^{-7}, \qquad \sigma_0 = 5 \cdot 10^{-30} \Gamma_d / \Delta \tag{11a}$$

if $R = 13 \cdot 10^{-13}$ cm for $A = 220$:

$$P_d = 10^{-4}, \qquad \sigma_0 = 3 \cdot 10^{-27} \Gamma_d / \Delta. \tag{11b}$$

In the latter case, the observed cross section of 10^{-28} cm² can therefore be explained by the plausible assumption $\Gamma_d = 0.03\Delta$, whereas, with the old nuclear radius, we would have to assume the "reduced" deuteron width Γ_d to be 20 times larger than the spacing of the levels which seems very unreasonable. This only confirms the almost obvious statement that the Bohr-Breit-Wigner theory of nuclear transmutations can only work if it is consistently applied, and in particular if the nuclear radius is calculated using the same theory.