

ording to whether the 1S state of the deuteron ($\pm 120,000$ volts) is assumed to be real or virtual the theoretical value of τ is 6.5×10^{-4} or 2.6×10^{-4} . The satisfactory agreement of this last value with the experimental result, seems to indicate that the 1S state is virtual.

We express our warmest gratitude to the Istituto di Sanita-Pubblica of Rome and in particular to Professor G. C. Trabacchi for having supplied us with the radon sources used in this research. Our thanks are also due to the Consiglio Nazionale delle Ricerche of Italy for a grant.

NOVEMBER 15, 1936

PHYSICAL REVIEW

VOLUME 50

Characteristics of the Compton Modified Band

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(Received August 31, 1936)

The shapes of Compton modified bands have been calculated for all scattering elements from hydrogen to argon. These shapes vary periodically with atomic number, atoms with one-valence electron being characterized by sharp, narrow lines while atoms with completed electron shells produce broadly rounded bands. Experimental results, though incomplete, confirm these variations except as to absolute breadth. Breadths of observed modified bands exceed calculated breadths on the average by 75 percent for carbon scatterers and 140 percent for beryllium. Variation of scattering angle has only a second-order effect upon the

observational breadth of wide modified bands. To an accuracy of about 10 percent the intensity ratio of modified to unmodified radiation from carbon (graphite) scatterers is found to agree with predictions based upon the Wentzel-Waller theory of scattering by bound electrons for wavelengths in the range 435 X.U. to 710 X.U. and scattering angles between 27 and 139°. Though previous measurements showed the magnitude of the wave-length shift of the maximum in the modified band to be less than that given by the Compton equation it is now argued that present theory predicts a still smaller shift.

ENERGY DISTRIBUTION IN THE MODIFIED BAND

JAUNCEY'S¹ suggestion that the distribution of energy with wave-length in Compton modified radiation is a direct result of the distribution of momenta among the electrons of the scatterer has been confirmed and greatly extended by others.²⁻⁴ This hypothesis has been used for the theoretical prediction of the shapes of shifted bands (or lines) upon bases of atomic theory, and while Jauncey's specific predictions are not now acceptable, being derived from Bohr orbits, DuMond^{2, 4} has shown that line shapes may be deduced from any given electron momentum distribution, and has illustrated the method by theoretical shapes pertaining to scattering by carbon and beryllium atoms with electron momenta distributed as required by modern atom models.

Bloch⁵ gave special attention to the modified radiations scattered by carbon and by beryllium and developed formulae for the calculation of the intensity distributions upon the basis of Wentzel's⁶ theory of the Compton scattering by bound electrons. Hydrogen scattering has recently been treated by Schnaidt⁷ and neon by Burkhardt,⁸ intensity (or energy) distributions being obtained in both cases. The interest of the present writers has been not only in the form of the distribution for a specific element but also in the nature of the variations of this form with change of the atomic number of the scatterer, and for investigating this matter the method of DuMond,⁴ utilizing the general momentum distribution formula of Podolsky and Pauling,⁹ is most appropriate, notwithstanding its restriction to atoms which may be regarded as hydrogen-like and which are free from the entangling alliances of the crystal state.

¹ G. E. M. Jauncey, *Phys. Rev.* **25**, 314 and 723 (1925).

² J. W. M. DuMond, *Phys. Rev.* **33**, 643 (1929).

³ J. W. M. DuMond and H. A. Kirkpatrick, *Phys. Rev.* **37**, 136 (1931); S. Chandrasekhar, *Proc. Roy. Soc. A* **125**, 231 (1929); J. W. M. DuMond and H. A. Kirkpatrick, *Phys. Rev.* **38**, 1094 (1936).

⁴ J. W. M. DuMond, *Rev. Mod. Phys.* **5**, 1 (1933).

⁵ F. Bloch, *Phys. Rev.* **46**, 674 (1934).

⁶ G. Wentzel, *Zeits. f. Physik* **43**, 1 (1927).

⁷ F. Schnaidt, *Ann. d. Physik* **21**, 89 (1934).

⁸ G. Burkhardt, *Ann. d. Physik* **26**, 567 (1935).

⁹ B. Podolsky and L. Pauling, *Phys. Rev.* **34**, 109 (1929)

Using screening data from Slater¹⁰ we have calculated the energy distributions in modified bands from scattering atoms with atomic numbers from 1 through 18. This method of calculation ignores all higher order effects productive of line asymmetry and it therefore suffices to plot half of each line only, as has been done in Fig. 1. Theories^{2, 5, 6} of the structure of the modified band agree in predicting line profiles whose functional forms to a good approximation depend only upon the nature of the scattering atom, being independent of wave-length λ or angle of scattering φ . These variables affect only the wave-length scale, that is, the breadth of the line. Since it is further indicated that line breadths should be practically proportional to $\lambda(1 - \cos \varphi)^{\frac{1}{2}}$ the curves of Fig. 1, though furnished with an abscissa scale correct for a particular wave-length and scattering angle, may be adjusted to represent theoretical expectations for any desired angle and for any wave-length within the usual observational range for the Compton effect.

The computed curves show how the modified line reflects the process of atom building by the addition of successive electrons. Generally speaking, a wide line or a line which is in part wide denotes the existence of electrons of large momentum, while a steep, narrow maximum is evidence of the presence of relatively slow electrons. The helium curve is broader than that of hydrogen since increase of nuclear charge causes the momentum per electron to increase. The lithium line exhibits a helium-like base with a narrow superposed spectrum due to scattering by the slower, single *L* electron. With the binding of additional electrons the scattered line loses its sharply peaked character since the *L* electrons are increasing in both individual momentum and numerical importance. At neon the contribution of the *K* electrons is inappreciable except at the wings of the curve, the profile as a whole being of the helium type. The next electron, however, produces, in sodium, a narrow maximum like that of lithium, after which the smoothing and broadening process ensues and at argon there recurs a smooth, broad line resembling those of the other rare gases.

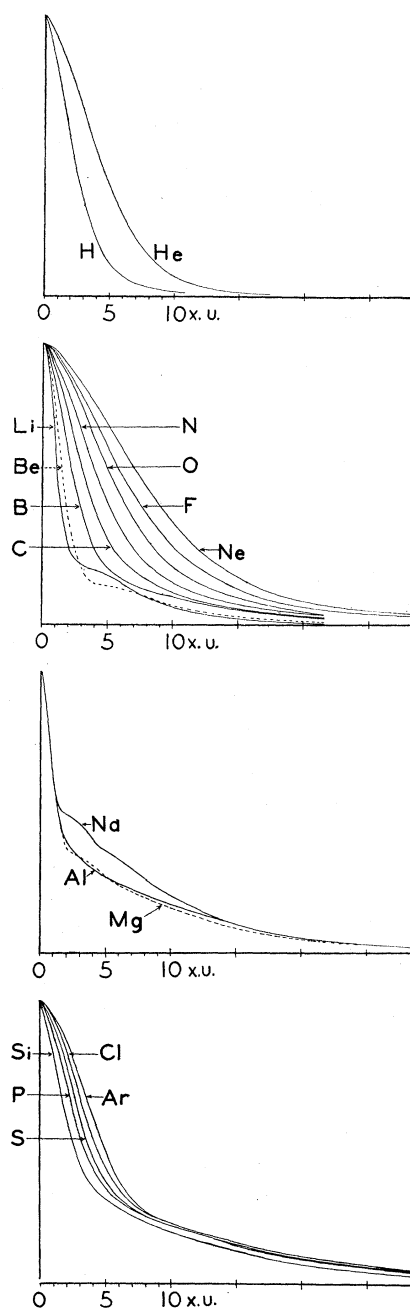


FIG. 1. Theoretical half-profiles of Compton modified bands from free atoms with consecutive atomic numbers from 1 to 18. Angle of scattering: 90° . Wave-length: 500 X.U. Abscissa scale is the same for all curves. Ordinate scales are arbitrarily selected to make curves coincide at maxima. Profiles for other wave-lengths and scattering angles may be obtained by simple alterations of the abscissa scale.

Experimental data are not now available for checking these curves in all particulars. All de-

¹⁰ J. C. Slater, Phys. Rev. **36**, 57 (1930).

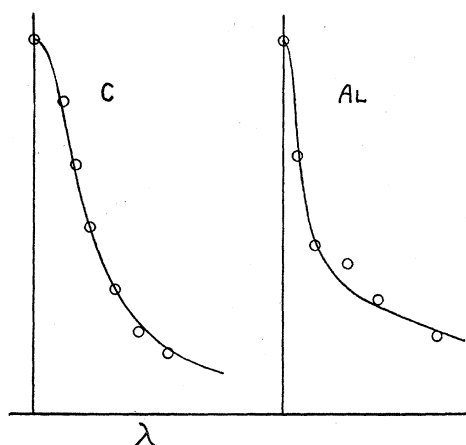


FIG. 2. Theoretical line shapes from Fig. 1 shown in comparison with corresponding spectrometer observations. The comparison is significant as to shape but not as to width since the experimental curves have been arbitrarily narrowed to fit the theoretical curves at half-maximum height.

tails of shape were missed by Woo¹¹ in his pioneer survey of scattered spectra from fifteen elements, since it suited his purposes to sacrifice resolving power for intensity. In general other observers have made the same choice. Kappeler,¹² however, has observed the scattering of Mo $K\alpha$ by gaseous neon under conditions favorable to the study of line form, and his results agree most satisfactorily in shape and width with their theoretical counterpart in Fig. 1.

When solid scatterers are used we find as did DuMond² that observed modified lines are broader than those predicted from theories of the free atom. This matter will come up in a later paragraph but it is relevant to note here that in spite of this broadening the modified lines from solid scatterers of differing atomic number, when observed with good resolving power, exhibit conspicuous differences in form such as appear in Fig. 1. In Fig. 2 the carbon and aluminum curves from Fig. 1 are repeated in conjunction with a few observed points for each case. The wavelength scales of the observations have been contracted relatively to that of the theoretical curves in order to make the comparative shapes more

conspicuous. The carbon and aluminum theoretical curves are quite different in type and cannot be made to coincide by any linear manipulation of their abscissa scales, and the observed lines from carbon and aluminum evidently differ similarly. The degree of agreement between the points and the curves is a little surprising in view of the fact that the observations were not corrected for the broadening and presumably somewhat distorting instrumental effects referred to in the next section. We were first impressed by the difference between the sharply peaked aluminum lines and the more broadly rounded lines from carbon upon discovering that the $K\alpha$ doublet of molybdenum, which always appears as a single, broad line in the modified radiation from graphite, may readily be resolved into two sharp and well separated maxima in the modified scattering from aluminum.

It may be concluded that differences in the form of the Compton band characteristic of different scattering elements are not only theoretically predicted but are also experimentally observable with solid scatterers, and that comparisons of predictions with observations may be expected to yield information about the momenta of electrons in conducting and nonconducting solids.

BREADTHS OF MODIFIED LINES

Upon examining line breadths at half the maximum height in Fig. 1 it is seen that this quantity varies periodically with atomic number, attaining maxima at the rare gases and decreasing abruptly thereafter with increasing Z . This behavior is in general agreement with Kappeler's¹² wide neon line and with spectra from solid scatterers of beryllium, carbon, sulfur and aluminum, obtained in this laboratory, but has not been precisely confirmed as yet throughout a wide range of atomic numbers.

Using the ring-target x-ray source¹³ and a double spectrometer¹⁴ we have secured some fifty spectrum curves representing three different wave-lengths, two angles of scattering, and two scattering elements, carbon (graphite) and beryl-

¹¹ Y. H. Woo, Phys. Rev. 27, 119 (1926).

¹² In Burkhardt's paper (reference 8) Kappeler's experimental line is compared with three theoretical shapes, one of which is equivalent to the neon curve in our Fig. 1. A full account of Kappeler's work on gaseous and solid scatterers has recently appeared in Ann. d. Physik 27, 129 (1936).

¹³ P. Kirkpatrick and P. A. Ross, Rev. Sci. Inst. 4, 645 (1933).

¹⁴ P. A. Ross, Rev. Sci. Inst. 3, 253 (1932).

lium. These spectra were previously¹⁵ discussed in another connection but we have found them suitable for breadth measurement and are presenting the results in figures and Table I. In addition to the modified line broadening which distinguishes crystalline scatterers from those composed of free atoms there are in practice breadth increments produced by the wave-length breadth of the incident radiation, the finite resolving power of the spectrographic or spectrometric apparatus, and the range of scattering angles simultaneously effective.

The first of these additional effects is entirely negligible in the present connection. The second produces a small effect whose magnitude depends upon what may be called the spectrometer transmission pattern, that is, the curve of transmitted intensity of monochromatic radiation *vs.* dihedral angle between the spectrometer crystals. The actual shape of this curve is not known for any spectrometer, but with the limited resolution afforded by our polished crystals it was permissible to take as its width the angular width of unmodified lines, which was 1.6'. Following Ehrenberg and Mark¹⁶ we assume that both the spectrometer transmission pattern and the modified line may be approximated by Gaussian curves, in which case, as these authors show, the square of the width of the observed line equals the sum of the squares of the corrected width and the width of the spectrometer transmission pattern. Upon this basis corrections for spectrometer effects have been made in Table I. Since the corrections are well within the probable errors of observation the approximations mentioned above are deemed unobjectionable.

Line broadening because of variation of scattering angle may sometimes be rendered negligible by a proper choice and restriction of the angle, but in the general case special consideration is necessary. The Compton equation shows that a variation of scattering angle, $d\phi$, causes the theoretical position of the center of the shifted line to occur simultaneously over a wave-length range

$$d\lambda = (h/mc) \sin \phi d\phi. \quad (1)$$

Analysis shows that the effect on the observed

line is to increase its half-maximum width *not* by $d\lambda$ but by a smaller amount which depends upon the true shape which the line would possess for a single-valued scattering angle. In order to correct observed lines for excess width from this cause it is then necessary to have some knowledge or assumption about the real shape of modified lines when freed from these experimental effects. Since only the approximate line form is required it has been assumed that the real line may be represented by $y = a/(1 + (x/b)^2)$, a convenient function¹⁷ which happens to fit the carbon curve of Fig. 1 rather well. In this expression a and b are constants and x is the wave-length variable, with origin at the center of the line. The ordinate of a modified line resulting from a variable scattering angle will then be represented by the integral of $y dx$ over a range of x sufficiently wide to correspond to $d\phi$. The equation of the resultant curve which is obtained upon integrating is found to possess a half-maximum breadth significantly greater than the breadth of the elementary (integrand) curves *only* when the latter breadth is small compared to $d\lambda$. In other words, the breadths of modified curves as observed are almost independent of variability of scattering angle so long as real line width is large compared to the wave-length range (Eq. (1)) introduced by the variability. As a special example, if the wave-length range due to variation of scattering angle is one-third the observed line width then it is found that a three percent correction is required to reduce the observed line width to the width for constant scattering angle.

It is not suggested that this analysis renders correction unnecessary but rather that the smallness of the correction makes unnecessary the frequently impracticable task of determining the range of scattering angles with high precision. We have calculated and applied the correction in all cases to follow in this section.

Line ordinates for locating the half-maximum height were measured from a somewhat indefinite background level whose determination probably constituted the most important source of error in our breadth measurements. The result of the various corrections is shown in Table I. Corrected breadths are thought to be good to one or two X units. These results are shown in comparison to

¹⁵ P. A. Ross and P. Kirkpatrick, Phys. Rev. **46**, 668 (1934).

¹⁶ W. Ehrenberg and H. Mark, Zeits. f. Physik **42**, 807 (1927).

¹⁷ A. Hoyt, Phys. Rev. **40**, 477 (1932).

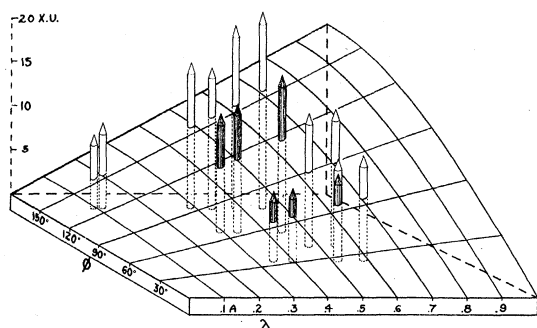


FIG. 3. Widths at half-maximum height of modified bands scattered by carbon, shown as a function of wavelength and angle of scattering. The cross-ruled surface gives theoretical widths for free carbon atoms (except that the assumptions underlying the theory are not well fulfilled for very short wave-lengths) while the vertical columns show measured widths obtained with graphite scatterers. Shaded columns represent the new measurements, corrected for instrumental broadening, while unshaded ones are from uncorrected measurements of DuMond and H. A. Kirkpatrick.

theoretical breadths in Figs. 3 and 4. Fig. 3 also includes uncorrected breadth measurements by DuMond and H. A. Kirkpatrick.^{3, 18}

It is evident that our experimental breadths with graphite and beryllium agree with free atom theory in respect to the linear variation of breadth with wave-length, and that they disagree but slightly in the functional variation of breadth with angle of scattering. As to absolute breadths the measured lines are found to be broader than

¹⁸ These quoted data are not in need of significant correction for variation of the scattering angle since this angular range was kept small. Small corrections for spectrograph characteristics, comparable to those which we have used in Table I, might appropriately have been applied, though they would have been somewhat smaller than what we judge to be the observational probable errors.

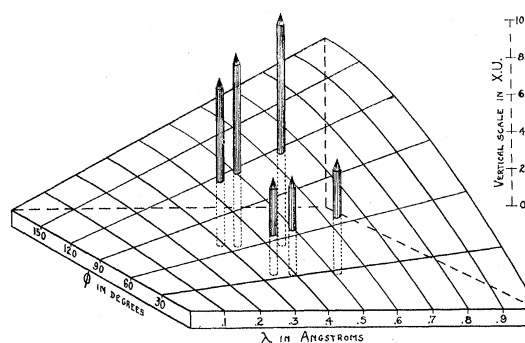


FIG. 4. Theoretical and observed modified band widths for beryllium scatterers, represented as in Fig. 3 (though with a different vertical scale).

those of theory by an average value of 75 percent with carbon and 140 percent in beryllium, the discrepancies for the two substances standing, perhaps significantly, in the order of their conductivities.

These results support and extend those which DuMond and H. A. Kirkpatrick¹³ obtained by very dissimilar experimental means. Since Mo $K\beta$ was one of the radiations used in both investigations it is possible for this case to compare the two sets of results on one graph, as in Fig. 5. This agreement is of some interest since experimenters have heretofore obtained very discordant results in the measurement of modified line breadths. Bearden¹⁹ and Gingrich²⁰ have reported modified lines from graphite which are very much too narrow to be at all consistent with the theoretical surface of Fig. 4, not to mention the associated experimental data. Gingrich obtained clear reso-

¹⁹ J. A. Bearden, Phys. Rev. **36**, 791 (1930).

²⁰ N. S. Gingrich, Phys. Rev. **36**, 1050 (1930).

TABLE I. Observed and corrected widths of the Compton modified bands for various lines and scatterers.

Radiation	Scatterer	Mean Angle of Scattering	Observed Width	Corrected for Spectrometer Effects		Corrected for range of angle of scattering
Sn $K\beta$	Be	64°	8.4 min.	8.2 min.	7.2 X.U.	5.2 X.U.
Sn $K\beta$	Be	116°	11.8	11.7	10.3	9.0
Ag $K\beta$	Be	64°	8.5	8.3	7.3	5.3
Ag $K\beta$	Be	116°	13.3	13.2	11.6	10.3
Mo $K\beta$	Be	64°	9.2	9.1	8.0	6.2
Mo $K\beta$	Be	116°	15.4	15.3	13.5	12.4
Sn $K\beta$	C	64°	11.0	10.9	9.6	8.1
Sn $K\beta$	C	116°	16.6	16.5	14.5	13.5
Ag $K\beta$	C	64°	11.3	11.2	9.9	8.5
Ag $K\beta$	C	116°	17.7	17.6	15.5	14.6
Mo $K\beta$	C	64°	12.8	12.7	11.2	10.0
Mo $K\beta$	C	116°	21.3	21.2	18.6	17.9

lution of the Mo $K\alpha$ doublet in the modified band upon scattering from graphite at about $165^\circ 25'$. The theoretical width of each component of this modified doublet, assuming perfect spectrometer resolving power, no variability of scattering angle and free carbon atoms, is 13 X.U. at half-maximum. Since the spectral separation is only 4.3 X.U. no resolution is to be expected even with these unrealizable ideal conditions.

INTENSITIES OF MODIFIED LINES

Theories of the Compton process provide many predictions about modified line intensity which are as yet untested by experiment. The areas of the spectrum curves just discussed were determined by planimeter measurement for the purpose of obtaining the relative intensities of modified radiation scattered at the available supplementary angles 64° and 116° . According to the Wentzel⁶-Waller²¹ theory (for free atoms) this ratio is not sensitive to wave-length variation and remains in the neighborhood of unity for the three wave-lengths and two scattering materials used in this work. Our data when corrected for absorption in the scatterer give, for all six cases, ratios equal to unity plus or minus five or six percent but do not permit more decisive checking chiefly because of the difficulty of locating the base lines of the curves.

The relative intensity of modified to unmodified radiation has been determined for a graphite

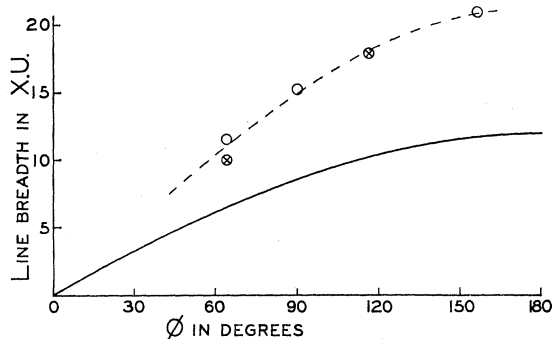


FIG. 5. Breadth at half-maximum height of Mo $K\beta$ ($\lambda = 631$ X.U.) modified by scattering from graphite, showing relation between present work (crosses) and previous measurements by DuMond and Kirkpatrick (open circles), who used very dissimilar experimental methods. A smooth curve is fitted to the points. The curve lying below the observed points presents theoretical expectations for a scatterer of free carbon atoms.

²¹ I. Waller, Phil. Mag. 4, 1228 (1927).

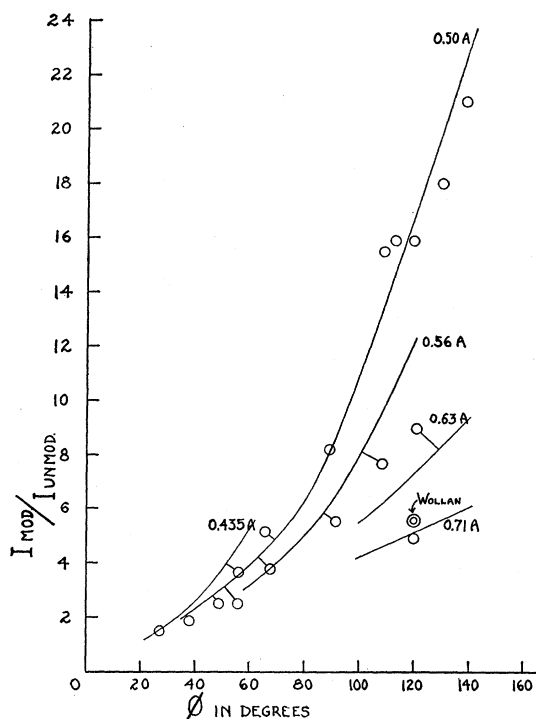


FIG. 6. Relative intensities of modified and unmodified scattering from carbon as a function of incident wave-length and angle of scattering. The curves show the predictions of the Wentzel-Waller theory of scattering while circles indicate observations with graphite scatterers.

scatterer and a variety of wave-lengths and angles, using some of the data already described and many additional spectra. As in the measurements of the previous paragraph, the shifted and unshifted lines obtained from spectrometer observations were plotted and the ratio of the areas of the lines was taken as the ratio of emitted intensities. Two slight systematic errors in this practice, due to differential absorption in the scatterer and to variation of spectrometer sensitivity with wave-length, are both smaller than the accidental observational errors, and since they are furthermore opposed as to sign we have applied no corrections for them.

For a wave-length λ scattered at the angle φ from atoms of atomic number Z , Wentzel's theory of scattering as extended by Waller furnishes the expression

$$I_{\text{mod}}/I_{\text{unmod}} = (Z - \sum f_n^2) / (1 + (h/mc\lambda) \text{vers } \varphi)^3 F^2.$$

Here F is the structure factor of the atom as a whole and $\sum f_n^2$ is a summation of the squares of

the individual structure factors of the atomic electrons, taken over the Z electrons present. The expression given omits a term, negligible here, which Waller deduced from the Pauli principle. Values of this modified to unmodified ratio have been computed, using carbon structure factor data from James and Brindley,²² with results which are plotted in Fig. 6 with the experimentally determined ratios.

Though a few individual points show striking percentage discrepancies the agreement between theory and experiment is on the whole as complete as the experimental and theoretical uncertainties could permit. For large scattering angles or small wave-lengths theoretical predictions may be in error by ten percent or more because of uncertainty as to F , and the scatter of the observed points in Fig. 6 suggests that the probable error of experiment may be of similar magnitude. Although a majority of the observations fall *below* the theoretical curves there is no definite systematic discrepancy here such as one finds in comparing observed and theoretical *widths* in Figs. 3 and 4. Since we are not in fact dealing with the free atoms of theory but with a crystal lattice, such a discrepancy might not cause surprise. However, it is to be realized that for the angles and wave-lengths represented here the unmodified scattering is mostly by the undisturbed K electrons while the intensity of modified scattering is not very responsive to electronic rearrangement. The *widths* of modified lines on the other hand are almost entirely controlled by L electron scattering and should indeed be sensitive to the alteration of momentum which accompanies this change of physical state.

Prior measurements of the ratio of modified to unmodified intensity have been made by Ross,²³ Defoe,²⁴ Woo,^{11, 25} Jauncey and Boyd,²⁶ Albrecht,²⁷ Wollan,²⁸ and Backhurst.²⁹ Though these data disclose no exception to the rule that the intensity ratio increases with angle and de-

creases with wave-length they are extremely discrepant as to numerical values. For carbon scatterers within the wave-length range of Fig. 6 published ratios are quite generally distinctly lower than the Wentzel-Waller values. Ross and Woo, using, respectively, photographic and ionization single-crystal spectrometers, obtained ratios less than half as large as those required by the theory. We are unable to account for the low results of these early experiments but cannot agree with Wollan that they are a reasonable consequence of the differing widths of the two lines. Defoe and Jauncey and Boyd used a single filter method in which unfortunately the errors of observation become enormously amplified in the modified-unmodified ratio as this ratio becomes large in comparison to unity. Where this difficulty does not apply their results (at least for carbon) are found to be below those of theory. Albrecht used the short wave-length 0.21A exclusively and his results are therefore not comparable with those of the present report. Wollan, using an extension of Ross'³⁰ balanced filter method, determined the carbon modified-unmodified ratio for a single wave-length and angle, obtaining a value in good agreement with theory and with the corresponding determination of this work. Backhurst's method was a refinement of that of Defoe and is subject to the same criticism for large values of the ratio. His single datum relevant to the range of this investigation (0.395A, scattered by carbon at 150°) gives a ratio of 12.5 which is obviously much too small to conform to either the observations or the theoretical curves of Fig. 6. Backhurst's neglect of the real breadth of the modified band has the effect of depressing his ratios to some slight extent.

We have also made a number of measurements of the intensity ratio for scatterers of Al, Mg and S, but these relatively poor scatterers present special difficulties which have restricted present conclusions to the statements that the modified-unmodified ratios for these elements are several times smaller than for carbon; that the ratios for Al and Mg are identical within a few percent (with $\lambda=0.5A$), and that the sulfur ratio is still lower.

Although present results are in agreement with Wollan in promoting confidence in the Wentzel-

²² R. W. James and G. W. Brindley, *Phil. Mag.* **12**, 81 (1931).

²³ P. A. Ross, *Proc. Nat. Acad. Sci.* **11**, 569 (1925).

²⁴ O. K. Defoe, *Phys. Rev.* **27**, 675 (1926).

²⁵ Y. H. Woo, *Phys. Rev.* **28**, 426 (1926).

²⁶ G. E. M. Jauncey and R. A. Boyd, *Phys. Rev.* **28**, 620 (1926).

²⁷ E. Albrecht, *Zeits. f. Physik* **57**, 326 (1929).

²⁸ E. O. Wollan, *Phys. Rev.* **43**, 955 (1933).

²⁹ I. Backhurst, *Phil. Mag.* **17**, 321 (1934).

³⁰ P. A. Ross, *Phys. Rev.* **28**, 425 (1926).

Waller theory as applied the relative intensities of modified and unmodified scattering it cannot be claimed that the experimental evidence is as yet either complete or satisfactorily concurrent.

WAVE-LENGTH OF THE MODIFIED BAND

Previously published measurements¹⁵ of the magnitude of the Compton shift showed that the wave-length of maximum intensity is separated from the unmodified line by a wave-length interval slightly less than that given by the Compton formula,³¹ the difference being found proportional to the square of the wave-length of the incident radiation. This shift defect was ascribed to the binding energy of atomic electrons³² and Bloch⁵ found it possible, upon certain approximately fulfilled assumptions, to calculate its

³¹ In a summary of evidence on the question of the correctness of the original Compton shift formula Compton and Allison (*X-Rays in Theory and Experiment*, p. 210) assigned chief weight to the results obtained by Gingrich (see reference 20). It is suggested here that consideration discussed above (p. 933) weaken the force of this evidence.

³² P. A. Ross and P. Kirkpatrick, *Phys. Rev.* **45**, 223 (1934).

magnitude. The defect has also been theoretically justified by Burkhardt.⁸

The quantitative agreement between theory and the observations, however, is now somewhat less satisfactory than it appeared to be in 1934, for the following reasons: (1) Errors in calculation have been discovered which when corrected increase the theoretical defects by some 10 percent. (2) The choice of numerical values of binding energies for use in the theoretical calculations has been questioned. Any alteration of the values used must be in such a sense as to increase further the theoretical defects. (3) Burkhardt has called attention to a neglected source of shift defect associated with the sign of the electron momentum, which requires a further increase in the theoretical value of the defect.

It is not certain that this widening gap between predicted and observed shift defects can be explained by the fact that the scatterers have actually been solids whereas theories have been concerned with free atoms. Precise shift measurements using monatomic gaseous scatterers are expected to clarify this point.

Columnar Ionization

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(Received September 5, 1936)

The ionization collected from single alpha-particles has been measured as a function of the angle between the path of the particle and the direction of the electric field. Data obtained for electric field strengths of 8, 100 and 500 volts per cm and for air pressures of one and two atmospheres are given. These results have been compared with curves based on the theory given by Jaffé. The agreement between theory and experiment is satisfactory and shows clearly that for low field strengths the loss of ions by recombination is appreciable even for large angles between particle path and electric field. In contrast with values of the recombination coefficient given by Jaffé, it is indicated in the present work that previously accepted values for this quantity are correct. A description of the FP-54 Pliotron circuit which has been used for this work is given.

THE theory for the fraction of ions collected from a strongly ionized column of gas by an electric field has been developed by Jaffé.^{1, 2} In this theory it is assumed that the electric field separates the intensely ionized column into two

parts, one of positive ions and the other of negative ions. During separation these columns broaden by diffusion, and, wherever they overlap, there is a loss of ions by recombination to the extent given by the recombination coefficient.

The expressions given by Jaffé for the ratio of the ions collected to those formed are as follows:

¹ Jaffé, *Ann. d. Physik* **42**, 303 (1913).

² Jaffé, *Physik. Zeits.* **30**, 849 (1929).