NOVEMBER 15, 1936 PHYSICAL REVIEW VOLUME 50

On the Absorption and the Diffusion of Slow Neutrons

E. AMALDI AND E. FERMI, *Royal University of Rome, Rome, Italy*

(Received September 5, 1936)

The study of the absorption properties of slow neutrons establish the existence of absorption bands which correspond to energy intervals sufficiently narrow to allow one to isolate relatively homogeneous neutron groups. In this work, we describe several experiments and considerations designed to disclose the characteristics of the absorption and scattering properties of the various groups. Summary: 1. Introduction. 2. Measurements and their reduction. 3. Selective absorptions. 4. Slow neutron groups. 5. Systematic absorption measurements. 6. Albedo. 7. Scattering of the neutrons of a single group. 8. Mean free path of thermal neutrons. 9. Groups as a function of the distance from the source; energy ratios. 10. Passage of the neutrons from one group to another. 11. Behavior of the groups in the neighborhood of the surface of the paraffin. 12. Width of the energy bands corresponding to the groups. Total number of neutrons. 13. Summary and discussion.

1. INTRODUCTION

THE purpose of this work is to describe systematically the researches we have carried out on the absorption and scattering properties of slow neutrons.[1] It is known that the simplest theory[2] on the probability of capture of a neutron by a nucleus leads to the prediction that the capture cross section is, for small neutron velocities, inversely proportional to the velocities. However, the constant of proportionality can vary between large limits from element to element.

If this were true, thicknesses of two different elements inversely proportional to their constants would be exactly equivalent as absorbers of slow neutrons independently of neutron velocity, and of the substance used as detector. That the phenomenon was not so simple was early observed by various authors,[3] who showed that generally the absorption by an element was greater when the slow neutrons were detected by means of the activity induced in the same element.

The thermal experiments of Moon and Tillman and others,[4] which showed that different detectors exhibit different sensitivities with a variation of the temperature of the paraffin, lead to a similar conclusion. Finally, more recently, Rasetti, Segrè, Fink, Dunning and Pegram[5] have measured the dependance of the absorption coefficient of thermal neutrons on their velocity by means of a mechanical arrangement, and have found that, at least for Cd, the $1/v$ law is not fulfilled. In a systematic study of these phenomena, we have found that the selective absorption of slow neutrons is accentuated by filtering with a sheet of cadmium of sufficient thickness.[6] This fact, observed independently by Szilard,[7] pointed to the possible existence of relatively narrow absorption bands characteristic of various elements.

A systematic description of these phenomena can be obtained by means of an analysis of the slow neutron radiation into groups which are approximately homogeneous with respect to their absorption properties.[8] Such analysis could be performed by decomposing the absorption

---

[1] E. Amaldi and E. Fermi, Ric. Scient. **VI-II**, 334, 443 (1935); **VII-I**, 56, 223, 310, 393, 454 (1936); see also: E. Fermi, E. Amaldi, O. D'Agostino, F. Rasetti and E. Segrè, Proc. Roy. Soc. **A146**, 483 (1934); E. Amaldi, O. D'Agostino, E. Fermi, B. Pontecorvo, F. Rasetti and E. Segrè, Proc. Roy. Soc. **A149**, 522 (1935).

[2] E. Amaldi, O. D'Agostino, E. Fermi, B. Pontecorvo, F. Rasetti and E. Segrè, Proc. Roy. Soc. **A149**, 522 (1935); H. A. Bethe, Phys. Rev. **47**, 747 (1935); F. Perrin and W. M. Elsasser, Comptes rendus **200**, 450 (1935).

[3] P. B. Moon and J. R. Tillman, Nature **135**, 904; **136**, 136 (1935); T. Bjerge and C. H. Westcott, Proc. Roy. Soc. **A150**, 709 (1935); L. Arsimovitch, I. Kourtschatow, L. Missowski and P. Palibin, Comptes rendus **200**, 2159 (1935); B. Pontecorvo, Ric. Scient. **VI-II**, 145 (1935); L. N. Ridenour and Don M. Yost, Phys. Rev. **48**, 383 (1935).

[4] P. B. Moon and J. R. Tillman, Nature **135**, 904 (1935); Dunning, Pegram, Fink and Mitchell, Phys. Rev. **48**, 265 (1935); P. B. Moon and J. R. Tillman, Proc. Roy. Soc. **A153**, 476 (1936).

[5] F. Rasetti, E. Segrè, G. A. Fink, J. R. Dunning and G. B. Pegram, Phys. Rev. **49**, 104 (1936); Rend. Linc. **23**, 343 (1936).

[6] E. Amaldi, E. Fermi, Ric. Scient. **VI-II**, 344 (1935).

[7] L. Szilard, Nature **136**, 950 (1935).

[8] E. Fermi and E. Amaldi, Ric. Scient. **VI-II**, 443 (1935).

curves into exponentials. We also studied different groups of neutrons by means of their scattering properties in hydrogenated substances.

In §2, we describe the methods used in performing the experiments and the reduction of the results. In §§3, 4, 5, we have collected some systematic data on the absorption properties and on the intensity of the activation produced by the various groups in several detectors. In §6, we describe the measurement of the reflection coefficient (albedo) for the neutrons of different groups; in §§7 and 8, we study the mean free paths and the average number of impacts of a neutron of a given group in paraffin. In §9, we show how the intensity of the activation due to several groups varies as a function of the distance from the source inside a large mass of water; from this relation we can deduce the energy sequence of the various groups and we are also able to evaluate the ratio of the energies of different groups. In §§10 and 11, we describe some diffusion experiments. In §13, all the results are collected and discussed.

In this work, we will often make use of some theoretical relationships that have been derived by one of us.[9] We shall refer to this paper in what follows by F.

2. Measurements and Their Reduction

As neutron source, we always used small glass bulbs about 4 mm in diameter and 15 mm in length which contained Rn (up to 800 mc) and powdered beryllium. Since our researches were extended over a period of several months, it was clearly necessary to establish some standard method of comparing results obtained with different sources and at different times.

We now describe the criteria employed for this purpose: The measurements of activity were made with an ionization chamber represented schematically in Fig. 1, which also contains the scale. The chamber was filled with carbon dioxide at three atmospheres and was closed by a window of aluminum 0.1 mm thick and 7 cm in diameter. Since the aluminum window is greatly curved by the pressure, it was covered by a Cellophane foil which served the double purpose of protecting the chamber from possible contamination and providing a flat surface upon which to place the detectors. The ionization was measured with an Edelmann electrometer, the wire of which was projected on a scale; it was possible to vary the sensitivity of the electrometer and the magnification of the image of the wire so that we obtained from 5 to 250 divisions per volt. To check the readings we used a uranium oxide preparation contained in a packet of aluminum 5×5 cm^2. We will designate this preparation and its activity by *U*.

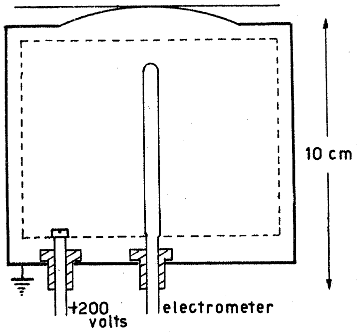

Fig. 1. Ionization chamber.

The ionization produced in our chamber by the *U* is equivalent to that obtained from a solution of 0.96 g of uranyl nitrate ($UO_2(NO_3)_2 + 6H_2O$) in 25 cm^3 of water contained in a dish of aluminum 0.1 mm thick and 5×5 cm^2 base. Correcting the activity of this solution for the absorption in the solution and in aluminum, we find that it is equal to 0.066 g of uranium element, that is 840 disintegrations per second.

In order to make the readings more rapidly and to be able to use different sensitivities of the electrometer and different scale intervals, we constructed a nomogram which permitted us to find directly, given the sensitivity of the electrometer and the number of divisions used in the reading, the time it would take the *U* to produce the same ionization. In this way one obtains automatically the correction for the nonlinearity of the instrument and for the dependance of the capacity on the sensitivity. Naturally the readings were corrected for the zero effect which was about 15 percent of the *U*. The activity of a given detector placed in a definite position with respect to the source and the surrounding objects is evidently proportional

---

[9] E. Fermi, "Sul moto dei neutroni nelle sostanze idrogenate," Ric. Scient. VII-II, 13 (1936).

to the number of neutrons emitted by the source. In order to have comparable data, it is always necessary to divide the activity by the intensity of the source measured in neutrons (neutron intensity). In order to measure the neutron intensity, it is not sufficient to assume it to be proportional to the quantity of radon contained in the source, which can be measured for example by the γ activity; in fact small differences in the size of the beryllium granules or in the preparation of the source produce considerable variations in the number of emitted neutrons. Because of this it is convenient to measure the neutron intensity directly by means of the radioactivity induced in a suitable detector. Our usual procedure was as follows: a sheet of rhodium of 5×5.6 cm^2 and 10.15 g was activated by placing it in the center of the top surface of a paraffin cylinder

$$25 \text{ cm diameter,} \qquad 15 \text{ cm height;} \qquad (1)$$

on top of the sheet of rhodium was placed a second cylinder of paraffin of 13 cm diameter and 10 cm height; the source was placed on the axis of the lower cylinder 3 cm below its upper surface.

We will designate by S the initial activity of the 44 sec. period of Rh obtained under these conditions after irradiation for an infinite time. We will call neutron intensity I the ratio of this initial activity S to the U:

$$\mathrm{I} = S/U. \qquad (2)$$

It should be noted that this definition depends, though not very markedly, on the ionization chamber used. The strongest source we have used had a neutron intensity of 144.9. A neutron intensity 1 corresponds to a source which contains from 5 to 6 mc radon.

We are now able to define what we shall call *activability* A of a given detector (referring, when necessary, to one of its periods) placed in a definite position with respect to the source and the surrounding objects.

Let a be the initial activity of a detector irradiated for an infinite time; we define the activability A of this detector in a given position by the expression

$$A = 1000a/S = 1000a/I \times U. \qquad (3)$$

In practice, naturally, we have calculated the activability by means of this last formula; the neutron intensity was measured once for each source with very high accuracy, and from this value the intensity at any later time was calculated by means of the period of radon.

The initial intensity a was measured by irradiating for a finite time, and was reduced to infinite time by well-known formulae.

In the case of elements of short period, it is convenient to establish a standard for the measurement of the initial activity in order to be as independent as possible, at least for relative measurements, from the possible errors in the value of the period. Therefore, in the case of rhodium (44 sec.) and of silver (22 sec.), which we have used as detectors in a very large number of experiments, we have performed the irradiation for 1 minute; in the case of silver, the readings were made from 20 sec. to 80 sec. after the end of the irradiation, and the activity so measured multiplied by an appropriate factor, gave the initial activity. In reality, a measurement of this kind gives not only the 22 sec. period, because there is a disturbance due to the 2.3 min. period; this disturbance was not very large and ordinarily was not corrected for (see also §5).

In the case of rhodium we have always irradiated 1 min. and we have taken the readings from about 30 sec. to 60 sec. after the end of irradiation; the initial activity was rapidly calculated by means of a nomogram. Also in this case we have neglected the small disturbance due to the 4.2 min. period.

In the course of this work we have performed a large number of measurements of absorption of slow neutrons in different substances. The criteria adopted in these measurements have been the following: The measurements were performed outside of the paraffin because otherwise a large fraction of the neutrons pass through the absorber many times (see §6). The source of Rn+Be was usually inserted in the paraffin cylinder (1), 3 cm below the center of the upper face on which were placed the various detectors. Most of the detectors were squares of 5 cm side.

We will call normal activability A_n of a detector its activability under these conditions. This furnishes an indication of the efficiency of the different detectors. As examples, in Table I we give the normal activabilities of some of the detectors which we have used.

For absorption measurements, the absorbers were inserted in layers as thin as possible between the paraffin and the detectors.

In order to correct the error due to the height h of the detector from the top of the paraffin, we have measured the variation of the activity of a rhodium detector as a function of h; the practical rule found for reducing the activability to $h=0$, consists in dividing the results of the measurements by $1-h/7$, where h is the height in cm; this rule is sufficiently correct for h up to a little more than 1 cm.

Rigorously, the correction for height would depend on the absorber used and on the group of neutrons with which we work. For the present, we have neglected these refinements. Even if the neutrons leaving the paraffin were homogeneous, the absorption curves obtained under the conditions which we have described, would not be exponential, because the neutrons which come out with different inclination θ traverse different thicknesses of the absorber. In the calculation of the absorption coefficients from the experimental data, it is necessary to take these facts into account; for a long time we have calculated this correction on the assumption that the neutrons leave the paraffin according to the cosine law.

On this assumption, the absorption curve with a thin detector, as a function of $K\delta$ ($K=$ absorption coefficient of the neutrons, $\delta=$ thickness of the absorber) instead of being the exponential $e^{-K\delta}$ (curve a of Fig. 2) is the function

TABLE I. *Activability A_n of various detectors.*

Substance	Period	Weight (in g)	Area (cm²)	A_n
MnO_2	2.5 hr.	30	26	30
Cu	5 min.	13	38	2
Ga	20 min.	2.6	7	1.3
As	26 hr.	7	20	24
NaBr	18 min.	26	25	9.2
Rh	44 sec.	10.15	28	282
Ag	22 sec.	8	27	135
In	54 min.	0.76	9	21.2
In	16 sec.	0.76	9	14.2
I	25 min.	25	25	16.9
Ir	19 hr.	14.5	25	78
Au	2.7 d	6.6	25	26

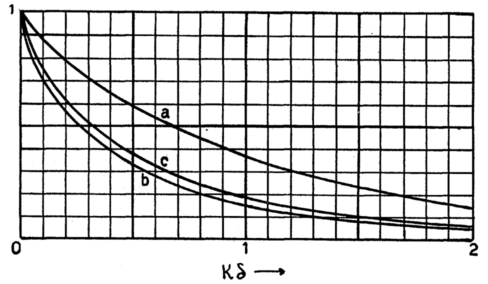

FIG. 2. Absorption curves following different laws. Curve (a) $a(K\delta)=e^{-K\delta}$; curve (b) $b(K\delta)=\int_0^1 e^{-K\delta/x}dx$; curve (c) $c(K\delta)=2/(2+\sqrt{(3)})\int_0^1 e^{-K\delta/x}(1+\sqrt{(3)}x)dx$.

$$b(K\delta)=\int_0^1 e^{-K\delta/x}dx \tag{4}$$

represented by the curve b of Fig. 2. We now believe that for the neutrons of group C, (see F §6) it is better to assume that the number of neutrons emerging per unit solid angle instead of being proportional to cos θ, is approximately proportional to

$$\cos\theta+\sqrt{(3)}\cos^2\theta. \tag{5}$$

If we take this angular distribution of the neutrons, the absorption curve as a function of $K\delta$ is given by the expression

$$c(K\delta)=\frac{2}{2+\sqrt{(3)}}\int_0^1 e^{-K\delta/x}(1+\sqrt{(3)}x)dx \tag{6}$$

represented by the curve c of Fig. 2. This curve probably represents the behavior of the absorption of the C neutrons fairly well; for the neutrons of other groups, it is probable that the absorption curve will be between b and c; however all reductions have been made with curve c. In the case of thick detectors, in other words when the absorption of the neutrons in the useful thickness was considerable, the data obtained from curve c was further corrected.

The differences between the absorption coefficients given in this work and those reported in preliminary communications, are due, in large part, to the fact that the reductions have been made with curve c instead of curve b of Fig. 2.

3. SELECTIVE ABSORPTIONS

We have already mentioned that, as was observed by different authors, the absorption of slow neutrons in an element is generally stronger when one uses a detector of the same element. For instance, Table II indicates the percentages of the activity transmitted by absorbers of 0.36 g/cm^2 of rhodium and 0.96 g/cm^2 of silver when the detector is a rhodium target of 0.36 g/cm^2 or a silver target of 0.80 g/cm^2. Data of this kind concerning a larger number of elements have been published, by various authors.[10]

These facts suggest the existence of selective absorptions of the slow neutrons. The most obvious method of studying these selective absorptions consists in measuring and analyzing the absorption curves. In Fig. 3, are given the absorption curves of cadmium with rhodium detector (0.36 g/cm^2) and silver detector (0.057 g/cm^2). It is clear from these curves that the radiations which are responsible for the activity of both these detectors are not homogeneous, but consist of a fraction which is absorbed very strongly in cadmium, and a fraction which is very weakly absorbed. Taking into account the fact that with our geometrical disposition the absorber is traversed obliquely by the neutrons, (see §2) it is possible to calculate the absorption coefficient of the strongly absorbed fraction from each of these curves.

In this way we obtain, from the curve with silver detector $K=16$ cm^2/g, and from the curve with rhodium detector $K=13.5$ cm^2/g. The absorption coefficient of the hard fraction is certainly less than 1/100 of these values; so, for instance, in the case of the rhodium detector we find $K=0.05$ cm^2/g.

From the examination of the curves of Fig. 3, it can be seen that in our silver detector half of the activity is due to the component which is strongly absorbed in cadmium and half to the hard component. Instead, in our rhodium detector 72 percent of the total activity is due to the soft component and the remainder to the hard component. Therefore the absorption by cadmium makes it possible to divide the slow neutrons which come out of a paraffin block containing a source of neutrons into two fractions: that fraction which is strongly absorbed by cadmium will be hereafter designated as group *C*. We will discuss its absorption properties in more detail in §5.

TABLE II. *Absorption of slow neutrons as measured by different detectors.*

Absorber	Rh	Ag
Detector		
Rh	54	67
Ag	68	45

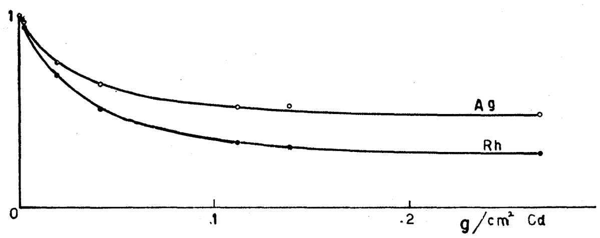

FIG. 3. Absorption curves for Cd as determined by Ag and Rh detectors.

We will now study the properties of the radiations which traverse a thickness of cadmium sufficient to totally absorb the C group. For this purpose we have repeated some absorption experiments on the radiations filtered by 0.27 g/cm^2 cadmium using rhodium and silver absorbers and detectors.[11] Table III, similar to Table II, is given as an example of the results of such experiments; the results summarized in Tables II and III were obtained with the same absorbers and detectors.

Comparison of Tables II and III shows that the selectivities are accentuated for the radiation filtered by cadmium, which indicates that the component weakly absorbed by cadmium is not homogeneous. In order to further investigate the properties of the radiation filtered by cadmium, we have measured the absorption curve of silver with silver detector (2.2 g/28 cm^2) both for the unfiltered radiation and for the radiation filtered by 0.27 g/cm^2 of cadmium. These curves are

TABLE III. *Absorption of radiations filtered by 0.27 g/cm^2 Cd.*

Absorber	Rh	Ag
Detector		
Rh	33	84
Ag	88	29

---

[10] P. B. Moon and J. R. Tillman, Nature **136**, 66 (1935); B. Pontecorvo, Ric. Scient. **VI-II**, 145 (1935); E. Amaldi and E. Fermi, Ric. Scient. **VI-II**, 344 (1935).

[11] E. Amaldi and E. Fermi, Ric. Scient. **VI-II**, 344 (1935); L. Szilard, Nature **136**, 950 (1935).

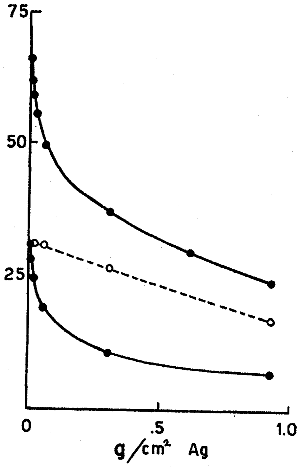

FIG. 4. Upper curve, absorption curve for Ag, unfiltered radiation. Lower curve, absorption curve for Ag, radiation filtered by 0.27 g/cm² of Cd. Ordinates are the activability of Ag detector. Dotted curve is difference of upper and lower curves.

given in Fig. 4 where the abcissa represents the thickness of the silver absorber and the ordinate represents the activability of the detector. The upper curve refers to the unfiltered radiation and the lower curve to the radiation filtered by 0.27 g/cm^2 of cadmium.

In both curves, we see that the radiation which is responsible for the activity of silver contains a component strongly absorbed in the same element, which component is present with the same intensity both in the filtered and in the unfiltered radiations. This means that this component is not strongly absorbed by cadmium. We will call this component group *A*. The absorption coefficient of the neutrons of group *A* in silver is $K=20$ cm^2/g.

Curves of similar character have also been found with a thicker silver detector; in this case however the percentage of activity due to the radiation strongly absorbed in silver is naturally smaller, since it is saturated in less than 0.1 mm of thickness of the detector.

The dotted curve of Fig. 4 is the difference of the two other curves; it therefore represents the behavior of that fraction of the silver activity that is absorbed by 0.27 g/cm^2 of Cd. It is, therefore, the absorption curve of silver with silver detector due only to the neutrons of group *C*. From this we obtain as value of the absorption coefficient of silver for the *C* neutrons, $K=0.3$ cm^2/g.

4. SLOW NEUTRON GROUPS

We have seen from the examples of the preceding paragraph, that the analysis of the absorption curves permits us to classify slow neutrons into components which are differently absorbed in different elements. It is well known that the analysis of absorption curves into exponentials can be considered reliable only when the accuracy of the individual measurements is very high, and also the geometrical disposition of the absorber and detector with respect to the source, corresponds to accurately calculable conditions. In our case, neither of these conditions were exactly satisfied, and therefore we seek the possibility of checking accurately the homogeneity of each component. For this reason, we have limited ourselves to the description of the properties of groups of approximately homogeneous neutrons.

The differences of behavior from group to group are so marked that it is possible to assign to each of them an individuality, despite the small internal inhomogeneity. The analysis into groups is made possible by the following circumstances:

(a) For some absorbers there exist very large differences in the absorption coefficients from group to group.

(b) It is possible to use different absorbers and detectors which behave quite differently.

(c) It is possible to filter the slow neutrons with absorbers so as to reduce the inhomogeneity.

The most obvious assumption concerning the physical nature of the difference between the various groups is that it is due to a difference in velocity. In a paraffin block the fast neutrons, which escape from the source suffer a progressive slowing down due to the successive collisions with the hydrogen atoms until they arrive at the energy of thermal agitation; therefore, in paraffin there are present at each instant neutrons of all velocities, from the energy of thermal agitation to the energy with which they leave the source. Likewise, neutrons of all these velocities diffuse out of the paraffin block. Hence we may think that the different groups correspond to neutrons of different energy intervals. Selective absorptions must be interpreted as due to irregular variations of the absorption coefficient as a function of the neutron velocity.

The possibility of an irregular dependance of the absorption coefficient on the neutron velocity has been demonstrated by recent considerations of Bohr, Breit and Wigner.[12] We will return to the discussion of the physical origin of the groups after we have described the absorption experiments and also the scattering properties of the various groups.

Already in the preceding paragraph, we have called the radiation strongly absorbed in cadmium ($K=13.5\div 16$ cm^2/g) group C; and the radiation strongly absorbed in silver ($K=20$ cm^2/g) group A.

We have also seen that only 50 percent of the activity of our silver detector (0.057 g/cm^2) is due to group C; of the remaining 50 percent about one-half is due to group A, while the rest is due to a radiation which is only slightly absorbed in cadmium and in silver (see Fig. 4); this radiation we have called group B, which is very probably complex. Up to now we have not been able to analyze it into approximately homogeneous components, since we have not found any elements which absorb group B with a high absorption coefficient.

In the preceding section, we have shown that 72 percent of the activity of our rhodium detector (0.36 g/cm^2) is due to group C; the remaining 28 percent constitutes a new group, which we have called group D, which is very slightly absorbed by cadmium. In studying a PbI_2 detector (19 g/25 cm^2, normal activability 11.4), we have found that only about 25 percent of its activity is due to group C and less than 10 percent is due to group A; the remainder is due to a component which is very weakly absorbed in all elements that we have studied up to date, except iodine; this radiation which is responsible or about 70 percent of the activity of our detector has been called the I group.

Frisch, Hevesy and McKay[13] have studied the absorption of gold with gold detector using the cadmium filtered neutrons and have obtained results similar to those we obtained for silver (see Fig. 4). We have tried to analyze the activity of a gold detector (1.75 g/25 cm^2) in terms of the groups already mentioned; we have found that the activity of gold is due in part to group C, A and B; the high absorption coefficient of gold with gold detector is probably due to a radiation distinct from all the preceding groups.

5. SYSTEMATIC ABSORPTION MEASUREMENTS

Group C

Of all groups the group C is the best known, because it is responsible in almost all detectors for a large percentage of the activity; there are many reasons for believing (see §13) that this group is composed, in large measure, of neutrons having energy of thermal agitation; for this reason we will often refer to it as the thermal group. The study of the C group is particularly easy because of the fact that a thickness of $0.3\div 0.4$ g/cm^2 of cadmium absorbs it practically completely without appreciably absorbing any of the other groups. In order to obtain the fraction of the activity due to the C group, it is sufficient to obtain the difference in the activity observed without and with a filter of cadmium of the above thickness.

In Table IV we give the absorption coefficients in cm^2/g for the C neutrons, observed with various detectors (always forming the difference between the activity without and with a cadmium filter) and various absorbers. It is to be noted that the absorption coefficients of the same element measured with different detectors are approximately the same. The differences are due in part to errors in measurements; however we believe that they are not completely accounted for by this reason.

Despite this, if we remember the large differences of the absorption coefficients of an element for the neutrons of different groups, we see that the C group actually has a notable internal homogeneity.

Group D

The D group is easily observed by using rhodium or indium as detector; with both these elements precise measurements are possible.

The fact that it is possible to characterize the D group by using one or the other of these detectors indicates that the corresponding absorption bands are nearly coincident; naturally there is no reason to believe that this coincidence

---

[12] N. Bohr, Nature **137**, 344 (1936); G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936).

[13] O. R. Frisch, G. Hevesy and H. A. C. McKay, Nature **137**, 149 (1936).

TABLE IV. *Absorption coefficients in cm²/g for group C neutrons (those strongly absorbed by Cd).*

Absorber	Rh	Ag	Cd	In	Ir	Au	Hg
Detector							
Mn 2.5 hr.	0.9	0.3	—	—	—	—	—
Rh 44 sec.	0.6	0.2	13.5	0.6	1.0	0.2	0.7
Ag 22 sec.	0.7	0.3	16	—	0.9	0.25	0.9
In 54 min.	0.6	0.2	14	0.9	1.0	0.2	0.6
Ir 19 hr.	1.0	0.3	—	—	—	—	0.6

TABLE V. *Absorption coefficients in cm²/g for group D neutrons (those strongly absorbed in Rh).*

Absorber	B	Rh	Ag	Cd	In	Ir	Au	Hg
Detector								
Rh 44 sec.	4.7	2.0	0.06	0.05	3.0	1.0	0.03	0.07
In 54 min.	—	1.6	0.09	—	3.8	1.0	0.04	0.04

is complete; we will see in §9 that of all the groups we have studied, the D group has smaller energy than any, except the C group.

We have already mentioned that 28 percent of the activity of our detectors of rhodium is due to the D group. In an indium detector of 0.76 g/9 cm², 42 percent of the activity of the 54 min. period is due to the D group; the remaining 58 percent is due to the C group. In order to study group D, it is therefore sufficient to use one of these detectors, filtering the radiation by $0.3 \div 0.4$ g/cm² of cadmium in order to eliminate the C group. In Table V some absorption coefficients in cm²/g of the D neutrons are given.

Group A

This group, characterized by the large absorption coefficient in silver ($K = 20$ cm²/g), is ordinarily studied by using a thin sheet of silver as detector.

As was mentioned above, 25 percent of the normal activability of a silver detector of 0.057 g/cm² is due to the A neutrons. In order to isolate this group it is sufficient to form the difference of the activability of a silver detector without and with a silver filter sufficiently thick to absorb completely the A group and not thick enough to appreciably absorb the other groups. In practice it is convenient to eliminate the C group by means of a cadmium filter.

The A group is also detected by gold; a gold detector of 1.535 g/25 cm² has a normal activability of 21, of which 13 percent is due to group A; the absorption coefficient of gold for this group is 4 cm²/g. The absorption coefficient of the A group in boron is 3 cm²/g.

Not much is known concerning the properties of the other groups mentioned in the preceding section. We have performed a few measurements of absorption coefficients which we give here:

Group B: boron $K = 2.3$ cm²/g; gold $K = 1.7$ cm²/g.
Group I: boron $K = 1$ cm²/g; iodine $K = 0.7$ cm²/g.

We have seen that despite the fact that the C group almost always constitutes more than 50 percent of the activity of the different detectors, placed outside of the paraffin, the other groups are found in a smaller number of detectors and are responsible for a smaller percentage of the activities.

This is not due to the fact that the coefficient of absorption of the different elements are greater for the neutrons of group C than for the neutrons of the other groups; we have already encountered numerous examples to the contrary. It is rather due to the fact that the number of C neutrons which come out of paraffin is much larger than the number of neutrons of the other groups.

Therefore, a very important datum for characterizing a group is its *numerosity*. We will define this quantity for practical purposes as follows: the numerosity N_0 of a group is the number of neutrons which are emitted per second per square centimeter and per unit of neutron intensity of the source, averaged over 25 cm² at the center of the top surface of the paraffin (1) containing the source (neutron intensity I) 3 cm below the center of the top.

The principle on which the measurement of the numerosity of a group is based is as follows: We place on the center of block (1) a thin detector of area s; the number of neutrons which strikes this detector is

$$N_0 I s.$$

If we designate the absorption coefficient of the detector by K, assuming that its thickness δ is very small, it is possible to see that the number of neutrons captured by the detector is

$$\sqrt{(3)} N_0 I s K \delta,$$

where the factor $\sqrt{(3)}$ is due to the obliquity of the neutrons, which is taken into account according to Eq. (5).

If we designate by η the efficiency of the ionization chamber, that is the ionization current due to one disintegration per second on the surface, the normal activability (see Eq. (3)) of our detector is

$$A = 1000\sqrt{(3)}sN_0K\delta(K_1/K)(\eta/840\eta_U), \quad (7)$$

where η_U is the efficiency of the chamber for the β-rays emitted by the uranium preparation we used, which, as already has been mentioned, corresponds to 840 disintegrations per second at the surface.

The factor K_1/K is introduced in order to take into account that when a detector absorbs neutrons of a given group (with an absorption coefficient K) it is possible that a fraction of these gives rise either to nonradioactive isotopes or to radioactive isotopes with periods different from that studied; we indicate by K_1 the absorption coefficient due only to the process which is responsible for the studied activity; it is evident that although K is the coefficient which determines the absorption of the neutron in the detector, K_1 determines its activity.

From (7) we find the numerosity of the group under examination.

$$N_0 = 0.485\frac{\eta_U}{\eta}\frac{K}{K_1}\frac{A}{sK\delta}. \quad (8)$$

In this expression for N_0, the quantities A, s, K, δ are directly measurable; in most cases it is possible to put K_1 equal to K; in the other cases where an element has more than one mean life, the ratio of their activabilities corrected for the absorption of the β-rays and for the different efficiencies η, allows one to evaluate K/K_1.

A considerable error in the measurement of N_0 is due to the insufficient knowledge of the ratio of the efficiency of the chamber for β-rays of different hardness. From the relatively small variations which are found in the calculated numerosities for different detectors on the assumption of a constant efficiency η, it can be concluded that this quantity does not vary between very large limits. In the following calculations we have always put $\eta = \eta_U$.

In Table VI we give the values of the numerosities of the various groups calculated using different detectors; as absorption coefficients we have used the averages of the data of the preceding tables. The values given in Table VI have been calculated with a formula similar to (8) in which however the absorption of the neutrons and the electrons in the detector have been taken into account. We have also introduced a small correction in order to take into account the reflection of the electrons from the upper layers of the detector.

The data for indium have been calculated by adding the contributions of the two periods of 16 sec. and 54 min. and by neglecting the weak period of 4 hours. For rhodium and for silver we have made the calculations by taking into account not only the 44 sec. and the 22 sec. periods, but also the 4.2 min. and 2.3 min. periods.

If we try to calculate the numerosity of group C from the activity of iridium (19 hr.), we find a value of about 10; this value seemed to us much too small to be attributed to errors in measurements. Therefore we have tried to find another activity strong enough to justify the previous small value of the numerosity. In reality, this suspicion was well grounded since we have found a second iridium activity with a period of 68 days whose activability, although not yet measured, is of the order of magnitude of the activability of 19 hr. period. The β-particles of this new activity have very small penetrating power.

It is very interesting to determine for elements which have two or more periods, both due to slow neutrons, whether or not the activities of the different periods are due to the same groups.

Therefore we have studied the following detectors; rhodium (0.36 g/cm^2 44 sec., 4.2 min.), silver (0.057 g/cm^2 22 sec., 2.3 min.), indium (0.065 g/cm^2 16 sec., 54 min.), sodium bromide (1 g/cm^2 18 min., 4.2 hr.) and we have compared for each of them the activability of the two periods with and without appropriate filters.

In the case of rhodium, the ratio of the activabilities of the 44 sec. and of the 4.2 min. periods remains the same (about 10) with and without a filter of cadmium sufficient to absorb the C group.[14] In the case of silver, however, we have

---

[14] E. Fermi and E. Amaldi, Ric. Scient. **VI-II**, 443 (1935); E. Segrè, Ric. Scient. **VII-I**, 389 (1936).

TABLE VI. *Numerosity (N_0) for various neutron groups.*

Detector	Group *C*	*D*	*A*	*I*
Rh	43	9	—	—
Ag	51	—	0.6	—
In	31	6	—	—
I	—	—	—	5
Au	33	—	0.5	—

found a marked difference in the behavior of the two periods; in group *C* the activability of the long period is 24 percent of the total activity; in group *B*, 19 percent; finally in group *A* we have found 3 percent which, within the limits of experimental error could be zero; we can therefore say that the group *A* is characteristic of the 22 sec. period of silver.

In the case of indium the percentage of activation of the two periods due to group *C* and *D* are approximately equal. Similarly for bromine, a cadmium filter does not considerably change the ratio of the activability of the two periods.

We have also replaced the paraffin cylinder on which the measurements were usually made, by an equal cylinder of water and using rhodium, silver and lead iodide detectors and appropriate filters, we have determined the percentages of activation due to the different groups. In this way we have been able to show that the ratios of the numerosity of the various groups are the same in water as in paraffin. Also the absolute values do not vary considerably. However it should be noted that the ratios of the intensities of the various groups undergo a variation at different distances from the source, as will be discussed in more detail in §9.

Finally we have tried to see whether the ratio of intensity of the various groups is affected by using different source of fast neutrons.[15] For this purpose we have determined the activability of rhodium and silver detectors with and without cadmium and silver filters, using a source newly prepared, in which the radium *C* was not yet in equilibrium; and we have followed the increase of activability during the formation of radium *C*. We have not found any difference in the ratio of the groups for silver and rhodium during the formation of radium *C*, although the activability increased from 40 (effect due to the particles of Rn+Ra A) to 100 (effect due to the particles of Rn+Ra A+Ra C). In Fig. 5 we give the curve of the increase of the activity as a function of the time. One should note the high efficiency of the radium *C* α-particles for producing neutrons from beryllium; their efficiency is three times greater than that of the α-particles of Rn and Ra A.

6. ALBEDO

The measurements of normal activability as also the absorption measurements of which we have spoken up to now are made outside of the paraffin block. In this section we will study the properties of the slow neutrons inside the paraffin.

The detector (rhodium 10 g/28 cm^2, or silver (2.2 g/38.5 cm^2) was placed in the center of the top of the normal paraffin block (1) and upon it was placed a second equal paraffin cylinder. The detector could be placed between appropriate filters.

In order to indicate rapidly the relative position of source, paraffin, detectors and absorbers, it is convenient to set up the following convention: We will indicate with the letter *S* the source, *P* the paraffin, *R* the detector, and with the symbols Cd and Ag the cadmium (0.27 g/cm^2) and silver (0.057 g/cm^2) absorbers. Thus *S P* Cd *R* Cd *P* means an experiment in which above the source (*S*) there are 3 cm of paraffin (*P*), followed by a layer of cadmium (Cd), the detector (*R*), a second cadmium layer (Cd), and finally the second paraffin cylinder (*P*).

In Table VII we give the activabilities of the two detectors of silver and rhodium in various experiments. If we compare the experiment *S P R* and *S P R P* we note the strong increase of the activity due to the superposition of the second block of paraffin. This large increase is due to the fact that neutrons, both fast and slow, which in the absence of the second block would escape, can now be reflected back from this block.

If we put the detector between appropriate absorbers, it is possible to analyze the radiation responsible for the activity into groups. Thus the comparison of experiments *S P R P* and *S P* Cd *R* Cd *P* shows that in the case of silver detectors the fraction $(417-66.5)/417=0.84$ of the activity is due to group *C* and in the case of rhodium detector, the fraction $(1025-140)/1025=0.86$ is

---

[15] E. Fermi and E. Amaldi, Ric. Scient. **VI-II**, 443 (1936).

due to group *C*. It is to be noted that the ratio of group *C* to the other groups is much larger inside of the paraffin than outside; we will soon see the reason for this behavior. As a check of the correctness of the analysis into groups of the activity of a detector, also inside of the paraffin, we have doubled the thickness of the cadmium absorbers in the experiment *S P* Cd *R* Cd *P*, and we have found only a very small further decrease in the activity. In the case of the silver detector, the addition of silver absorbers to those of cadmium causes a reduction of the activity to one-half (experiments *S P* Cd *R* Cd *P* and *S P* Cd Ag *R* Ag Cd *P*); this fact shows that also inside the paraffin it is possible to speak of a group *A* intensely absorbed by silver. In agreement with what is obtained outside the paraffin, the thin absorbers of silver added to those of cadmium do not produce a further reduction of the rhodium activity.

Let us now study the behavior of group *C*. Let us consider the results of the measurements *S P R P*, *S P* Cd *R P*, *S P R* Cd *P*, *S P* Cd *R* Cd *P*; this last measurement indicates what part of the activity is not due to the group *C*, so that if we subtract this value from the results of the other three measurements, we will obtain the fraction of the activity due only to group *C*.

The difference between *S P* Cd *R P* and *S P R* Cd *P* can easily be interpreted by taking into account the fact that the density of slow neutrons inside the paraffin decreases with the distance from the source in such a way that a sheet of cadmium which screens the detector on the side of the source stops more neutrons than an equal sheet on the other side. The mean value of these two experiments may be considered as the value which would be obtained in the case of a uniform density distribution of slow neutrons inside the paraffin, when the detector is screened on one side by cadmium.

Therefore we find for the silver detector, as activability due only to group *C*, $A_c=417-66.5=350.5$. The activability due only to group *C* when the silver is screened only on one side by a layer of cadmium is $B_c=(92.2+113.1)/2-66.5=36.1$.

The corresponding values for rhodium are $A_c=885$, $B_c=167.5$. The ratio A_c/B_c is for silver 9.7 and for rhodium 5.3.

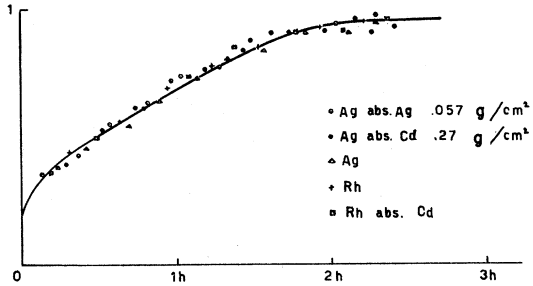

FIG. 5. Increase of activity as a function of time.

An elementary consideration allows us to understand the values of this ratio. We will call β the probability that a neutron that is incident on a flat surface which limits a very large paraffin block, comes out after having suffered several collisions in the paraffin.

We call this value β *albedo*, which corresponds to the coefficient of diffuse reflection of slow neutrons from a paraffin surface. Actually it depends upon the angle of incidence of the slow neutrons, and it is evidently smaller, the smaller the angle of incidence (see *F*, §5).

In this elementary consideration we will neglect this dependence.

We call ζ the probability that a neutron which traverses the detector is captured by it. Also for ζ we will neglect the dependence on the angle of incidence.

The method for the measurement of the albedo is based on the comparison of the activabilities A_c and B_c defined above.

Let *N* be the number of *C* neutrons which are incident on a detector of group *C* when it is screened on one side by a total absorber of *C* neutrons. The number of neutrons captured by the detector is $N\zeta$ and its activability B_c will be proportional to $N\zeta$.

TABLE VII. *Activabilities for various arrangements.* *S* = source, *P* = paraffin, Cd = cadmium, Ag = silver, *R* = detector.

Arrangement	Silver	Rhodium
S P R	66.1	282
S P Cd *R*	32.2	80
S P Cd Ag *R*	17.5	78
S P R P	417	1025
S P Cd *R* Cd *P*	66.5	140
S P Cd *R P*	92.2	244
S P R Cd *P*	113.1	371
S P Cd Ag *R* Ag Cd *P*	34.5	144
S P Cd Ag *R* Cd *P*	45.6	—
S P Cd *R* Ag Cd *P*	54.8	—

Let us now remove the absorber and let us calculate the increase of the activity of the detector.

First, we must take into account the fact that the neutrons can impinge on the detector on both sides; this gives rise to a factor 2.

But a much more important increase is due to the fact that when one places immediately behind the detector a sheet which absorbs all neutrons, a C neutron which impinges on the detector traverses it only once and therefore has the probability ζ of being captured; when the absorber is not present the neutron can traverse the detector many times.

Actually, the first time the neutron has the probability ζ to be captured and the probability $1-\zeta$ to go through; the probability that the neutron goes back to the detector is therefore $\beta(1-\zeta)$ and the probability that it is captured in the second passage is $\beta(1-\zeta)\zeta$ and so on. Thus we find that the probability of capture is

$$\zeta+\zeta\beta(1-\zeta)+\zeta\beta^2(1-\zeta)^2+\cdots = \frac{\zeta}{1-\beta(1-\zeta)} = \frac{\zeta}{1-\beta+\beta\zeta}.$$

The number of captured neutrons is

$$2N\zeta/(1-\beta+\beta\zeta)$$

and hence the ratio A_c/B_c is

$$A_c/B_c = 2/(1-\beta+\beta\zeta). \quad (9)$$

If ζ is very small, i.e., in the case of very thin detector, this reduces to

$$A_c/B_c = 2/(1-\beta). \quad (10)$$

Therefore the measurement of the ratio A_c/B_c allows us to determine the albedo β.

In the case of our silver detector, ζ is very small but not negligible. We can evaluate it from its thickness and absorption coefficient, also taking into account the obliquity of the neutrons. Thus we find $\zeta=0.03$ and therefore $\beta=0.82$.

The rhodium detector is much thicker; for it ζ is about 0.35; it is clear that a small error in this large value of ζ gives rise to a great error in the value of β. Therefore we cannot use the measurements with rhodium in order to determine β, but we can rather calculate for this detector the ratio A_c/B_c, using the value of the albedo found by silver measurements. We have $A_c/B_c=4.3$ in sufficient agreement with the experimental value 5.3.

We have observed that the concept of albedo, as we have introduced it, is not completely defined, since the percentage of neutrons reflected by a paraffin surface depends on the angular distribution of the incident neutrons. In order to give a more definite meaning to the concept of albedo, we can define β by means of Eq. (10), which holds in the case of an infinitely thin detector (experimental albedo; see *F*, §7).

The experimental albedo depends on the number N of free paths which a neutron C can traverse on the average before it is captured by protons in paraffin.

It is possible to show (*F*, formula 63) that the relation connecting these two values is

$$\beta = 1-2/\sqrt{\mathrm{N}}. \quad (11)$$

This equation holds on the assumption that the energy of thermal agitation is negligible with respect to the quantum $h\nu$ of the elastic bond of hydrogen atom in paraffin; otherwise it is necessary to introduce a small correction (see §13).

From the value $\beta=0.82$, we find from Eq. (11) $\mathrm{N}=124$.

We have performed some experiments similar to those just described, in order to determine the albedo of the neutrons of group A, using a detector of silver of 0.010 g/cm^2, so that ζ for the A group is not too large.

The behavior of the group A is, in this respect, quite different from that of group C, as we found that the albedo of group A is practically zero. This fact must not be interpreted in the sense that the A neutrons are not at all reflected from paraffin, but it shows that when A neutrons are reflected they undergo such a change in velocity that they no longer belong to the A group.

Because of the low albedo of group A, the increase in activity produced by A neutrons when a second block of paraffin is placed on the detector is only of the order of 2. This increase is not due to the albedo but to the fact that the neutrons can enter the detector on both sides.

Groups B, D and I also have a negligible albedo. Hence it follows that for all the groups, except group C, the change in the velocity due to one impact is normally sufficient to remove the neutron from the energy band corresponding to the group (see §12).

The high value of the albedo of the group *C* compared to the albedo of the other groups is responsible for the fact that inside the paraffin the percentages of the activations due to the *C* group are always much higher than outside the paraffin.

Similarly, one can interpret the experiments of Tillman[16] on the different behavior of various detectors lying on a block of paraffin, as a function of the thickness of a second superimposed layer of paraffin. A strong relative increase of the activity of a detector is to be expected, for detectors which are very sensitive to group *C*; naturally it is necessary to take into account in the interpretations of these experiments also the coefficient ζ of the detector according to relation (9).

7. Scattering of the Neutrons of a Single Group

From the results of the preceding paragraph we deduced that the *C* neutrons can traverse a large number of free paths. Their motion in paraffin is therefore analogous to a diffusion, provided that the destruction of the neutrons by proton capture is taken into account. Let N be the average number of free paths of a *C* neutron and λ its mean free paths; evidently the neutron will diffuse through a length of the order of magnitude of $\lambda\sqrt{N}$; therefore the diffusion experiments will enable us to measure this magnitude.[17] We shall call the expression

$$l=\lambda(N/3)^{\frac{1}{2}} \tag{12}$$

diffusion length; this magnitude can be obtained directly from the measurements we are going to describe.

Let us consider a *C* neutron inside a paraffin block limited by a plain surface and let x be the distance of the neutron from this surface.

It can be proved by the diffusion theory (see *F*, §3) that the probability $p(x)$ for the neutron to get out of the paraffin before it is captured is

$$p(x)=e^{-x/(D\tau)^{\frac{1}{2}}}, \tag{13}$$

where D is the diffusion coefficient for thermal neutrons in paraffin, and τ is their mean life for the capture process. In the case of thermal neutrons the kinetic energy is small compared to the quantum of the frequencies due to the elastic bond of the hydrogen in paraffin. We shall therefore make the assumption that the hydrogen atoms can be considered as fixed centers of isotropic diffusion, and that the mean free path λ is independent of the velocity (see *F*, §10, 11). These assumptions are not quite correct, as shall be discussed in detail in §13.

In the case of isotropic diffusion, the diffusion coefficient D is given by

$$D=\tfrac{1}{3}\lambda v, \tag{14}$$

v being the average velocity of the neutrons; we have furthermore

$$N=v\tau/\lambda; \tag{15}$$

hence

$$D\tau=\tfrac{1}{3}\lambda^2 N. \tag{16}$$

We find finally (see *F*, formula 34)

$$p(x)=e^{-x/(\lambda^2 N/3)^{\frac{1}{2}}}=e^{-x/l}. \tag{17}$$

By determining the probability $p(x)$ we shall be able to measure l and therefore also λ^2N.

The principle on which the determination of $p(x)$ is based is the following: let *R* be a detector of the *C* group, large enough to cover all the plane surface of a large paraffin block from which *C* neutrons can emerge. We will assume that this detector absorbs all the outcoming thermal neutrons in a thickness small compared to the absorption of the β-rays. The activity of this detector is proportional to the number of neutrons emerging from the paraffin, and is independent of their angular distribution. If we now destroy *Q* thermal neutrons at a depth x below the paraffin surface, the activity of the detector shall decrease by an amount proportional to $Q\cdot p(x)$; in order to destroy these neutrons we can put an absorber of thermal neutrons at the depth x; we shall then be able to deduce *Q* from the activity induced in the absorber.

The experiment was performed by putting the detector on a cylinder of paraffin 24 cm in

---

[16] J. R. Tillman, Nature **137**, 107 (1936).

[17] T. Bjerge and C. H. Westcott, Proc. Roy. Soc. **A150**, 709 (1935); E. Fermi and E. Amaldi, Ric. Scient. **VI-II**, 443 (1935); **VII-I**, 56 (1936).

diameter and 15 cm high. The source was 3.5 cm below the center of the top face. The upper part of the cylinder was sectioned into slices so as to enable us to insert the absorber at the various depths. As detector we used a rhodium plate (10 g/28 cm^2); as this plate covered only a small part of the upper paraffin surface, we took measurements placing the detector in nine different positions and adding the results.

As absorber of thermal neutrons, we used a plate of Cd−Sn alloy containing 0.019 g/cm^2 cadmium. As a small thickness of cadmium practically absorbs only the *C* neutrons, the decrease in activity of the detector is due to *C* neutrons only, as can be easily verified. Since the absorption of neutrons in cadmium does not activate this element we could not measure directly the number Q of the absorbed neutrons; therefore we measured the activity induced by *C* neutrons in a rhodium plate having for the *C* group the same absorption as our Cd absorber. We measured the activity on the two faces of this rhodium plate and added the results in order to take into account that the neutrons entered from both sides as explained in §11.

The absorber was put at the distances $x=0$, 1, 2, 3 cm. In Table VIII are collected the results of the measurements. Column one gives the depth of the absorber; column two gives the decrease in the activability of the detector due to the presence of the absorber; column three gives the sum of the activabilities of the rhodium plate equivalent to the absorber measured on both sides for the group *C* only; column four gives the ratio of column two to column three. In column five the values of these ratios are corrected in order to take into account the fact that our detector does not satisfy the condition of totally absorbing the *C* neutrons in a thickness small compared to the absorption of electrons (absorption coefficient for electrons 7.3 cm^2/g in rhodium; absorption coefficient for neutrons 0.7 cm^2/g; thickness 0.36 g/cm^2). From this it follows that our detector is more sensitive to the neutrons of larger obliquity. The absorber, when in the position $x=0$, absorbs more intensely those neutrons that get out very obliquely; the decrease in activity of the detector is therefore in this case relatively larger than in the case when the absorber is placed deep inside the paraffin, since in this latter case there is no coherence between the direction of the emergent neutrons and the direction that they had when traversing the absorber. In order to take into account this factor we must reduce the value of the ratio for $x=0$ in order that it may be compared to the other ratios.

A quantitative calculation shows that the first ratio must be corrected by a factor 0.832.

The values of the corrected ratios given in the last column are proportional to $p(x)$; it can be seen that they depend with good approximation exponentially on x. The diffusion length is the inverse of the coefficient of this exponential; thus we find

$$l=2.1 \text{ cm}.$$

From this we obtain (12)

$$\lambda^2 N=13 \text{ cm}^2.$$

We performed similar experiments also for groups *D* and *A*. In the case of the *D* group, the source was 3.4 cm below the center of the upper face of a cylinder of paraffin similar to that used in the preceding experiments. In the case of group *A* the distance was 2.4 cm.

As detectors, we used, respectively, for the groups *D* and *A* indium (0.065 g/cm^2) and silver (0.057 g/cm^2), both screened with cadmium filters of about 0.5 g/cm^2. As absorbers we used for the *D* group rhodium (0.36 g/cm^2) and for the *A* group silver (0.057 g/cm^2).

In Table IX we have collected the results of these measurements; in column one we give the depth x of the absorber; columns two and five give for the groups *D* and *A* the decreases in activability of the detectors; column three is the sum of the activability measured at various depths on both faces of the rhodium absorber screened by cadmium so as to measure the activity due to the *D* group only; column six contains similar data for the silver absorbers of group *A*. Columns four and seven give the activabilities of the absorber measured on the lower face and due only to the neutrons of group *D* or *A* that enter the absorber from the lower face (for the method of performing this measurement see §11). The numbers in brackets of the last column have been interpolated using some results similar to those mentioned in §11.

TABLE VIII. *Data for the determination of the probability $p(x)$.*

x (cm)	Decrease of Activability	Q	Ratio	Corrected Ratio
0	89	358	0.249	0.207
1	139	1135	0.123	0.123
2	128	1580	0.081	0.081
3	98	1950	0.050	0.050

From the low albedo of the groups D and A, we have already deduced in the preceding paragraph that the neutrons of these groups generally traverse as such only one free path. We cannot therefore apply to these groups the theory of diffusion. It is nevertheless illuminating to reduce the data of the measurements on the groups D and A by the same method that we used for the C group. For this, we examine, as a function of x, the ratios of the second to the third column, and the ratios of the fifth to the sixth column. One finds that both ratios decrease exponentially with a good approximation; in both cases the inverse coefficient of the exponential is 0.42 cm. We notice the analogous behavior of these two groups in contrast with the markedly different behavior of the C group.

It is more correct, however, to reduce the data of Table IX on the extreme assumption that the neutrons of groups D and A can traverse only one free path. Assuming this, we can deduce the mean free path of the neutrons of these two groups. If this assumption is not correct, the values that we shall obtain would represent only upper limits for the mean free path.

Let us compare the activities of the detector of group D with or without the absorber at a depth x. Both activities are the sum of two terms: (a) activity due to D neutrons that have not crossed the plane at depth x; (b) activity due to D neutrons that have crossed this plane.

From our assumption that the D neutrons can traverse only one free path, it follows that all the neutrons that contribute to the part (b) of the activity have traversed the plane x from below. The term (a) is not changed by the presence of the absorber in the position x, while the term (b) is decreased by the absorber. The total decrease in activity is hence equal to the decrease of the term (b) only. For simplicity we shall in this discussion denote as neutrons of class b the D neutrons that arrive on the plane x from below.

The decrease of the activity of the detector given in column two, Table IX, can thus be calculated by taking into account only the neutrons of class b. From our assumption, it follows that the layer of paraffin of thickness x that the neutrons of class b must traverse in order to reach the detector must be considered as an absorbing (and not as a scattering) layer, having absorption coefficient $1/\lambda_D$.

When there is no absorber, the activity of the detector due to the neutrons of class b only is thus proportional to

$$N_b c(x/\lambda_D),$$

where N_b is the number of the neutrons of class b, and c is the function defined by (6) (absorption curve taking into account the obliquity of the neutrons).

We have implicitly made the assumption that the angular distribution of the neutrons of class b is independent of the depth x; in reality this assumption is not quite correct, but we believe that it will not introduce an appreciable error.

Instead, when the absorber is in the plane x, the activity of the detector due only to the neutrons of class b is proportional to

$$N_b c\left(\frac{x}{\lambda_D}+K_D\delta\right),$$

where δ is the thickness of the absorber and K_D its absorption coefficient for the D neutrons.

Therefore the difference of the two activities is proportional to

$$N_b\left\{c\left(\frac{x}{\lambda_D}\right)-c\left(\frac{x}{\lambda_D}+K_D\delta\right)\right\}.$$

TABLE IX. *Data for D and A neutrons.* x = depth of absorber.

	Group D			Group A		
x	Decrease of Activity	Activity of Absorber	Lower Activity	Decrease of Activity	Activity of Absorber	Lower Activity
0	12.0	102	68	15.3	27.3	17.4
0.16	9.6	105	71	11.6	31.2	(18)
0.34	6.4	124	75	8.3	34.6	(19)
0.87	2.7	173	93	3.8	51.5	(22)
1.87	0.5	244	101	0.5	61.5	23.1

This expression corresponds to the data of column two, Table IX.

In column four is given the activability of the absorber due only to the neutrons of class b; this is evidently proportional to N_b (see §11). Therefore the ratios of the corresponding data of the second and fourth columns are proportional to

$$c\left(\frac{x}{\lambda_D}\right)-c\left(\frac{x}{\lambda_D}+K_D\delta\right). \tag{18}$$

For our absorber of rhodium $K_D=1.8$ cm²/g; $\delta=0.36$ g/cm² so that $K_D\delta=0.65$. Similar argument holds for group A ($K_A=20$ cm²/g; $\delta=0.057$ g/cm²).

In Fig. 6 are plotted the curves (18) for groups A and D, assuming their values equal to 100 for $x=0$; the points represent the experimental values of the ratio of columns two and four, Table IX. The abscissae were calculated assuming $\lambda_D=\lambda_A=1.1$ cm.

Therefore it seems that the neutrons of these two groups have the same mean free path which is a little larger than 1 cm. We have already noted that this method is based on the assumption that the neutrons of these groups traverse only one mean free path. Though this assumption is quite reliable for group A, it is more doubtful for group D (see §12). However, the fact that we have found approximately the same value for λ_D and λ_A can be considered an argument for the validity of this assumption.

8. Mean Free Path of Thermal Neutrons

In the preceding paragraphs, we have found the value of the mean free path of the groups D and A. The value found depends on the assumption that the neutrons of these groups traverse only one mean free path (see §12).

In order to check this assumption, one could measure directly the mean free path of these groups. Up to now, we have not been able to perform such a measurement because of feeble intensity. However, we were able to measure directly the mean free path λ of thermal neutrons in paraffin.

This result, plus the measured value of λ^2N of the preceding paragraph, allows one to calculate the average number N of mean free paths of a thermal neutron. We must remember that the albedo measurements also permit one to calculate the value of N (see §6).

The measurement of the mean free path λ can be made using a source and a detector of thermal neutrons placed at a distance as large as possible with respect to their dimensions, and interposing between them various thicknesses of paraffin, the dimensions of which are just sufficient to screen the detector from the source. In ideal geometric conditions, when the solid angles under which the detector is seen from the source, and the source from the detector, the activity of the detector would be proportional to $e^{-x/\lambda}$ where x is the thickness of the scatterer of paraffin.

In practice, in order not to get too small an intensity, we were very far from these ideal conditions, and therefore it is necessary to correct the results in order to take into account the imperfection of the geometry.

The disposition used was as follows:

Source

A paraffin cylinder of 12 cm diameter and 13 cm in height contained the source of Rn+Be, 2 cm below the center of the upper face. This cylinder was completely covered with the cadmium sheet of 0.5 g/cm²; in the center of the upper face there was a window of 5×5.5 cm² in the cadmium, which could be opened or closed. If we measure the activity of a detector of slow neutrons, placed outside such a paraffin block, once with open window and once with closed window, and we take the difference of the two activities so measured, we will obtain the part of the activity due only to neutrons C coming out through the window. This difference is therefore equivalent to the activity that would be due to a source of only neutrons C having the same position and dimensions as the window. In this sense we will speak in following paragraphs of source of neutrons C, or simply source C.

Detector

In order to be able to have reasonably good geometrical conditions, and sufficient intensity, we have constructed[18] some small cylindrical

---

[18] E. Amaldi and E. Fermi, Ric. Scient. **VII-I**, 393 (1936).

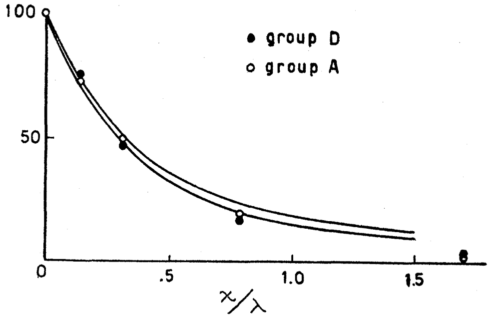

FIG. 6. Ratio of activities (Eq. 18). Dots are the experimental values. $\lambda_A = \lambda_D = 1.1$ cm.

ionization chambers of 3.5 cm inside diameter and 10 cm in length, full of oxygen at a pressure of 75 atmospheres. The element which must be irradiated (Rh 0.125 g/cm^2) was placed, as a cylindrical sheet of 5 cm length and 3.5 cm diameter, inside the chamber, and was itself the electrode at high potential. The other electrode, connected to the electrometer, was a metal rod placed on the axis of the chamber. The wall of the chamber was of steel 4 mm in thickness and it practically absorbed none of the slow neutrons which, going through the wall of the chamber, could impinge on the detector.

This arrangement has the advantage, with respect to the methods we used up to now, that due to the high pressure, almost all the energy of the β-rays is spent inside the chamber; besides the electrode has a large useful surface and the geometrical conditions are very good. In fact, with this disposition it is possible to obtain almost the same sensitivity of observation as made with counters, and it has the advantage of the greater stability of the ionization chambers.

The chamber connected to the electrometer was irradiated for 2 minutes, and 15 seconds after having taken away the source we started the reading; i.e., we measured the number of scale divisions covered in 2 minutes.

We have performed two series of measurements: In the first one the distance of the source C from the axis of the ionization chamber was 20 cm; in the second one, 10 cm; in both series the paraffin scatterers were put at half distance between the source C and the detector. In order to avoid random diffusions, the ionization chamber and all the path between the source C and the detector was screened by a cadmium layer of 0.5 g/cm^2.

In Table X are given in arbitrary units the results of the two series of measures (each value is the average of three readings); the error is about one unit.

In the first column are given the thicknesses of the scatterers of paraffin (density 0.9) in centimeters. In columns two and three are given the measurements of activity at the distance of 20 cm, with the cadmium window of the source opened and closed. The activity due only to the neutrons C coming out from the window is, as we have said above, the difference of the values of these columns. Columns four and five are similar to columns two and three, for the case that the distance between the source and the detector is 10 cm.

In order to deduce from these measurements the value of the mean free path of the thermal neutrons, it is necessary to take into account the corrections due to the imperfections of the geometry, which are especially large in the measurements at 10 cm distance. Calculation of the corrections for thick scatterers are rather unreliable; instead, it is possible to obtain a fairly accurate evaluation of the correction for thin scatterers.

For the distance of 20 cm and a thin scatterer, the correction was calculated as follows: the cross section of the detector was 17.5 cm^2, and the area of the scatterer was 22.3 cm^2; the neutrons that reach the detector are in part those that did not undergo collision in the scatterer, and in part those that had collided once in the scatterer (the case of multiple scattering can be neglected when the thickness x of the scatterer is small).

The first number of neutrons is evidently proportional to the area (17.5 cm^2) of the detector, and to the probability $e^{-x/\lambda}$ that a neutron did not collide in the scatterer. For small x, the number of these neutrons is proportional to

$$17.5(1-x/\lambda).$$

The small effect due to the obliquity of the neutrons' path in the scatterer has been neglected.

On the other hand, the number of neutrons that strike the scatterer is proportional to 4

TABLE X. *Measurement of activity for different thicknesses* (x) *of paraffin with detector at 10 cm and 20 cm from the source.*

x	20 cm		10 cm	
	without Cd	with Cd	without Cd	with Cd
0	45.1	23.6	175	92.5
0.047	40.5	22.1	162	89.5
0.099	36.8	21.7	158	90
0.203	29.9	19.3	140.5	87
0.38	24.8	17.8	127	86
0.68	21.2	16.7	116.5	87

times its area, because its distance from the source is one-half the distance of the detector from the source. This number is hence proportional to

$$4\cdot 22.3=89.2.$$

The fraction x/λ of these neutrons has a collision in the scatterer; assuming that the C neutrons are scattered isotropically by the paraffin hydrogens (see §13 and *F*, §§10 and 11), we find that the fraction of the scattered neutrons that impinge on the detector is equal to the ratio of the area of the detector to the area of a sphere with radius equal to the distance between the scatterer and the detector (10 cm).

The total number of scattered neutrons which hit the detector is hence proportional to

$$89.2\cdot(x/\lambda)\cdot(17.5/4\pi 10^2).$$

Adding this number to the number of neutrons that hit the detector without being scattered by the scatterer, we obtain the total number of neutrons that impinge on the detector as a function of x. This number is proportional to

$$17.5(1-0.929(x/\lambda)+\cdots).$$

Therefore, assuming as 1 the activity for $x=0$, we find that the activity of the detector as a function of the thickness x of the scatterer is given by

$$1-0.929(x/\lambda)+\cdots.$$

From this relation we obtain that the tangent of the diffusion curve for $x=0$ cuts the abscissae axis at

$$x_1=(\lambda/0.929)\text{ cm}.$$

From a graph of the experimental data we have found, by means of the method of the secants, the tangent for $x=0$; this cuts the axis of the abscissae at the point

$$x_1=0.29\text{ cm}.$$

Thus we find

$$\lambda=0.29\cdot 0.929=0.27\text{ cm}.$$

We have also verified that all the entire experimental diffusion curve could be brought to coincide, also for thick scatterer, with the calculated curve (calculation, of which we do not give details, is performed by means of the methods explained in *F*, §5).

A little more complicated is the calculation of the corrections for the experiments at 10 cm, because in this case one cannot neglect the obliquity of the neutrons.

The calculation was performed with a numerical procedure from which we have found as expression of the initial shape of the diffusion curve

$$1-0.84(x/\lambda)+\cdots.$$

From the experimental curve we have found, as above,

$$x_1=0.84\text{ cm}$$

so that from this measurement we have

$$\lambda=0.84\cdot 0.4=0.34\text{ cm}.$$

From these two combined measurements we can obtain, as the more probable value of the mean free path of thermal neutrons (see §13),

$$\lambda=0.3\text{ cm}.$$

9. GROUPS AS A FUNCTION OF THE DISTANCE FROM THE SOURCE; ENERGY RATIOS

In a preceding work[19] we have measured the activity of a rhodium detector in a water tank as a function of the distance from the source. The data given there correspond to a mixture of groups C and D, which are both efficient in activation of rhodium. It is evidently interesting to study, as a function of the distance from the source, the behavior of each single group.

The measurements were performed in a cylindrical water tank 95 cm deep and 90 cm in diameter. As detectors, we used:

---

[19] E. Amaldi, O. D'Agostino, E. Fermi, B. Pontecorvo, F. Rasetti and E. Segrè, Proc. Roy. Soc. **A149**, 522 (1935).

For groups C and D: rhodium (0.36 g/cm^2), area 5×5.5 cm^2): this detector, screened by 0.5 g/cm^2 of cadmium, is sensitive only to the D neutrons; instead the difference between the activities measured without and with cadmium gives the activity due only to group C. A detector of silver of 0.057 g/cm^2 and 5×5 cm^2 area, screened by cadmium, was used as a detector of groups $A+B$, which were not studied separately. Finally, for the group I we have used a detector of 0.068 g/cm^2 of PbI_2 screened by cadmium, the area of which was 5×5 cm^2.

The activities of the detectors were measured on both sides at various distances from the source; the sum of the two activities is proportional to the number of neutrons of the various groups which enter the detector (see §11).

In order to keep as small as possible the errors due to the finite dimensions of the tank, the measurements were performed, maintaining in the center of the water cylinder the mean point between the source and the detector.

The dimensions of our detectors, and also the bulb containing the Rn+Be, are not small enough that one can consider the distance between the centers of the source and the detectors equal to the distance between these two objects, especially for small distances. Therefore we have used average distances. The experimental results for groups C, D, $A+B$, and I are given in Tables XI, XII, XIII, XIV.

The numbers are the sum of the activities measured on both sides of the detectors. The data on groups C, D and $A+B$ are obtained as average values of three series of measurements on each side; for group I, we have made only one series of measurements on each side, since the period of iodine is long enough to get sufficiently accurate results.

In Fig. 7 the activations multiplied by r^2 (r = distance from the source) are plotted against r. The data are taken from Tables XI, XII, XIII, XIV; the units of the ordinates have been chosen so that each curve incloses the same area.

Although the differences among the curves D, $A+B$, and I are small, the accuracy of the measurements is sufficiently high to distinguish among them. By inspecting these curves, one can see that the activity due to the various groups decreases as a function of r according to different laws; the activity decreases more rapidly for groups in the following order: C, D, $A+B$, I. This fact enables us to dispose the groups in order of the increasing energy. We notice that neutrons of lower energy have, as an average, collided more times after they have been emitted, than neutrons of high energy; on the other hand, neutrons that have collided more times will as an average have diffused to a larger distance from the source. It follows that groups for which the intensity decreases more rapidly as a function of r, are those of greatest energy. The order of increasing energy is therefore, for the groups that we have investigated, the following:

$$C,\ D,\ A+B,\ I.$$

These considerations can be made quantitative, as follows: a convenient parameter, in order to characterize the rapidity of the decrement of the various groups as a function of the distance, is the mean value $(r^2)_{\text{av}}$ of the distances from the source for the neutrons of the various groups. $(r^2)_{\text{av}}$ was calculated for each group by

$$\int_0^\infty f(r)r^2dr \Big/ \int_0^\infty f(r)dr$$

where $f(r)$ is the curve of Fig. 7 referring to the group under consideration. The values of $(r^2)_{\text{av}}$ are given in Table XV.

Although the curves of the activities as a function of the distance have been measured very accurately, we were not able to measure the intensity of groups D, $A+B$, and I up to a long distance. In the calculation for $(r^2)_{\text{av}}$ we had therefore to extrapolate the intensities of these groups for great distances. These extrapolations are relatively reliable as for great distances the law of decrease becomes quite similar for all the groups so that the end part of the curves of Fig. 7 can be represented satisfactorily by exponential with coefficient 0.106 cm^{-1}.

Nevertheless, a considerable error might be due to this extrapolation.

It can be proved (see F, §2) that for groups of energy larger than the quantum $h\nu$ of the frequence of the elastic bond of hydrogen in paraffin (in practice, for groups of energy larger than 1 volt) the difference $(r'^2)_{\text{av}}-(r''^2)_{\text{av}}$ in the average square distances of two groups is connected to the ratio W''/W' of the corresponding energies by the equation

$$\log\frac{W''}{W'}=\frac{(r'^2)_{\mathrm{av}}-(r''^2)_{\mathrm{av}}}{6\lambda_g^2},\tag{19}$$

λ_g being the mean free path of the neutrons of the two groups in paraffin. The assumption has been made that λ_g is equal for the two groups (see §7).

The preceding equation can easily be understood qualitatively, but for the numerical factor in the denominator; the left-hand side represents the average number of impacts necessary for the reduction of the energy from W'' to W'. On the other hand, the mean square of the displacement of a neutron in a free path is $2\lambda_g^2$ (the factor 2 is due to having performed mean square and not the square of the mean). If the orientations of successive free paths were incoherent, $(r^2)_{\mathrm{av}}$ would then increase by $2\lambda_g^2$ for each impact, and then $\log W''/W'$ impacts would produce an increase by $2\lambda_g^2 \log W''/W'$; this assumption would then give a formula similar to (19) with a factor 2 instead of 6 in denominator. Factor 6 is obtained by taking into account the coherence of successive free paths, and also the fluctuations in the number of impacts necessary for reducing the energy from W'' to W'. We can use (19) in order to determine the energy ratio of two groups (except group C, for which this formula does not hold). This method is, however, very inaccurate, since small errors in λ_g and $(r^2)_{\mathrm{av}}$ affect the ratio W''/W' considerably.

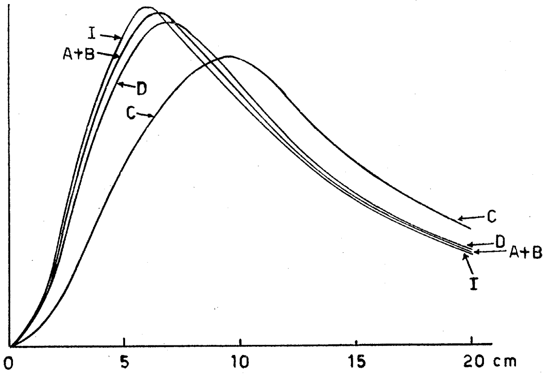

FIG. 7. Ordinates = activation × r^2. Abscissae = r. The units of the ordinates are chosen so that each curve includes the same area.

TABLE XI. *Group C (difference of the activity of Rh detector without and with cadmium).*

r (cm)	2.5	3.6	5.4	7.3	10.2	15.1	20.1	25	30	35	40
Activity	2121	1898	1441	1006	523	167	58	23	9	3.3	1.7

TABLE XII. *Group D (Rh detector screened by cadmium).*

r (cm)	2.5	3.6	5.4	7.3	10.2	15.1	20.1
Activity	306	252	166	96	40	10.6	3.8

TABLE XIII. *Groups A+B (Ag detector screened by cadmium).*

r (cm)	2.5	3.6	5.4	7.3	10.2	15.1
Activity	144	119	73	39.6	15.7	4.3

TABLE XIV. *Group I (PbI_2 detector screened by cadmium).*

r (cm)	2.62	3.36	5.1	7.0	9.9
Activity	24.9	19.7	12.0	6.28	2.46

As we measured $(r^2)_{\mathrm{av}}$ in water, and not in paraffin, we shall have to put for λ_g the mean free path in water; we can deduce this from the value in paraffin (1.1 cm, for nonthermal neutrons, see §7) by assuming the mean free path to be inversely proportional to the concentration of hydrogen; we find then $\lambda_g=1.27$ cm.

Assuming this value, we find from (19) that the difference in $(r^2)_{\mathrm{av}}$ corresponding to a ratio e in the energies is $6\lambda_g^2=9.7$ cm^2.

From Table XV we obtain the following ratios of the energies for the groups I, $A+B$, and D:

$$W_I : W_{A+B} : W_D = 4.4 : 1.85 : 1.$$

It has been already mentioned that the error in these ratios can be very large.

It is perhaps more appropriate to deduce from (19) the values of the mean free path in water by assuming that the energies of the groups are inversely proportional to the square of their absorption coefficients in boron[20] (see also §13).

Taking the following absorption coefficients in boron

$$K_I=1;\quad K_B=2.3;\quad K_A=3;\quad K_D=4.7;\quad K_c=38\ \mathrm{cm^2/g}$$

---

[20] O. R. Frisch and G. Placzek, Nature **137**, 357 (1936); D. F. Weekes, M. S. Livingston and H. A. Bethe, Phys. Rev. **49**, 471 (1936).

we obtain the following energy ratios:

$$W_I : W_B : W_A : W_D : W_c = 1440 : 270 : 160 : 65 : 1.$$

Assuming[21] $W_c = kT = 0.025$ volts
we obtain

$$W_I = 36;\quad W_B = 7;\quad W_A = 4;\quad W_D = 1.6 \text{ volts}.$$

From these values and (19), we find the mean free path in water

$$\lambda_g = 0.87 \text{ cm}$$

instead of 1.27 cm. The difference is within the limits of the experimental error.

We notice further that $(r^2)_{\text{av}}$ for the C group given in Table XV is considerably different from the values of $(r^2)_{\text{av}}$ for the other groups. For instance,

$$(r_C{}^2)_{\text{av}} - (r_D{}^2)_{\text{av}} = 50 \text{ cm}^2.$$

This difference is partly due to the large number of free paths of the neutrons when their energy is already reduced to the thermal energy; the amount due to this process of diffusion is (see §7)

$$2\lambda^2 N = 26 \text{ cm}^2.$$

The remaining 24 cm² correspond to the slowing down of the neutrons from the energy corresponding to the D group to the energy of thermal agitation. This process cannot be easily calculated, because it is complicated by the effect due to the chemical bond of hydrogen (see F, §19).

10. Passage of the Neutrons from One Group to Another

In the preceding section, we arranged the groups in order of decreasing energy. It is now obvious that a neutron of a group of high energy, after some impacts, will have a smaller energy, and may thus go over into a group of less energy. We must, to be more precise, expect that the neutrons of all the groups, after a certain number of impacts, become thermal neutrons.

Table XV. *Mean values of r^2.*

Group	$(r^2)_{\text{av}}$ (in cm²)
C	326.7
D	276.6
$A+B$	270.6
I	262.2

We tried, therefore, to observe the transformation of D neutrons into C neutrons.[22]

The principle on which these experiments are based can be clearly understood if we assume that we have a detector R that can be activated only by C neutrons and an absorber A for the D neutrons only. We put the detector R on the center of the upper face of a paraffin cylinder, containing the source S; if the absorber is inserted between the upper face of the paraffin and the detector, no reduction of the activity of the detector will occur, since the C neutrons that came out of the paraffin are not absorbed by A. If we put the absorber A inside the paraffin, 1 or 2 cm below the upper face, it can happen that the absorber captures some of the D neutrons which could be transformed into C neutrons by impacts in the paraffin before reaching the detector.

If D neutrons can be really transformed into C neutrons, we must expect that the absorber A produces no effect when put between the paraffin and detector, but that it reduces the activity of A when put 1 or 2 cm inside the paraffin.

The experiment cannot be performed in this ideal way, because we have no detector of group C only, and no absorber of group D only. We have, therefore, used as detector a rhodium plate, 0.36 g/cm² thick; about 70 percent of its activity is due to the C group and the rest to the D group. In order to measure only the activity due to the C group, all measurements were performed without and with a cadmium absorber 0.54 g/cm² thick, that was put immediately below the rhodium plate. The difference in activities without and with cadmium gives the activity due to the C group only.

As absorber for the D group, we used an indium plate 1.83 g/28.5 cm²; the absorption coefficient of indium for the D group is 3.4 cm²/g, and for the C group 0.7 cm²/g. It follows that the absorption of C neutrons in our indium plate is far from negligible, the more so if we keep in mind the high numerosity and albedo of this group. In order to correct for this, we prepared a plate of Sn − Cd alloy that was

---

[21] H. H. Goldsmith and F. Rasetti, Phys. Rev. **50**, 328 (1936) show that it is more correct to use kT instead of $3kT/2$ as thermal energy in this calculation.

[22] E. Amaldi, E. Fermi, Ric. Scient. **VII-I**, 56 (1936).

equivalent to the indium plate as absorber for the C group, and practically did not absorb the D neutrons. Taking the difference of the activities produced by group C in the detector with the indium plate or the cadmium plate, we obtain the effect due only to the absorption of the D group by indium.

In Table XVI are collected the activities induced in the rhodium plate with different setups; the different arrangements are denoted by a notation similar to that used in §6; the number added in parentheses after the symbol P of paraffin, gives in centimeters the thickness of paraffin layers; since we used two different cadmium sheets, one of 0.54 g/cm^2 thickness for complete absorption of group C, and the other 0.0036 g/cm^2 thickness equivalent to the indium sheet as absorber in the C group, we denoted these two absorbers, respectively, with Cd and cd. The figures given in the table are averages of ten readings.

The first five measurements are performed in order to control the identity of the thin cadmium (Cd) and the indium (In) as absorbers of the C neutrons. Their result is that the absorption of the indium plate is probably slightly larger than the absorption of the cadmium sheet; the difference $280.4-68.7-(230.2-45.6)=27.1\pm1.2$ represents the activity of the rhodium detector due to group C that is absorbed by the indium plate; the difference $280.4-245.8=25.6\pm1$ is the analogous absorption of the thin cadmium sheet.

The next four measurements are performed with the absorber (Cd or In) 1 cm inside the paraffin. The difference $253.0-71.1=181.9\pm0.9$ is the activity of the detector due to the group C with the absorber Cd 1 cm inside the paraffin. The difference $241.5-64.9=176.6\pm0.9$ is the similar activity with the indium absorber. The difference between these two activities is 5.3 ± 1.2 and is considerably larger than the difference that we might expect from the small difference between the two absorbers when placed close to the detector.

The next four measurements are analogous to these last, with the only difference that the absorbers are 2 cm below the paraffin surface. Also in this set of measurements there is a difference analogous to that observed in the preceding set; this difference is now 4.0 ± 1.2.

TABLE XVI. *Activities induced in Rh plate under different arrangements.* S=source, P=paraffin (cm thick), R=detector.

$S\,P(3.5)\,R$	280.4±0.7
$S\,P(3.5)$ In R	230.2±0.7
$S\,P(3.5)$ In Cd R	45.6±0.4
$S\,P(3.5)$ Cd R	68.7±0.5
$S\,P(3.5)$ cd R	254.8±0.7
$S\,P(2.5)$ cd $P(1)\,R$	253.0±0.7
$S\,P(2.5)$ In $P(1)\,R$	241.5±0.7
$S\,P(2.5)$ cd $P(1)$ Cd R	71.1±0.5
$S\,P(2.5)$ In $P(1)$ Cd R	64.9±0.5
$S\,P(1.5)$ cd $P(2)\,R$	254.0±0.7
$S\,P(1.5)$ In $P(2)\,R$	248.9±0.7
$S\,P(1.5)$ cd $P(2)$ Cd R	69.2±0.5
$S\,P(1.5)$ In $P(2)$ Cd R	68.1±0.5

By discussing together all these data, one might conclude that the observed effect has a sign corresponding to a transformation of D neutrons into C neutrons, and is 2.3 times larger than the quadratic error.

Although we have a serious presumption of a real effect in this sense, we cannot deduce from this experiment a final conclusion.

An evaluation of the effect that one must expect in this experiment shows that it is really of the order of magnitude of the observed effect; the evaluation was made as follows:

We have measured the induced activities in the indium sheet of the above experiment, placed inside the paraffin 2 cm below the upper face, separately in group C and D, and we have found, respectively, 257 and 47.5. These numbers are proportional to the numbers of neutrons of the two groups absorbed by our indium sheet.

Let p_1 be the probability that a neutron C which is in paraffin in the place of the indium absorber comes out from the upper face and is captured by the rhodium detector; similarly, let p_2 be the probability that a neutron D which is in the place of the indium sheet, comes out from the upper face transformed into a C neutron, and is captured by the rhodium.

The decrements of the activities of the rhodium sheet in group C, due, respectively, to the absorption by indium sheet of neutrons D and C are in the ratio $47.5\,p_2/257\,p_1$. Assuming p_1 and p_2 equal or at least of the same order of magnitude, we find that these decrements are about in the ratio 47.5/257, and, since the decrement due to the absorption of C neutrons is 26.9 (see Table XVI), the effect which we must expect in the preceding experiment is equal to 5, and therefore of the same order of magnitude as the really observed effect.

Similar experiments were performed by Preiswerk and von Halban,[23] who found a genetic relation between groups I and A.

11. Behavior of the Groups in the Neighborhood of the Surface of the Paraffin

In §9, we have studied the activity due to the various groups inside a water tank so large that we could neglect the disturbances due to the limitations of size. We will now study the behavior of the activity due to different groups close to the surface of the paraffin.

First, we shall consider the behavior of group C. From the theory of diffusion of thermal neutrons, it follows that, independently of the position of the source, the density of thermal neutrons close to the surface of the paraffin decreases toward the outside (see F, §6, formula 57); one can furthermore show that if we indicate with x the depth below the surface of the paraffin, the density $n(x)$ is, for small x, approximately proportional to

$$x+\lambda/\sqrt{(3)},$$

so that if we extrapolate the curve of $n(x)$ towards the outside of the paraffin ($x<0$), $n(x)$ is zero at a distance from the surface of the paraffin equal to

$$x_0=-\lambda/\sqrt{3}.$$

This is just the behavior of the density of thermal neutrons close to the surface which determines the law of angular distribution (5) of the outcoming C neutrons. If the density would be constant close to the surface of the paraffin, one would find the cosine law.

In order to measure $n(x)$, it is necessary to use a detector of group C thin enough not to disturb the neutrons' distribution. Its activity will then be proportional to the density $n(x)$ of thermal neutrons.

We have used two detectors which were obtained by electrolytic deposit of rhodium on a nickel plate (which served only as an inactive carrier) of 29 cm^2 area; the weights of rhodium deposited on the two detectors were respectively 0.087 and 0.169 g. Taking into account the absorption coefficient of rhodium for the C group (0.7 cm^2/g) $K\delta$ for these two detectors is equal, respectively, to 0.002 and 0.004; to these values there corresponds a mean probability ζ for the capture of a thermal neutron which traverses the detector, of about 0.004 and 0.008, respectively (see §6). These probabilities are very small, and therefore the detectors used can be considered as thin.

The measurements have been performed using a paraffin cylinder 24 cm in diameter and 10 cm in height, containing the source 3.2 cm below the center of the upper face. The layer of paraffin of 3.2 cm thickness between the source and the upper surface was cut into sheets in order to be able to put the detector at various depths.

In order to find the activity due to the group C only, we have always made the difference between the activities of the detectors without and with two cadmium screens 0.44 g/cm^2 thick, between which could be placed the detectors.

In Fig. 8 the activity due only to the C group of our two detectors is plotted against the depth x. We give also the curve extrapolated towards negative values of x. The intersection, with the axis of the abscisse is close to the point

$$x_0=-0.18 \text{ cm}.$$

From this value we find

$$\lambda=0.31 \text{ cm}$$

in good agreement with the values obtained from the direct measurements (see §8).

We have already noticed that this experiment has a very simple interpretation, provided the detector is very thin. Instead, a thick detector disturbs strongly the density of thermal neutrons, so that the curve of the activity as a function of x is very different.

Thus, using a rhodium detector 0.36 g/cm^2 thick, and adding the activities due to C group measured on both sides, in order to get data comparable to that of the preceding experiment, we have found that the curve of the activity,

---

[23] Preiswerk and von Halban, Comptes rendus **202**, 840 (1936).

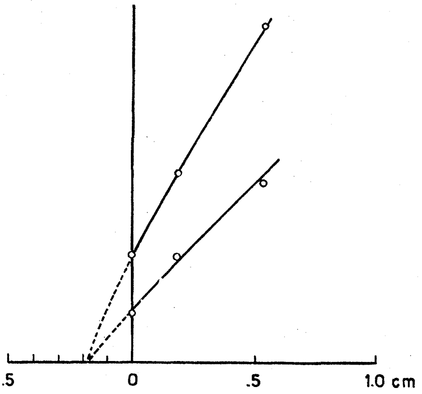

FIG. 8. Activity of C group plotted against the depth (x) in paraffin.

extrapolated toward negative values of x, cuts the x axis at

$$x_0 = -0.4 \text{ cm},$$

i.e., at a distance more than double than in the case of thin detectors.

In this experiment, we have added the activities measured on both sides of the detector, since this sum is proportional to the total number of thermal neutrons which entered the detector. Let N_1 and N_2 be the numbers of neutrons which enter the detector, respectively, through the face 1 and face 2; the activities measured on the faces 1 and 2 are, respectively, given by

$$A_1 = N_1 a + N_2 b, \qquad (20)$$
$$A_2 = N_1 b + N_2 a,$$

where a and b are constants characteristic of the detector, which depend on its absorption coefficients for neutrons and electrons, and on its thickness. The ratio a/b can be obtained by measuring the ratio A_1/A_2 when the detector is placed outside the paraffin so that $N_2 = 0$.

For the rhodium detector, 0.36 g/cm² thick, $a/b = 4/3$ in the C group, and 2/1 in the D group. This difference depends on the larger absorption coefficient for D neutrons. For a thin detector, one would have $a = b$.

Summing up the two equations in (20), we find

$$A_1 + A_2 = (a+b)(N_1 + N_2),$$

and therefore the sum of the activities measured on both sides of a detector is always proportional to the total number $N_1 + N_2$ of neutrons which traverse the detector.

By (20) we find

$$N_1 = \frac{aA_1 - bA_2}{a^2 - b^2}; \quad N_2 = \frac{aA_2 - bA_1}{a^2 - b^2}. \qquad (21)$$

Therefore, measuring A_1 and A_2, and the ratio a/b, we can find values proportional to N_1 and N_2 (this method was used in §7 in order to measure separately N_1 and N_2). In Table XVII are given the values of N_1 and N_2 measured with our rhodium detector, 0.36 g/cm² thick for groups C and D separately. In order to be able to compare these two sets of data, we have taken as 1 in both groups the value of N_1 for $x = 0$. The source in these experiments was at $x = 2.4$ cm. The data on group C do not permit simple interpretation, since the detector cannot be considered as thin.

Instead, in group D the detector, although thick, does not disturb the distribution of the D neutrons, since the neutrons of this group traverse on the average only one mean free path (see §§6, 7, 12), and therefore each of them hits only once against the detector.

By extrapolation of the shape of the curve $N_1 + N_2$ for D group towards negative values of x, we find the intersection with the x axis at

$$x_0 = -0.9.$$

The fact that these intersections occur much farther from the surface of the paraffin than for group C ($x_0 = -0.18$ cm with thin detector), is due on one hand to the greater mean free paths (see §7), and on the other hand to the coherence of the orientation of successive free paths, which is very large for neutrons of energy larger than the quantum of the elastic bond of hydrogen in paraffin.

By comparing columns two and four, Table XVII, we notice that the shape of the curve N_1

TABLE XVII. *Number of neutrons that enter faces (1) and (2) of detectors.* x = thickness of paraffin.

	Group C		Group D	
x	N_1	N_2	N_1	N_2
0	1	0	1	0
0.16	1.10	0.28	1.07	0.07
0.34	1.15	0.63	1.17	0.14
0.53	1.28	0.91	1.25	0.28
0.87	1.60	1.19	1.31	0.45
1.87	2.19	2.00	1.35	1.00

close to the surface, for our detector, is not very different in groups C and D, instead, at greater depth the C group increases much more rapidly than group D. One can justify this behavior by taking into account the increase of the albedo of the layer of paraffin, superimposed on the detector, as a function of its thickness.

12. Width of the Energy Bands Corresponding to the Groups. Total Number of Neutrons

We shall discuss in this section a method for determining the width of the bands[24] of energy which correspond to the various groups; more precisely, the ratio $W_{\max}/W_{\min}$ of the maximum to the minimum energy which limits a band. In the calculation the bands were assumed to be sharply defined. It would be possible to perform a similar calculation, assuming for the band a resonance form; however, this seems scarcely worth while, owing to the small accuracy of the experimental data.

In the last part of this section we will give an evaluation of the total number of neutrons emitted by the source of Rn+Be.

The method for determining the width of the band which corresponds to a group g of energy larger than 1 volt, is based on the comparison of the two following values: the activability A_g of a detector due to the neutrons of the considered group g, and the activability B_c of the same detector due to the thermal neutrons and obtained by screening the detector on one side with a cadmium sheet thick enough to absorb completely the thermal neutrons hitting on it. B_c can naturally be obtained as the difference of the activabilities of our detector with a cadmium sheet on one face only or on both sides.

If the distribution of the fast neutrons inside a large block of paraffin were uniform, it is clear that A_g and B_c would have values independent of the position and orientation of the detector. However, the source of fast neutrons is small and therefore we must compare the mean values of A_g and B_c averaged for all positions and orientations inside the paraffin; i.e., we must compare

$$\int \tilde{A}_g d\tau; \quad \int \tilde{B}_c d\tau,$$

where the integrands $\tilde{A}_g$ and $\tilde{B}_c$ are the mean values of the activabilities of the detector measured with the two opposite orientations; these mean values are practically identical with the average taken over all directions. By using the said integrals we eliminate all the complication arising from the nonuniform distribution of the neutrons.

We suppose now that in the paraffin around our detector, q fast neutrons are produced per cubic centimeter and per second. These neutrons are slowed down in such a way that we shall have in the neighborhood of our detector neutrons of all velocities.

It can be shown (see F, §1) that for energies larger than 1 volt, the number of neutrons having velocity between v and $v+dv$ is

$$(2q\lambda(v)/v^2)dv. \tag{22}$$

From this we can easily calculate the activity A_g as a function of the energy $W_{\min}$ and $W_{\max}$ limiting the energy band g, of the mean free path λ_g and the absorption coefficient K_g of the detector for the group g and of the surface s and thickness δ of the detector. We find (see F, §8)

$$A_g = \eta s q \lambda_g K_g \log \frac{W_{\max}}{W_{\min}} \times \int_0^\delta b(K_g x) e^{-\mu x} dx + \int_0^\delta b(K_g x) e^{-\mu(\delta - x)} dx \Big/ 2, \tag{23}$$

where μ is the absorption coefficient of the β-rays of the detector in the detector itself and $b(K_g x)$ is the function (4); η is the efficiency of the ionization chamber for the β rays of the detector. The last factor in (23) represents the effect of the absorption of the neutrons and of the β-rays in the detector. This factor would be equal to δ for a very thin detector.

Also the activity B_c due to the thermal neutrons can be calculated in terms of the diffusion length $(D\tau)^{\frac{1}{2}} = (\lambda^2 N/3)^{\frac{1}{2}}$ (see §7), of the ab-

---

[24] E. Amaldi and E. Fermi, Ric. Scient. **VII-I**, 310 (1936).

sorption coefficient K_c of the detector; taking into account the angular distribution (5) of the neutrons impinging on the detector, we find

$$B_c=\eta sq\lambda(\mathrm{N})^{\frac{1}{2}}K_c\int_0^\delta c(K_cx)e^{-\mu x}dx, \tag{24}$$

where $c(K_cx)$ is the function (6). We have disregarded the variation of the absorption coefficient of our detector for thermal neutrons of different velocities. For a very thin detector, the integral is equal to δ.

We must now integrate (23) and (24), as already mentioned, and we obtain

$$\int A_gd\tau=\eta sQK_g\lambda_g\log\frac{W_{\max}}{W_{\min}}\times\int_0^\delta b(K_gx)e^{-\mu x}dx+\int_0^\delta b(K_gx)e^{-\mu(\delta-x)}dx\Big/2, \tag{25}$$

$$\int B_cd\tau=\eta sQK_c\lambda(\mathrm{N})^{\frac{1}{2}}\int_0^\delta c(K_cx)e^{-\mu x}dx,$$

where $Q=\int qd\tau$ is the total number of neutrons emitted by the source per second. From (25) we obtain

$$\log\frac{W_{\max}}{W_{\min}}=\frac{K_c}{K_g}\frac{\lambda(\mathrm{N})^{\frac{1}{2}}}{\lambda_g}\frac{\int A_gd\tau}{\int B_cd\tau}\times\frac{2\int_0^\delta c(K_cx)e^{-\mu x}dx}{\int_0^\delta b(K_gx)e^{-\mu x}dx+\int_0^\delta b(K_gx)e^{-\mu(\delta-x)}dx}. \tag{26}$$

We performed the experiments on the groups D, A, and I. For D group we used a rhodium detector (0.36 g/cm^2); for the group A a silver detector (0.057 g/cm^2); and for group I a lead iodide detector (0.76 g/cm^2); since the small absorption coefficient of iodine in the C group is not too well known, we used the rhodium data also in the calculation of B_c for the I group, assuming equal efficiency of the chamber for β-rays of rhodium and iodine. In order to calculate the integrals

$$\int\tilde{A}_gd\tau,\quad\int\tilde{B}_cd\tau$$

we determined $\tilde{A}_g$ and $\tilde{B}_c$ for a fixed distance, and deduced the values for all the distances by using the curves of Fig. 7.

In the following equations, the numerical values are written in the same order in which the various magnitudes appear in formula (26)

Group D

$$\log\frac{W_{\max}}{W_{\min}}=\frac{0.7}{1.8}\frac{\surd(13)}{1.1}\frac{4.58\cdot10^5}{1.5\cdot10^6}\frac{2\cdot0.108}{0.087+0.052}=0.60,$$

Group A

$$\log\frac{W_{\max}}{W_{\min}}=\frac{0.25}{20}\frac{\surd(13)}{1.1}\frac{7.8\cdot10^4}{2.04\cdot10^5}\frac{2\cdot0.048}{0.021+0.019}=0.038,$$

Group I

$$\log\frac{W_{\max}}{W_{\min}}=\frac{0.7}{0.38}\frac{\surd(13)}{1.1}\frac{4.16\cdot10^4}{1.5\cdot10^6}\frac{2\cdot0.108}{0.085+0.049}=0.27.$$

From these values we may derive the ratios $W_{\max}/W_{\min}$ for each group; we shall call the magnitude $\log W_{\max}/W_{\min}$ logarithmic width; this magnitude has a simple physical meaning because it represents the average number of impacts of a neutron while it belongs to a given group. For example, group A that has a logarithmic width 0.04 is such a narrow energy band that only 4 percent of the neutrons pass through it during the process of slowing down. In the case of group A, the detector used cannot be considered as thin with respect to the absorption of the neutrons; if one would take into account the resonance form of the band, one would find, by a simple numerical evaluation, that the logarithmic width of group A as calculated from (26) is too large by a factor of about 2. For the other detectors, this correction is much smaller.

The logarithmic width of group I, and even more of group D, is considerably larger, as shown also by the larger numerosity (see §5) of these groups. Even for group D, the probability that a neutron belongs to the group for more than one free path is relatively small and amounts to 0.27. These facts explain our results that the albedo of all nonthermal groups is practically zero.

Instead, one might expect a small difference in the behavior of groups D and A with respect to the diffusion experiments descriped in §7. The fact that we failed to find any such difference might, however, be explained by inaccuracy of our measurements, as the expected difference is fairly small.

If we assume that the energy of the groups are those deduced in §9 by the boron method, we obtain for the width of group D, A and I:

1; 0.15; 10 volts.

Finally, we can use the second of (25) in order to calculate the total number Q of the neutrons emitted from the source. In (25), B_c is the initial activity of the detector used. In order to get the activability (3), it is necessary to multiply B_c given in (25) by the factor

$$1000/I\cdot U=1000/I\cdot 840\eta_U.$$

So that we find

$$\frac{Q}{I}=0.84\frac{\eta_U}{\eta}\frac{\int B_c d\tau}{s\lambda(\mathrm{N})^{\frac{1}{2}}K_c\int_0^\delta c(K_c x)e^{-\mu x}dx}. \qquad (27)$$

In this formula, the same elements enter which we have used in the calculation of the logarithmic width. Using, for instance, the data on the activation of the rhodium, we find

$$Q/I=160{,}000(\eta_U/\eta).$$

Assuming $\eta=\eta_U$ we have that one of our neutronic units corresponds to 160,000 neutrons per second, and taking into account that 1 neutronic unit corresponds to about 6 mc of Rh+Be we have finally

27,000 neutrons per sec. and per mc.

This value is considerably larger than the values found with different methods;[25] this fact can be only partially accounted for by a difference between η and η_U.

13. Summary and Discussion

From the results we have explained, it seems that one must conclude that in elements which are sensitive to slow neutrons, the capture cross section is often an irregular function of the energy of slow neutrons, with sharp maxima representing some sort of absorption bands.

The analysis of the absorption curve of the various elements with several detectors enables us to identify several absorption bands (groups A, B, C, D, I).

There are several reasons for believing that group C (radiation strongly absorbed by cadmium) corresponds, at least for its largest part, to neutrons having a velocity of thermal agitation, while the other groups correspond to larger velocities.

The most direct proof of this statement is the experiment with a velocity selector of cadmium[26] (so as to observe just group C) that enabled a direct measurement of the velocity of the C neutrons that was found equal to the thermal velocity.

The same result can be derived from the experiments by Preiswerk and von Halban and others,[27] who found that the radiation filtered by cadmium is not sensitive to temperature variations; this shows that the neutrons not belonging to group C have energy larger than thermal agitation energy.

Finally, our albedo measurements (see §6) show that only the C neutrons can traverse many free paths while belonging to this group. This fact can be easily understood on the assumption that the C group consists of neutrons in thermal equilibrium, since then successive impacts do not alter the average energy; instead, a neutron belonging to an energy band $\gg kT$, has a high

---

[25] Cf., e.g., R. Jaeckel, Zeits. f. Physik **91**, 493 (1934); F. A. Paneth and H. Loleit, Nature **136**, 950 (1935).

[26] J. R. Dunning, G. B. Pegram, G. A. Fink, D. P. Mitchell and E. Segrè, Phys. Rev. **48**, 704 (1935); Lincei Rend. **23**, 340 (1936).

[27] P. Preiswerk and H. von Halban, Nature **136**, 1027 (1935); F. Rasetti and G. Fink, Phys. Rev. **49**, 642 (1936).

probability of getting out of the band after one impact.

In all the cases investigated by us, a large percentage of the activity (in most cases larger than 50 percent) is due to thermal neutrons; this does not mean, however, that the absorption coefficient for thermal neutrons is, as a rule, larger than the absorption coefficient for neutrons having an energy of some volts. This fact can be explained to some extent by the large number of thermal neutrons that get out of a paraffin block containing the source (see Table VI).

The problem of determining the energy bands corresponding to the known thermal groups can be attacked by the following method, first used by Frisch and Plazcek and by Weekes, Livingston and Bethe. They assumed that the capture cross section of a slow neutron in boron is inversely proportional to the velocity v of the neutron. Assuming this, we obtained from our measurements the following energies:

Group	*C*	*D*	*A*	*B*	*I*
Volts	0.037	1.6	4	7	36.

The error in this type of measurements can be, however, fairly large.

A quite independent determination of the energy order of the groups, and also an estimate of the energy ratio (this last only for nonthermal groups) was given by us in §9 by considerations based on the behavior of the intensity of the various groups as a function of the distance from the source. The energy order found by us coincides with the one deduced from the boron absorption, and the values of the energy ratios found by us, although smaller than the ratios obtained by the boron method, are not inconsistent because of the fairly large error involved in these determinations.

Our method, although very inaccurate, has the advantage of being very direct.

We were also able to derive, by analogous methods, the width of the energy bands (more exactly, the relative width $\Delta W/W$) for the nonthermal groups; we found that the narrowest band is group *A* (radiation strongly absorbed by silver), for which $\Delta W/W=0.04$. The other groups (see §12) are somewhat wider; nevertheless, the probability that a neutron remains in the same group after one free path is fairly low.

Connected to the width of the absorption bands, is the numerosity of the various groups, which corresponds essentially to the number of neutrons coming out per second from the surface of a paraffin block containing the source (see §5). The most numerous group of all is the thermal one (numerosity 40), while the least numerous is group *A*) (numerosity 0.5), which, as we have seen, is the narrowest.

These facts are qualitatively in agreement with the consideration of Bohr and Breit and Wigner.

Some further information on the properties of these absorption bands can be obtained by comparing the behavior of two periods of the same element, both sensitive to slow neutrons. We were able to investigate in this respect only silver, rhodium, indium and bromine. Only in the case of silver we found a different behavior for the two periods of this element. In all the other cases, no difference was found within the limits of our accuracy. It may be noticed that rhodium, indium and bromine are the three cases for which the number of observed periods produced by slow neutrons is larger than the number of known isotopes.

Sections 6 to 11 are devoted to a study of the diffusion properties of slow neutrons in hydrogenated substances. The diffusion process can be analyzed in two successive phases. First (slowing-down phase), the neutron loses energy by means of successive impacts until it reaches the energy of thermal agitation; afterwards (diffusion phase), the energy does not decrease further on the average and the neutron is scattered until it is captured by the protons or other nuclei.

During the phase of slowing down, the mean free path decreases very soon to a value of the order 1 cm, afterwards it remains approximately constant until the energy of the neutron is comparable with the quantum $h\nu$ of the frequencies of the elastic bond of the hydrogen atom in paraffin.

We can say that all the studied groups except the thermal one, belong to the interval of energy where the mean free path λ_0 in paraffin is about 1 cm.

From the theory of the impact of slow neutrons and hydrogen atoms, taking into account the chemical bond of these last (see *F*, §§10 and 11), one finds that when the energy of the slow neutrons passes from values larger than $h\nu$ to values smaller, the mean free path decreases and tends to a limit equal to $\lambda_g/4$. This variation of the mean free path is connected also to a different angular distribution of the neutrons after an impact; when $W \gg h\nu$ the angle between the directions of the motion of the neutrons before and after the impact is always sharp, while for $W \ll h\nu$ the angular distribution after an impact tends to become isotropic.

In agreement with this theoretical result, one observes a really remarkable decrease of the mean free path in going from the nonthermal to the thermal group.

For this last, one can take $W/h\nu$ equal to about 1/10; although this value is pretty small, one cannot consider the thermal neutrons as neutrons of energy practically zero. Thus one finds (see *F*, formula 102) that for $W/h\nu = 1/10$ the mean free path is

$$\lambda = \lambda_g/3.3 \tag{28}$$

while at the limit for $W=0$, one would have $\lambda = \lambda_g/4$.

We can also evaluate how much the coherence is conserved between the directions of the motion of a thermal neutron before and after an impact. As a quantititative expression of the coherence we can take $(\cos\theta)_{\text{av}}$, i.e., the mean value of the cosine of the diffusion angle of the neutrons after an impact; $(\cos\theta)_{\text{av}} = 1$ means complete coherence, while for an isotropic diffusion, $(\cos\theta)_{\text{av}} = 0$. For the neutrons of energy larger than 1 volt, we find $(\cos\theta)_{\text{av}} = 2/3 = 0.67$; for the thermal neutrons, assuming $W/h\nu = 1/10$, we deduce (see *F*, formula 103) $(\cos\theta)_{\text{av}} = 0.067$, i.e., one-tenth of the preceding value.

These results justify the approximations that were made sometimes in this work, in considering the diffusion of thermal neutrons as isotropic. We must, however, notice that in such a way we have introduced a small error which is in the sense that the diffusion length, given by (12), is too small, while the albedo, given by (11), is too large. It would also be possible to calculate these corrections, if one knew the value of $(\cos\theta)_{\text{av}}$; this value is, however, subordinate to the knowledge of $W/h\nu$ of which we can give only the order of magnitude.

In §§6–11 five different magnitudes have been measured, which can be expressed by means of only two, using the relations obtained from the theory of the diffusion of the neutrons (see *F*).

These five magnitudes are: the mean free paths λ and λ_g of thermal neutrons (see §8) and of neutrons of groups *D* and *A* (see §7); the albedo β of thermal neutrons (see §6); the diffusion length l of thermal neutrons (see §7); and the length $|x_0|$ relative to the thermal neutrons, considered in Section 11, which is bound to λ by the relation

$$|x_0| = \lambda/\sqrt{(3)}.$$

By means of (28), (11), (12), and (29), the five above-mentioned magnitudes can be expressed as functions of λ and N (number of mean free paths traversed on the average by thermal neutrons).

In Table XVIII we compare the measured values of these five magnitudes with the values calculated by means of the above-mentioned formulae, assuming

$$\lambda = 1.0 \text{ cm}; \quad \text{N} = 140.$$

As we see, the agreement is good.

From these values of λ and N, we find the following values for thermal neutrons in paraffin:

elastic cross section with hydrogen	$\sigma_e = 43 \cdot 10^{-24}$ cm^2
capture cross section	$\sigma_c = 0.31 \cdot 10^{-24}$ cm^2
mean life	$\tau = 1.7 \cdot 10^{-4}$ sec.

This last value is in very good agreement with the measure of τ performed with a mechanical set-up[28] and with the theoretical value deduced from the theory given in *F*, §12, based on the assumption that the capture of slow neutrons by protons is due to the emission of a γ-quantum by magnetic dipole radiation.[29] It is shown there that ac-

TABLE XVIII. *Comparison of measured and calculated values.* λ and λ_g = mean free paths of thermal neutrons and of group *D* and *A*, respectively. β = albedo for thermal neutrons, l = diffusion length, $(x_0) = \lambda/\sqrt{(3)}$.

λ	0.3	0.30
λ_g	1.1	1.0
β	0.82	0.83
l	2.1	2.05
$\|x_0\|$	0.18	0.174

---

[28] E. Amaldi, O. D'Agostino, E. Fermi, B. Pontecorvo and E. Segrè, Ric. Scient. **VI-I**, 581 (1935).

[29] A preliminary account of this theory has been given by E. Fermi, Phys. Rev. **48**, 570 (1935).

cording to whether the ^1S state of the deuteron ($\pm$120,000 volts) is assumed to be real or virtual the theoretical value of τ is 6.5×10^{-4} or 2.6×10^{-4}. The satisfactory agreement of this last value with the experimental result, seems to indicate that the ^1S state is virtual.

We express our warmest gratitude to the Istituto di Sanita-Pubblica of Rome and in particular to Professor G. C. Trabacchi for having supplied us with the radon sources used in this research. Our thanks are also due to the Consiglio Nazionale delle Ricerche of Italy for a grant.

NOVEMBER 15, 1936 PHYSICAL REVIEW VOLUME 50

Characteristics of the Compton Modified Band

Paul Kirkpatrick, P. A. Ross and H. O. Ritland, *Department of Physics, Stanford University, California*

(Received August 31, 1936)

The shapes of Compton modified bands have been calculated for all scattering elements from hydrogen to argon. These shapes vary periodically with atomic number, atoms with one-valence electron being characterized by sharp, narrow lines while atoms with completed electron shells produce broadly rounded bands. Experimental results, though incomplete, confirm these variations except as to absolute breadth. Breadths of observed modified bands exceed calculated breadths on the average by 75 percent for carbon scatterers and 140 percent for beryllium. Variation of scattering angle has only a second-order effect upon the observational breadth of wide modified bands. To an accuracy of about 10 percent the intensity ratio of modified to unmodified radiation from carbon (graphite) scatterers is found to agree with predictions based upon the Wentzel-Waller theory of scattering by bound electrons for wavelengths in the range 435 X.U. to 710 X.U. and scattering angles between 27 and 139°. Though previous measurements showed the magnitude of the wave-length shift of the maximum in the modified band to be less than that given by the Compton equation it is now argued that present theory predicts a still smaller shift.

Energy Distribution in the Modified Band

JAUNCEY'S[1] suggestion that the distribution of energy with wave-length in Compton modified radiation is a direct result of the distribution of momenta among the electrons of the scatterer has been confirmed and greatly extended by others.[2-4] This hypothesis has been used for the theoretical prediction of the shapes of shifted bands (or lines) upon bases of atomic theory, and while Jauncey's specific predictions are not now acceptable, being derived from Bohr orbits, DuMond[2, 4] has shown that line shapes may be deduced from any given electron momentum distribution, and has illustrated the method by theoretical shapes pertaining to scattering by carbon and beryllium atoms with electron momenta distributed as required by modern atom models.

Bloch[5] gave special attention to the modified radiations scattered by carbon and by beryllium and developed formulae for the calculation of the intensity distributions upon the basis of Wentzel's[6] theory of the Compton scattering by bound electrons. Hydrogen scattering has recently been treated by Schnaidt[7] and neon by Burkhardt,[8] intensity (or energy) distributions being obtained in both cases. The interest of the present writers has been not only in the form of the distribution for a specific element but also in the nature of the variations of this form with change of the atomic number of the scatterer, and for investigating this matter the method of DuMond,[4] utilizing the general momentum distribution formula of Podolsky and Pauling,[9] is most appropriate, notwithstanding its restriction to atoms which may be regarded as hydrogen-like and which are free from the entangling alliances of the crystal state.

---

[1] G. E. M. Jauncey, Phys. Rev. **25**, 314 and 723 (1925).

[2] J. W. M. DuMond, Phys. Rev. **33**, 643 (1929).

[3] J. W. M. DuMond and H. A. Kirkpatrick, Phys. Rev. **37**, 136 (1931); S. Chandrasekhar, Proc. Roy. Soc. **A125**, 231 (1929); J. W. M. DuMond and H. A. Kirkpatrick, Phys. Rev. **38**, 1094 (1936).

[4] J. W. M. DuMond, Rev. Mod. Phys. **5**, 1 (1933).

[5] F. Bloch, Phys. Rev. **46**, 674 (1934).

[6] G. Wentzel, Zeits. f. Physik **43**, 1 (1927).

[7] F. Schnaidt, Ann. d. Physik **21**, 89 (1934).

[8] G. Burkhardt, Ann. d. Physik **26**, 567 (1936).

[9] B. Podolsky and L. Pauling, Phys. Rev. **34**, 109 (1929)