TABLE II. *Comparison of results with values calculated from Nishina's formula.*

X-ray tube voltage (kv)	Observed Polarization		Estimated wave-length A.U.	Corrected polarization results	Calculated from Nishina's formula
	1st series of experiments	2nd series of experiments			
	Percent	Percent		Percent	Percent
80	89.0	—	—	—	—
200	—	78.6	0.100	89.6	92.6
400	72.8	73.5	.060	84.5	84.2
500	63.9	68.9	.052	79.9	80.9
600	62.2	67.5	.050	78.5	80.0
700	60.8	63.2	.042	74.2	74.7
800	59.9	58.6	.039	69.6	73.1

radium tests throughout the period taken to do the experiments. Special precautions were also made to keep the x-ray tube operating under constant conditions. Due to the conditions under which the first set of results was obtained, they are not averaged with the results of the second set of experiments. The corrected results were obtained by adding a correction of 11 percent to each of the results of the second series of experiments. This is the estimated value for the two sources of error mentioned above. Each of the given experimental results represents an average of several separate trials. The value given for 80 kilovolts was obtained at Ryerson Laboratory with carbon scattering blocks before the apparatus was moved to the hospital. The corrected result at this voltage is not given because no effort was made to find the effect of multiple scattering in the carbon blocks. It would seem reasonable to assume that the correction in this case should be of the same order of magnitude as that at other voltages, and consequently the corrected result at this voltage should be approximately 100 percent. This would be in accord with earlier precision experiments on polarization, such as those of Compton and Hagenow.[9]

The last column in Table II gives the percentages of polarization predicted by Nishina's theory for the assumed wave-lengths. The estimated wave-lengths are rather uncertain, but they do not differ greatly from those which had previously been determined by Dr. Schmitz from absorption measurements in the primary beam. It is believed that these experiments furnish conclusive evidence that an unpolarized component of the predicted order of magnitude does exist.

The writer wishes to express his appreciation to Professor A. H. Compton for suggesting the problem and for much valuable advice, to Dr. Henry Schmitz of Mercy Hospital through whose cooperation this work was made possible, to the diagnostic x-ray department of Mercy Hospital for furnishing a room in which to keep the apparatus, and to Mr. F. W. Eims for many hours spent in operating the large x-ray tube.

[9] A. H. Compton and C. F. Hagenow, J. O. S. A. and R. S. I. **8**, 487 (1924).

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Scattering of X-Rays by a Spinning Electron

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A theory based upon classical electrodynamics is presented for the scattering of x-rays by an electron which is spinning and has a magnetic moment. The radiation scattered by such a magnetic doublet is found to be almost completely unpolarized. Being proportional to ν^2, it is negligible for ordinary x-rays, but should comprise the major part of the scattering according to classical theory for wave-lengths as short as hard gamma-rays. The rays thus scattered by one electron should be incoherent with those from every other. In all of these features the radiation thus magnetically scattered is closely similar in properties to that described by the added term of Klein and Nishina's quantum theory of scattering. Only in the distribution of scattered rays with angle does there appear a fundamental difference between the results of the two theories. Insofar as the two theories agree, we may consider the classical interpretation of Klein and Nishina's added term to be scattering due to the electron's spin. In particular, an interpretation according to classical electron theory of Rodger's experimental discovery of an important unpolarized component in scattered x-rays of very high frequency is that the electron is a magnetic doublet, which for these frequencies is comparable in importance with its electric charge.

THE concept of a spinning electron[1] which because of its spin has a magnetic moment has been useful in accounting for several physical phenomena, especially the fine structure of spectral lines.[2] When such an electron is traversed by an electromagnetic wave it should scatter radiation in a distinctive manner. The characteristics of the radiation thus scattered, as calculated according to classical electrodynamics, are similar to those of the scattered radiation described by Klein and Nishina's quantum-mechanics theory of the phenomena.[3] This theory, it will be recalled, added a term, which to the second power of α is

$$I_{KN}=I_e\alpha^2\frac{(1-\cos\varphi)^2}{1+\cos^2\varphi}, \tag{1}$$

to that given by the earlier formula of Breit, Gordon and Dirac.[4] Here I_e is the scattering per electron as calculated by Thomson's classical electron theory

$$I_e=I_0\frac{e^4}{m^2r^2c^4}(1+\cos^2\varphi), \tag{2}$$

φ being the angle between the primary and the scattered rays, r the distance from the electron to the observer, e, m, c and h have the usual significance, and

$$\alpha\equiv h\nu/mc^2=h/mc\lambda, \tag{3}$$

where λ is the wave-length of the incident rays. This term arose from the use of Dirac's relativistic form of quantum mechanics, whose classical analog contains a spin term and an imaginary electric moment term. Presumably a part at least of the added scattering predicted by Klein and Nishina should thus be attributable to electron spin. The fact that classical electrodynamics, when applied to a spinning electron gives rise to a very similar expression (Eq. (9)), supports this presumption.

Following the successes of the Klein and Nishina formula in describing the intensity of scattering and the absorption of x-rays of short wave-length, Rodgers has recently[5] shown also that for these short wave-lengths an unpolarized component of the scattered x-rays appears. According to the Klein and Nishina theory,[6] it is only the component described by Eq. (1) which is incompletely polarized when scattered at 90°. On classical theory, as is well known, the scattered rays due to the electron's charge are completely polarized, while, as we see below, those due to its magnetic moment have an important unpolarized component. Rodger's experimental discovery of an unpolarized component thus gives specific confirmation of the new term in the Klein-Nishina formula, and affords new and direct evidence for the spin and the magnetic moment of the electron.

Precession of a Spinning Electron

Let $\mathbf{p}$ be the angular momentum of the electron's spin, and $\boldsymbol{\gamma}$ be its magnetic moment. The torque exerted on the electron by the electromagnetic wave which traverses it is then

$$\mathbf{T}=\boldsymbol{\gamma}\times\mathbf{H},$$

where $\mathbf{H}$ is the magnetic vector of the wave. The result is a precession of the spin axis about $\mathbf{H}$ such that

$$d\mathbf{p}/dt=\mathbf{T},$$

and a change of magnetic moment at the rate

$$\dot{\boldsymbol{\gamma}}=(\gamma/p)[\boldsymbol{\gamma}\times\mathbf{H}].$$

If now the magnetic vector of the primary is expressed by

$$\mathbf{H}_0=\mathbf{A}_0\cos 2\pi\nu t$$

we have, assuming that the rate of rotation of the spinning electron is high compared with ν,

$$\begin{aligned}\ddot{\boldsymbol{\gamma}}&=(\gamma/p)[\boldsymbol{\gamma}\times(d\mathbf{H}/dt)]\\&=-2\pi\nu(\gamma/p)[\boldsymbol{\gamma}\times\mathbf{A}_0]\sin 2\pi\nu t.\end{aligned}$$

But the magnitude of the magnetic vector of the wave radiated by this changing magnetic doublet is

$$H_s=(\ddot{\gamma}/rc^2)\sin\theta,$$

[1] Cf. A. H. Compton, Phil. Mag. **41**, 279 (1921).
[2] E.g., L. Pauling and S. Goudsmidt, *The Structure of Line Spectra* (1930), p. 54.
[3] O. Klein and Y. Nishina, Zeits. f. Physik **52**, 853 (1929).
[4] For a summary of this work, cf. e.g. A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (1935), p. 234.
[5] E. Rodgers, preceding paper, this issue.
[6] A detailed discussion of the polarization characteristics of the rays scattered according to their theory has been given by Y. Nishina, Zeits. f. Physik **52**, 869 (1929).

where θ is the angle between $\ddot{\gamma}$ and $\mathbf{r}$, i.e., between $\mathbf{T}$ and $\mathbf{r}$. The instantaneous intensity of the scattered wave is then

$$I' = \frac{c}{4\pi} H_s^2 = \frac{c}{4\pi} \frac{4\pi^2\nu^2}{r^2c^4} \frac{\gamma^2}{p^2} [\boldsymbol{\gamma}\times\mathbf{A}_0]^2 \sin^2\theta \sin^2 2\pi\nu t.$$

The average value of this intensity taken over a complete cycle, assuming that the period of the electron's precession is long compared with the period of the wave, is

$$I'' = I'/2 \sin^2 2\pi\nu t.$$

The corresponding average intensity of the primary wave traversing the electron is

$$I_0 = (c/8\pi)A_0^2.$$

We may thus write

$$I'' = I_0 \frac{4\pi^2\nu^2}{r^2c^4} \frac{\gamma^4}{p^2} \sin^2\xi \sin^2\theta, \tag{4}$$

where ξ is the angle between $\boldsymbol{\gamma}$ and $\mathbf{A}_0$.

We next proceed to average I'' over all possible orientations of the spin axis. Let us refer to Fig. 1, in which the X axis is the direction of propagation of the primary beam, and the XY plane is chosen to include OP, the direction of the scattered ray under consideration. The primary magnetic vector $\mathbf{H}$ is then normal to OX and at an arbitrary angle ζ with the Y axis, and the magnetic doublet $\boldsymbol{\gamma}$ is at an angle ξ with $\mathbf{H}$. The torque $\mathbf{T}$ is perpendicular to $\boldsymbol{\gamma}$ and $\mathbf{H}$. The angle θ between $\mathbf{T}$ and $\mathbf{r}$ is given by

$$\cos\theta = -\cos\varphi \sin\psi - \sin\varphi \cos\psi \sin\zeta.$$

Thus,
$$\sin^2\theta = 1 - \cos^2\varphi \sin^2\psi - 2\sin\varphi\cos\varphi \times \sin\psi \sin\zeta - \sin^2\varphi \cos^2\psi \sin^2\zeta. \tag{5}$$

The average intensity for all orientations of the spin axis is,

$$I_s = \frac{1}{4\pi}\int_0^\pi \int_0^{2\pi} I'' \sin\xi d\psi d\xi.$$

Substituting the value of I'' and of $\sin^2\theta$ given in Eqs. (4) and (5), we obtain on integration

$$I_s = I_0 \frac{4}{3}\pi^2 \frac{\nu^2\mu^4}{r^2c^4p^2}[1 + \sin^2\varphi \cos^2\zeta]. \tag{6}$$

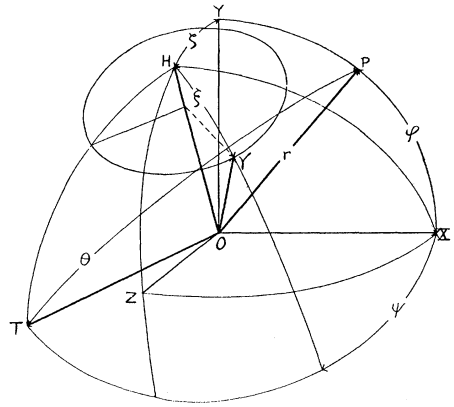

FIG. 1.

STATE OF POLARIZATION

For the ray scattered at $\varphi = 90°$, if the primary ray is polarized with the magnetic vector along the Y axis, $\zeta = 0$, and the factor in brackets becomes equal to 2. In this case reference to Fig. 1 shows that the direction of $\mathbf{T}$ and hence of $\ddot{\gamma}$ is perpendicular to the Y axis, which is the direction of scattering, and that the probability distribution of this direction is axially symmetrical about Y. Thus the scattered ray is in this case completely unpolarized.

If the primary ray has its magnetic vector along the Z axis, $\zeta = 90°$, and the bracketed factor becomes 1. In this case $\ddot{\gamma}$ lies in the XY plane. This gives rise to a component of the ray scattered along the Y axis which is polarized in the same plane as is the ray scattered by the electric charge of the electron.

Since for an unpolarized primary beam the average magnitudes of the Y and Z components of the magnetic vector are equal, this means that when an unpolarized ray is scattered at 90° the scattered ray may be thought of as consisting of an unpolarized component of intensity 2 upon which is superposed a polarized component of intensity 1. If this ray is scattered again at 90° by a magnetic doublet, as in the usual technique for studying polarized x-rays, it can be shown that the ratio of $I_{\parallel}$ to $I_{\perp}$ is 5/4. Thus the magnitude of the polarization of rays scattered only by the magnetic moment of the electron, as defined in the usual manner, should be

$$P \equiv (I_{\parallel} - I_{\perp})/(I_{\parallel} + I_{\perp}) = 1/9. \tag{7}$$

Nishina has shown[6] that the corresponding polarization of the added term in the Klein-Nishina formula is 0.

Intensity

In the theory of spectra, it is found necessary to assume that the effective value of the electron's angular momentum and magnetic moment are, respectively,[2]

$$p=\sqrt{(\tfrac{3}{4})}h/2\pi$$

and

$$\gamma=\sqrt{(\tfrac{3}{4})}(h/2\pi)(e/mc). \tag{8}$$

Substituting these values in Eq. (6), and taking the average value of $\sin^2 \zeta$ as $\frac{1}{2}$ for unpolarized radiation, we obtain,

$$I_\gamma=\frac{1}{8}\frac{e^4}{m^2r^2c^4}\frac{h^2\nu^2}{m^2c^4}(3-\cos^2\varphi).$$

Using relations (2) and (3), this may be written,[7]

$$I_\gamma=I_e\alpha^2\frac{3-\cos^2\varphi}{4(1+\cos^2\varphi)}. \tag{9}$$

Noting that $\alpha=0.024/\lambda$, if λ is expressed in angstroms, this means that for wave-lengths shorter than about 0.024A, in the middle gamma-ray region, the magnetic moment of an electron is responsible for more scattering than is its electric charge.

[7] This result, except for an error of a factor of 2, has been given in footnote 166, p. 237, of A. H. Compton and S. K. Allison's *X-Rays in Theory and Experiment* (1935), where a comparison with the results of Klein and Nishina's theory is also given. The derivation of the expression has, however, not been presented.

It will be seen that this magnetic scattering is of the same order of magnitude as the added term (1) of Klein and Nishina's theory. It is noteworthy however that while both theories give the same type of variation of intensity with frequency, the variation with angle of scattering is fundamentally different. The factor $(1-\cos\varphi)$ present in Klein and Nishina's formula suggests on classical theory an incoherent scattering which vanishes at $\varphi=0$ because of exactness of phase relationships. Such a process has no place in the present theory of scattering by a magnetic doublet.

Phase Relationships

It will be seen from Fig. 1 that for every orientation of **T** there is an equally probable orientation in the opposite sense. It follows that if the squares of the sum of the amplitudes of the waves scattered by a group of spinning electrons is averaged over all possible orientations, all the product terms must vanish, leaving only the sum of the squared terms. Thus each doublet scatters a wave which is incoherent with that from every other doublet. Because of this incoherence it is legitimate to assume, as we have done, that the scattering by an electron's magnetic doublet is independent of that by its electric charge.

This feature is also characteristic of the scattering represented by Klein and Nishina's added term. In their case incoherence follows from the fact that the initial and final states of the scattering atoms are of different energy, implying an increased wave-length, so that no exact phase relationships can exist.