Spin-Orbit Coupling in Nuclei

In considering the spin-orbit coupling of a proton or a neutron in a nucleus, we may accept as a first approach the classical model of a spinning particle, in terms of which the analogous atomic problem was first understood.[1] The part of the energy dependent on the relative orientation of the orbital angular momentum **L** and spin **S** consists of two terms. The magnetic term, which is the more important in the atomic case, may be described as $\boldsymbol{\omega}_L\cdot\mathbf{S}$, $\boldsymbol{\omega}_L$ being the Larmor precession of the spin axis in the coordinate system of the particle. The relativistic term may similarly be described as $\boldsymbol{\omega}_T\cdot\mathbf{S}$, $\boldsymbol{\omega}_T$ being the Thomas precession, the angular velocity, in the nuclear coordinate system, of the coordinate system (and spin axis) of the particle due to its velocity **v**, acceleration **a**, and the finite speed of light. It was shown by Thomas[1] that $\boldsymbol{\omega}_T=-\mathbf{v}\times\mathbf{a}/2c^2$. In the atomic case, due to the circumstance that the magnetic field causing $\boldsymbol{\omega}_L$ arises from a transformation, to the electron coordinate system, of the same electric field which makes the acceleration (and due also to the normal spin gyromagnetic factor, $g=2$, of the electron) there is a remarkably simple relation $\boldsymbol{\omega}_T=-\frac{1}{2}\boldsymbol{\omega}_L$. The partial cancelation of the magnetic term by the relativistic term leaves the sign determined by the magnetic term, which gives rise to *regular* doublets. The relativistic term alone is *inverted*.

Studies[2] in nuclear stability indicate that a proton or neutron in a nucleus experiences short range binding forces, probably of an exchange nature and not electromagnetic in any simple way, which are so strong as to leave the electric forces comparatively unimportant, and especially so in the lighter nuclei. The relativistic term, which is proportional to the acceleration, is the weaker by only a factor $\frac{1}{2}$ in the atomic case, and may therefore be expected to dominate in the nuclear case, leading to rather wide *inverted* doublets (and other multiplets) for *protons and neutrons alike*.

In this comparison of nuclei with atoms, both terms have been reduced by the mass ratio 1/1840, which enters $\boldsymbol{\omega}_T$ in **a** and $\boldsymbol{\omega}_L$ in the magneton. An electric field sufficiently strong to cause the acceleration of a proton would transform into a magnetic field, in the proton coordinate system, strong enough to preponderate over the relativistic effect; in actual nuclei, however, the electric field is both weak and repulsive. We assume that the nuclear binding forces in effect do not transform into such a magnetic field. For the proton, $\boldsymbol{\omega}_L$ is enhanced by an anomolous[3] gyromagnetic factor, $g\approx 6$; the excess of the binding forces over electric forces may be expected to enhance $\boldsymbol{\omega}_T$ even more (at least in light nuclei). The magnetic term strengthens the coupling for protons, weakens it for neutrons (g of the neutron being apparently negative). Even without exact knowledge of the effectiveness of the strong binding forces *in accelerating the particle*, it seems very unlikely that the actual exchange forces would lead to an expected acceleration of smaller order of magnitude than would be had by substituting an effective central potential, adjusted to give the correct size of the nucleus and average potential energy per particle. Introducing as a rough approximation such an effective potential energy $V(r)$, we may put $\mathbf{a}=-\mathbf{r}(dV/dr)/Mr$. The average of the spin-orbit energy, considered as a perturbation term in the Hamiltonian, is then roughly $\boldsymbol{\omega}_T\cdot\mathbf{S}=-\overline{(dV/dr)/r}\mathbf{L}\cdot\mathbf{S}/2M^2c^2$.

Feenberg and Wigner have indicated theoretically (in press) that Li^7 should have a ${}^2P$ as lowest states. Although it involves a mixture of configurations, the important spin-orbit coupling is that of the "extra" proton. A preliminary estimate of the relativistic term for this proton gives ${}^2P_{1/2}-{}^2P_{3/2}\approx 0.2$ Mev (neglecting antisymmetry, the size of the one-particle wave function $(x+iy)e^{-\nu r^2}$ being taken from a perturbation calculation[4] of the binding energy of Li^6, and V being the quadratic potential appropriate to the wave function). This agrees in sign and order of magnitude with the experimental result, 0.4 Mev, of Rumbaugh and Hafstad for Li^7 from proton ranges in the reaction $Li^6+H^2\rightarrow Li^7+H^1$ (also in press). A similar estimate of the corresponding magnetic term is 0.03 Mev.

Although there appears to be some difficulty with the anomalous magnetic moments of protons and neutrons, the Dirac equation does contain implicitly the transformation properties of their spin angular momenta, and these lead to the relativistic term discussed above. If, in a classical limit, the velocity of a proton (a "Dirac proton" with $g=2$) were known, we could derive the term thus: Consider three equally strong fields; (a) electric, attractive, (b) electric, repulsive, and (c) nonelectric, attractive; assuming additivity of their contributions to the instantaneous energy of the proton. If $\Delta\nu$ is the result of the usual Dirac method for the doublet splitting in (a), we find for (b)+(c) a magnetic splitting, $-2\Delta\nu$, by considering the Pauli spin in the proton coordinate system, which is not accelerated in this case. Adding, the splitting for (c) alone is $-\Delta\nu$, the nonelectric nature of the force reversing the sign. An elegant and direct derivation of the result from the Dirac equation will be discussed by Dr. W. H. Furry.

Professor Gregory Breit, to whom I am also indebted for generous discussion, and for acquaintance with the experiment mentioned, has pointed out that one may obtain the Thomas result (without any question of the properties of a classical model) by requiring that Pauli spin wave functions transform between Lorentz frames of different velocity in such a way as always to be derivable by reducing the Dirac wave functions to two components. If they are also to satisfy, to order v^2/c^2, a Pauli wave equation, it must be corrected by addition of the relativistic term of Thomas to the Hamiltonian.

D. R. Inglis

University of Pittsburgh,
Pittsburgh, Pennsylvania,
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[1] Thomas, Phil. Mag. **3**, 1 (1926); Frenkel, Zeits. f. Physik **37**, 243 (1926); see following letter.
[2] Heisenberg, Wigner, Majorana, Feenberg, Young, *et al.*, see Rev. Mod. Phys. **8**, 226 (1936).
[3] Estermann, Frisch and Stern, Zeits. f. Physik **85**, 17 (1933); Rabi, Kellogg and Zacharias, Phys. Rev. **46**, 163 (1934); **50**, 396 (1936).
[4] As outlined in Inglis, Phys. Rev. **50**, 399 (1936).