The Heat Conductivity of Tungsten and the Cooling Effects of Leads upon Filaments at Low Temperatures

IRVING LANGMUIR AND JOHN BRADSHAW TAYLOR, *General Electric Company*

(Received May 4, 1936)

The theory and the equations governing the temperature distribution, resistance, and heat flow in a tungsten filament as affected by its leads are given for the low temperature range (<600°K), both for the general case and for several special cases. *A low temperature vs. current scale for tungsten* is calculated from these equations using measurements of heat conductivity given in this paper and previously obtained data on the radiating properties and resistance of tungsten. It is given in the form of tables and formulas from which, knowing the current, filament dimensions, and lead and bulb temperatures one can find the maximum temperature, T_1, of the filament. Methods are described for calculating the effect on T_1 of a spring attached to one end of the filament. *The heat conductivity of tungsten*, λ, was determined experimentally for this low temperature range. λ at 273°K is 1.66 watts cm^{-1} deg.$^{-1}$ and decreases with rising temperature according to the equation $\log \lambda \pm 0.9518 = 0.30 \log T$ to 1.31 at 600°K.

I. INTRODUCTION

IN a recent paper[1] we have described experiments with an evacuated tube containing a tungsten filament attached to leads which could be maintained at any desired temperature (220° to 600°K). In the experiments already considered, the lead temperature, T_0 (°K), was higher than the bulb temperature T_B; and the current A through the filament was adjusted so that the filament was also at the temperature T_0, as indicated by its resistance, the resistance-temperature curve having been previously determined. In this way the cooling effect of the leads was eliminated and thus the power input gave directly the difference between the power radiated and that absorbed from the back-radiation from the bulb.

The results were accurately expressible by the equation

$$W = K(T^{\omega} - T_B^{\omega-\epsilon}T^{\epsilon}), \qquad (1)$$

where $\omega = 5.332$; $\epsilon = 0.87$; $\log_{10} K = 83.7105 - 100$; and W is the net radiation in watts cm^{-2} at a temperature T in a bulb at T_B.

It is the object of the present paper to describe experiments with the same tube in which the current A is no longer held at the value which makes the filament temperature equal to T_0. Measurements of the resistance and voltage input enable us to calculate the heat conductivity of the filament and the temperature distribution along the filament.

Forsythe and Worthing[2] have measured the temperature distribution along incandescent filaments near the leads and have calculated the heat conductivity λ. Their results are reproduced (within 0.2 percent) by the equation

$$\lambda = 0.840(T/1000)^{0.4} \text{ watts cm}^{-1} \text{ deg.}^{-1}. \qquad (2)$$

Langmuir, MacLane, and Blodgett[3] used this relation in developing equations for calculating the change in any of the characteristics of a filament which results from the cooling effect of the leads. The temperature distribution, determined by an optical pyrometer, over the central part of short filaments agreed well with that calculated and thus confirmed the accuracy of the Forsythe-Worthing values of the heat conductivity at temperatures above 1500°K. The analysis of the data, however, led to the conclusion that at lower temperatures the heat conductivity must be greater than is given by Eq. (2).

The heat conductivity of tungsten at 0°C is given by Barratt[4] as 1.60 watts cm^{-1} deg.$^{-1}$ and by Kannuluik[5] as 1.66. The experimental method of Kannuluik appears to be very accurate, but it should be noted that he "annealed" the tungsten only at 1300°C, a temperature which is quite insufficient to bring drawn tungsten wires into a steady state. He gives the specific electric re-

[1] I. Langmuir and J. Bradshaw Taylor, J. Opt. Soc. Am. **25**, 321 (1935). (Referred to as Paper I.)

[2] W. E. Forsythe and A. G. Worthing, Astrophys. J. **61**, 146 (1925).

[3] I. Langmuir, S. MacLane and K. B. Blodgett, Phys. Rev. **35**, 478 (1930). References to previous literature on the cooling effects of leads are given.

[4] T. Barratt, Proc. Phys. Soc., London **26**, 347 (1914).

[5] W. G. Kannuluik, Proc. Roy. Soc. **A131**, 320 (1931); **A141**, 159 (1933).

sistance of the wires as 6.0×10^{-6} after annealing at 200°C and 5.65×10^{-6} after annealing at 1300°C. These resistances are 20 and 13 percent, respectively, higher than normal values for well aged tungsten. Langmuir has shown[6] that a drawn tungsten wire undergoes a 15 to 20 percent decrease in cold resistance when first heated to 1500° for one minute and a further decrease of 2 to 4 percent upon aging for 24 hours at 2400°K.

There is therefore evidently a need to know λ more accurately in the range of temperatures below 1500°K. In some studies of the adsorption of caesium on thoriated tungsten filaments by methods already described[7] we have needed to know accurately the relation between the temperature at the midpoint of the filament and the heating current even when the filament temperature only slightly exceeds the bulb temperature.

II. Theory[8] of the Effect of Leads on the Temperature Distribution and Resistance

The general equation for the temperature distribution along a filament of nonuniform temperature is given by

$$\pi r^2 \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) = 2\pi r W - A^2 R/\pi r^2. \tag{3}$$

Here r is the radius of the filament, λ is the heat conductivity, T is the absolute temperature at a point x along the filament, W is the net radiation from the filament at the point x in watts cm^{-2}, A is the current through the filament in amperes, and R is the specific resistance of the filament in ohm cm.

It is desirable to express Eq. (3) in terms of dimensionless quantities. For this purpose we will replace T by θ defined as follows:

$$\theta = (T - T_0)/T_0; \quad \text{or} \quad T = (1+\theta)T_0, \tag{4}$$

where T_0 is the temperature of the leads.

[6] I. Langmuir, Phys. Rev. **7**, 302 (1916), see p. 313.

[7] J. Bradshaw Taylor and I. Langmuir, Phys. Rev. **44**, 423 (1933).

[8] The theory and equations in this part and in Part III enabled us to develop a temperature current scale for tungsten (Part V) in a convenient form but need not be referred to when calculating temperatures since Part V has been made complete in itself.

Let A_0 be the current in amperes, which must be passed through the filament in order to maintain the filament at the uniform temperature T_0 when the bulb is at 0°K, or at any temperature so low that there is no appreciable back-radiation from the bulb. The value of A_0 is evidently given by Eq. (3) if we place $dT/dx = 0$, so that

$$A_0^2 = 2\pi^2 r^3 W_0/R_0, \tag{5}$$

where W_0 and R_0 are the values of W and R which correspond to the case that the bulb is at 0°K and the whole filament is at T_0.

The current A is conveniently expressed in terms of a dimensionless quantity β defined by

$$\beta = (A/A_0)^2. \tag{6}$$

When the current A is small so that θ is small compared with unity, the values of θ should increase approximately in proportion to A^2, or in other words θ should vary approximately linearly with β.

If we insert these values from Eqs. (4), (5), and (6) into Eq. (3), we obtain

$$H\frac{d}{dx}\left(\frac{\lambda}{\lambda_0}\frac{d\theta}{dx}\right) = \frac{W}{W_0} - \beta\frac{R}{R_0}, \tag{7}$$

where H, a quantity having the dimensions of the square of a length, is given by

$$H = r\lambda_0 T_0/2W_0. \tag{8}$$

Here λ_0 is the value of λ which corresponds to T_0.

By Eq. (1) we see that

$$W_0 = KT_0^{\omega} \tag{9}$$

and

$$W/W_0 = (1+\theta)^{\omega} - (1+\theta_B)^{\omega-\epsilon}(1+\theta)^{\epsilon}, \tag{10}$$

where θ_B is the value of θ corresponding to the bulb temperature T_B. We have seen that to maintain the filament at T_0 when $T_B = 0$ requires that $\beta = 1$. Let β_0 be the value of β which corresponds to the current required to maintain the filament at T_0 when the bulb is at any temperature T_B. In this case we may put in Eq. (7) $d\theta/dx = 0$ and $R = R_0$ and so obtain

$$\beta_0 = W/W_0 \quad \text{when} \quad \theta = 0$$

and then by Eq. (10) we find

$$\beta_0 = 1 - (1+\theta_B)^{\omega-\epsilon}. \tag{11}$$

By using this relation, Eq. (10) may be written

$$W/W_0=(1+\theta)^{\omega}-(1-\beta_0)(1+\theta)^{\epsilon}. \qquad (12)$$

Over very wide ranges of temperature the specific resistance R of tungsten increases in proportion to T^{ρ} where ρ is a constant. Therefore, we have

$$R/R_0=(1+\theta)^{\rho}. \qquad (13)$$

We shall see that λ does not have a large temperature coefficient. Within a reasonably large range of temperatures we may assume that λ varies in proportion to a power of the temperature, so that

$$\lambda=\lambda_0(1+\theta)^k, \qquad (14)$$

where k is a constant.

Instead of expressing our equations in terms of x, the absolute length along the filament, let us now use a new variable φ defined by

$$\varphi=(W_0/r\lambda_0 T_0)^{\frac{1}{2}}x=x(2H)^{-\frac{1}{2}}. \qquad (15)$$

With these substitutions into Eq. (7) we have

$$\frac{1}{2}\frac{d^2\theta}{d\varphi^2}+\frac{k}{2(1+\theta)}\left(\frac{d\theta}{d\varphi}\right)^2=(1+\theta)^{\omega-k} -(1-\beta_0)(1+\theta)^{\epsilon-k}-\beta(1+\theta)^{\rho-k}. \qquad (16)$$

By replacing $(d\theta/d\varphi)^2$ and $\log(1+\theta)$ by new variables, this equation can be brought to the linear form and one integration can be performed. In this way we obtain

$$(1+\theta)^k(d\theta/d\varphi)=2(F-F_1)^{\frac{1}{2}}, \qquad (17)$$

where

$$F=[(1+\theta)^{\omega+k+1}-1]/(\omega+k+1) -(1-\beta_0)[(1+\theta)^{\epsilon+k+1}-1]/(\epsilon+k+1) -\beta[(1+\theta)^{\rho+k+1}-1]/(\rho+k+1). \qquad (18)$$

The quantity F_1 may be regarded as an integration constant.

In general we shall measure x and φ from the point where the filament temperature is a maximum, but if the two leads which cool the filament are at the same temperature T_0, then the maximum is also the midpoint. For this condition then, at the center of the filament the temperature gradient is zero and therefore $d\theta/d\varphi=0$. Let the value of θ at the maximum (center in this case) be θ_1. Then the value of F_1 is given by Eq. (18) if we replace θ by θ_1.

Let x_0 and φ_0 be the values which correspond to the ends of the filament where $T=T_0$. These quantities then represent the half-length of the filament. By integration of Eq. (17) we obtain

$$\varphi=(1/2)\int_{\theta}^{\theta_1}(1+\theta)^k(F-F_1)^{-\frac{1}{2}}d\theta. \qquad (19)$$

We can thus express θ as a function of φ and so obtain the temperature distribution along the filament.

We are also interested in knowing the resistance of the filament. Let Ω be the resistance of the half-length x_0 of the filament when the current A is passing and let Ω_0 be the value of Ω when the filament is at the temperature T_0. Then by Eq. (13) we have

$$\Omega/\Omega_0=\int_0^{x_0}(1+\theta)^{\rho}(dx/x_0). \qquad (20)$$

By Eq. (15) φ is proportional to x, and therefore

$$dx/x_0=d\varphi/\varphi_0=(d\theta/\varphi_0)(d\varphi/d\theta). \qquad (21)$$

Substituting this into Eq. (20) and combining with Eqs. (17) and (19) gives

$$(\Delta\Omega)/\Omega_0=(1/2\varphi_0)\int_0^{\theta_1}[(1+\theta)^{\rho}-1] \times(1+\theta)^k(F-F_1)^{-\frac{1}{2}}d\theta. \qquad (22)$$

In this equation $\Delta\Omega$ has been used to denote the increase in resistance $(\Omega-\Omega_0)$ caused by increasing the current from A_0 to A.

Relation between φ_0 and the voltage V_0

Let us consider again the filament maintained at the lead temperature T_0 by the current A_0 while the bulb is at 0°K. Then according to the definitions already given, the radiation in watts per cm² is W_0 and the specific resistance is R_0. Let V_0 be the voltage drop along the half-length, x_0, of the filament. Then we have

$$W_0=A_0V_0/2\pi r x_0 \qquad (23)$$

and

$$R_0=\pi r^2 V_0/A_0 x_0. \qquad (24)$$

Multiply these equations and solve for V_0 and obtain

$$V_0=(2W_0R_0/r)^{\frac{1}{2}}x_0. \qquad (25)$$

Eq. (15) furnishes us with another relation involving W_0/r. If we eliminate this factor between Eqs. (15) and (25) we get

$$V_0 = h_0\varphi_0, \tag{26}$$

where
$$h_0 = (2R_0T_0\lambda_0)^{\frac{1}{2}} \tag{27}$$

and h_0 is the value, at $T = T_0$, of a quantity h defined by

$$h = (2RT\lambda)^{\frac{1}{2}}. \tag{27a}$$

This parameter h is a specific property of tungsten as it depends on the temperature but not on the dimensions of the filament. We see from Eq. (26) that it has the dimensions of a voltage.

According to the Wiedemann-Franz law, λR, at a given temperature, is nearly the same for all metals and according to Lorenz λR increases in proportion to T. If these laws are applicable, h should increase in proportion to T but should be the same for all metals.

Flow of heat a long the filament

Let Q be the heat flux along a filament in watts; then

$$Q = \pi r^2 \lambda dT/dx. \tag{28}$$

By Eqs. (4), (14), and (27) this becomes

$$Q = (\pi r^2 h_0^2/2R_0)(1+\theta)^k d\theta/dx. \tag{29}$$

In Eq. (15) we can eliminate $\lambda_0 T_0$ by using Eq. (27) and by Eq. (5) we obtain

$$x = (\pi r^2 h_0/A_0R_0)\varphi \tag{30}$$

By combining Eqs. (17), (30), and (29) we find

$$Q = A_0h_0(F+c)^{\frac{1}{2}}, \tag{31}$$

where we have used c to represent the integration constant.

Eq. (31) is applicable to the flow of heat through a spring or lead to which a filament is attached. In such a case the heat flowing into the hot end may be such that there is no value of θ which makes $d\theta/d\varphi = 0$. A similar case arises if a filament is attached to two leads, one of which is at a temperature above T_M while the other is below T_M, where T_M is the temperature to which the current A would heat the central part of the filament if it were infinitely long. The value of Q is then not zero even when $\theta = \theta_M$. In such cases the integration constant c is to be determined by the boundary conditions.

With a filament having two leads at the temperature T_0, c becomes $-F_1$, and the heat that flows into each lead is given by

$$Q_0 = A_0h_0(-F_1)^{\frac{1}{2}} \tag{32}$$

since, by Eq. (18), $F=0$ when $\theta=0$.

Eqs. (19), (22), and (31) represent the general solutions of the problems of the temperature distribution, the resistance and the heat flow in filaments. There are, however, certain special cases in which the equations can be so simplified as to facilitate the calculations. We shall consider some of these.

Case 1: Low Values of θ_1. When β differs only little from β_0, the temperature of the whole filament is not far above T_0 so that we may take λ as constant and then put $k=0$. Furthermore, in Eq. (16) we may expand the terms involving powers of $1+\theta$. If we omit terms involving powers of θ higher than the first, the equation becomes linear and can be integrated. Choosing the integration constants so that $d\theta/d\varphi = 0$ when $\varphi = 0$, and at the leads where $\varphi = \varphi_0$, $\theta = 0$, we thus find that the temperature distribution along the filaments is given by

$$\theta = \frac{2(\beta-\beta_0)}{C^2}\left[1 - \frac{\cosh C\varphi}{\cosh C\varphi_0}\right], \tag{33}$$

where
$$C^2 = 2[\omega - \epsilon + \beta_0(\epsilon - \rho)]. \tag{34}$$

The temperature at the center of the filament is found by putting $\varphi = 0$:

$$\theta_1 = [2(\beta-\beta_0)/C^2](1 - \operatorname{sech} C\varphi_0). \tag{35}$$

By expanding the factor $(1+\theta)^\rho$ in Eq. (20), combining with Eq. (21) and inserting the value of θ as given by Eq. (33) we find

$$\Delta\Omega/\Omega_0 = \frac{2\rho(\beta-\beta_0)}{C^2}\left[1 - \frac{\tanh C\varphi_0}{C\varphi_0}\right]. \tag{36}$$

Short Filament: When φ_0 is sufficiently small, we can expand the cosh factors in Eq. (33) and so reduce the equation to

$$\theta = (\beta-\beta_0)(\varphi_0^2 - \varphi^2)[1 - (C^2/12)(5\varphi_0^2 - \varphi^2)]. \tag{37}$$

Thus when φ_0^2 is small compared to unity the temperature distribution is parabolic, and the heat transfer by radiation is small compared to that conducted.

A similar expansion applied to Eq. (36) gives

$$\Delta\Omega/\Omega_0=(2/3)(\beta-\beta_0)\rho\varphi_0^2(1-2C^2\varphi_0^2/5). \quad (38)$$

Long Filament: When the filament is so long that $C^2\varphi_0^2>6$, then the relationship of Eq. (33) may be sufficiently accurately expressed (within 1 percent) by

$$\theta=\theta_M[1-e^{C(\varphi-\varphi_0)}], \quad (39)$$

where
$$\theta_M=2(\beta-\beta_0)/C^2. \quad (40)$$

In this case the temperature is practically uniform at θ_M over the central part of the filament.

When $C^2\varphi_0^2\gg 6$ the hyperbolic tangent in Eq. (36) becomes unity so that

$$\Delta\Omega/\Omega_0=\frac{2\rho(\beta-\beta_0)}{C^2}-B_\infty/\varphi_0, \quad (41)$$

where
$$B_\infty=2\rho(\beta-\beta_0)/C^3. \quad (42)$$

Thus when the filament is long, $\Delta\Omega/\Omega_0$ varies linearly with $1/\varphi_0$.

Case 2: Large values of β (or $\epsilon=\rho$). If a filament is very long, the cooling effect of the leads becomes inappreciable in the central part of the filament and for this region we may put in Eq. (16) $d\theta/d\varphi$ and $d^2\theta/d\varphi^2$ both equal to zero and thus obtain

$$(1-\beta_0)+\beta(1+\theta_M)^{\rho-\epsilon}=(1+\theta_M)^{\omega-\epsilon}, \quad (43)$$

where θ_M is the value of θ over the central part of a very long filament corresponding to a temperature T_M.

Let us now introduce into Eq. (18) a new variable defined by

$$\sigma=(1+\theta)/(1+\theta_M)=T/T_M. \quad (44)$$

If we then eliminate β by Eq. (43), we find for the value of $F-F_1$, which is needed for Eq. (19),

$$F-F_1=(1+\theta_M)^{\omega+k+1}(H_1-H), \quad (45)$$

where

$$H=\frac{\sigma^{\rho+k+1}}{\rho+k+1}-\frac{\sigma^{\omega+k+1}}{\omega+k+1}+\left(\frac{\sigma^{\epsilon+k+1}}{\epsilon+k+1}-\frac{\sigma^{\rho+k+1}}{\rho+k+1}\right)\bigg/\left(1+\frac{(1+\theta_M)^{\rho-\epsilon}\beta}{1-\beta_0}\right) \quad (46)$$

and H_1 is obtained from this equation by putting $\sigma=\sigma_1=(1+\theta_1)/(1+\theta_M)$.

The last term in Eq. (46) becomes negligible when $\epsilon=\rho$; $\sigma\ll 1$; or when $\beta/(1-\beta_0)$ is very large. This occurs acccording to Eq. (11) if T_B is small compared to T_0 or if β is very large compared to unity. The data previously published[1] have given $\omega=5.332$; $\rho=1.23$; and $\epsilon=0.87$ for the temperature range from 250° to 600°K. Because of this small difference of 0.36 between ρ and ϵ the errors made by replacing ϵ by ρ in Eq. (46) are small even when β is compable with unity.

The greatest errors in φ_0 with small values of β occur with filaments so long that σ_1 is nearly unity and may approach a limiting fractional error of $(\rho-\epsilon)/2(\omega-\rho)$ or about 4.5 percent. With shorter filaments for which σ_1 is considerably less than unity, the errors are much smaller.

Replacing ϵ by ρ in Eq. (46), substiuting in Eqs. (45) and (19), we are led to the following equations:

$$\varphi=\sigma_0^{(\omega-k-1)/2}(Y_1-Y), \quad (47)$$

where
$$\sigma_0=1/(1+\theta_M), \quad (48)$$

$$Y=(1/2)\int_0^\sigma \sigma^k d\sigma(G_1-G)^{-\frac{1}{2}}, \quad (49)$$

and
$$G=\frac{\sigma^{\rho+k+1}}{\rho+k+1}-\frac{\sigma^{\omega+k+1}}{\omega+k+1}. \quad (50)$$

Similarly from Eqs. (20) and (21) we obtain

$$\Omega/\Omega_0=\sigma_0^{-\rho}(Z_1-Z_0)/(Y_1-Y_0), \quad (51)$$

where
$$Z=(1/2)\int_0^\sigma \sigma^{\rho+k}d\sigma(G_1-G)^{-\frac{1}{2}}. \quad (52)$$

The temperature distribution and the resistance of filaments is thus given by Eqs. (47) and (51) in terms of the functions Y and Z. These calculations are most readily made by series expansions in terms of powers of σ or of $(1-\sigma)$. Although general expansions can be obtained for the coefficients of the terms of these series, they are too complicated to justify their presentation here. In the third part of this paper we shall give such series for the particular values of ω and ρ which have been adopted.

Case 3: A general method[9] for calculating φ_0 and Ω by power series in θ. It is seen from Eqs. (19) and (22) that the expressions for φ_0 and Ω/Ω_0 can be brought into the general form

$$2\psi(\theta_1)=\int_0^{\theta_1} N(\theta)[F(\theta)-F(\theta_1)]^{-\frac{1}{2}}d\theta, \quad (53)$$

where N and F represent any arbitrary functions which for $0<\theta<\theta_1$ can be expanded by Taylor's theorem into a series in powers of $\theta-\theta_1$ with coefficients that are functions of θ_1. The coefficients themselves can be expanded in powers of θ_1. In this way by carrying out the integration we obtain

$$(-F_1)^{\frac{1}{2}}\psi=A_1S+A_3S^3+A_5S^5+A_7S^7+\cdots, \quad (54)$$

where

$$S=(\theta_1)^{\frac{1}{2}},$$
$$A_1=N_0,$$
$$A_3=(2/3)N_1-(5/12)N_0F_2/F_1,$$
$$A_5=(4/15)N_2-(3/10)N_1F_2/F_1-(11/60)N_0F_3/F_1+(43/160)N_0(F_2/F_1)^2,$$
$$A_7=(8/105)N_3-(13/105)N_2F_2/F_1-(29/210)N_1F_3/F_1+(23/112)N_1(F_2/F_1)^2-(31/560)N_0(F_4/F_1)+(27/112)N_0(F_2/F_1)(F_3/F_1)-(177/896)N_0(F_2/F_1)^3.$$

Here N_n and F_n are the nth derivatives of $N(\theta)$, $F(\theta)$ with respect to θ at the point $\theta=0$.

Thus in calculating φ_0 and Ω/Ω_0 we may take $F(\theta)$ as given by Eq. (18) so that

$$F_1=\beta_0-\beta,$$
$$F_2=\omega-\epsilon+(\beta_0-\beta)(\rho+k)-\beta_0(\rho-\epsilon) \text{ etc.}$$

In calculating φ_0 by Eq. (19) we place

$$N(\theta)=(1+\theta)^k,$$
$$N_0=1;\ N_1=k;\ N_2=k(k-1) \text{ etc.},$$

while for $\Delta\Omega/\Omega_0$ by Eq. (22) we split the second member into two integrals placing $N(\theta)=(1+\theta)^{\rho+k}$ for the first and $N(\theta)=(1+\theta)^k$ for the second.

III. General Equations for Tungsten Filaments up to 600°K

We have shown[1] that the radiation and resistance of tungsten filaments between 220° and 600°K are accurately represented by Eqs. (1) and (13) if we place

$$\omega=5.332;\ \rho=1.23;\ \text{and } \epsilon=0.87. \quad (55)$$

In order to determine λ from our experimental data we at first assumed $k=0$, and were thus able to calculate φ_0 and $\Delta\Omega/\Omega_0$ in terms of β and θ_1 by the methods we shall outline. Experiments with low values of θ_1 (which justify the assumption $k=0$) and with various values of T_0 soon showed that

$$k=-0.30 \quad (56)$$

gave the variation of λ with temperature to a satisfactory approximation.

Values of Y and Z.

When the bulb temperature T_B is low compared to T_0 or when β is either large or very small, the temperature distribution is given accurately by Eq. (47), but even for intermediate values of β this equation gives a rather good approximation. Let us therefore derive methods for calculating Y.

Introducing the numerical values of ω, ρ, and k in Eqs. (49) and (50), expanding and integrating, we obtain

$$Y=+1.2033\sigma^{0.70}+0.2355\sigma^{2.630}J+0.1498\sigma^{4.560}J^2+0.129\sigma^{6.490}J^3-0.02944\sigma^{6.732}J+0.1279\sigma^{8.420}J^4-0.05046\sigma^{8.662}J^2+0.1380\sigma^{10.35}J^5-0.0724\sigma^{10.59}J^3, \text{ etc.}, \quad (57)$$

where $\quad J=(1.4705\sigma_1^{1.93}-0.4705\sigma_1^{6.032})^{-1}. \quad (58)$

A similar method applied to Eq. (52) gives

$$Z=+0.4364\sigma^{1.93}+0.1604\sigma^{3.86}J+0.1180\sigma^{5.79}J^2+0.1084\sigma^{7.72}J^3-0.0249\sigma^{7.962}J+0.1115\sigma^{9.65}J^4-0.0412\sigma^{9.892}J^2, \text{ etc.} \quad (59)$$

For values of σ approaching σ_1, it was possible to develop other expansions giving Y_1-Y in terms of powers of $(\sigma_1-\sigma)/\sigma_1$. By adding the values of Y and Y_1-Y thus obtained in the range in which both series were accurate, the values of Y_1 were obtained and were found for $\sigma_1>0.6$ to be accurately given by

[9] We wish to thank Dr. H. Poritsky for obtaining the expressions involved in this method and for helpful discussions of other sections of this paper.

$$Y_1 = 1.0312 - 0.8039 \log_{10} y_1 + 0.390 y_1 + 1.573 y_1^2 - 1.31 y_1^3 + 0.050 y_1^4 + 1.5 y_1^5, \quad (60)$$

where $y_1 = 1 - \sigma_1$.

Similar calculations for Z_1, satisfactorily accurate for $\sigma_1 > 0.1$, give

$$Z_1 = 0.08782 - 0.8039 \log_{10} y_1 + 0.3029 y_1 - 0.448 y_1^2 + 0.06 y_1^3. \quad (61)$$

Infinitely long filament

For the special case that $\sigma_1 = 1$, which corresponds to $T_1 = T_M$, the value of Y, which we may denote by Y_M, is given accurately by the following equation over the range from $\sigma = 0.3$ to 1.0.

$$Y_M = 0.7892 - 0.8039 \log_{10} y - 0.3935 y - 0.1475 y^2 - 0.0641 y^3 - 0.0330 y^4 - 0.021 y^5 \cdots, \quad (62)$$

where $y = 1 - \sigma$. An analogous expression for Z_M, very accurate for $\sigma > 0.2$, is

$$Z_M = -0.1542 - 0.8039 \log_{10} y + 0.0360 y + 0.0698 y^2 + 0.0340 y^3 + 0.0108 y^4 + 0.00227 y^5. \quad (63)$$

By means of the foregoing methods, using 15 terms in Eqs. (57) and (59), tables were prepared giving Y and Z as functions of σ and σ_1 to an accuracy of about 1 in 3000. The family of curves in Fig. 1 represents the values of σ as functions of $Y_1 - Y$ for various values of σ_1. These curves give the temperature distributions, since the ordinates, according to Eq. (44), are proportional to the temperatures, while the abscissas $Y_1 - Y$ by Eq. (47) are proportional to distances measured along the filament from its center. These distances may also be expressed in terms of φ by Eq. (47) which takes the form

$$\varphi = \sigma_0^{2.316}(Y_1 - Y). \quad (64)$$

To calculate σ_0, the value of σ at the leads, we can use the following equation derived from Eqs. (43) and (48):

$$\beta = \sigma_0^{-4.102} - (1 - \beta_0)\sigma_0^{0.36}. \quad (65)$$

For the case that $\beta_0 = 0$ (that is, $T_B = T_0$), the values of β as a function of σ_0 are given in the first two columns of Table I (see Part V). Values of β for the case $\beta_0 = 1$, or $T_B = 0$, are given in the 6th column.

The temperature distribution near the central part of a long filament for which $\sigma_1 > 0.95$ is given by

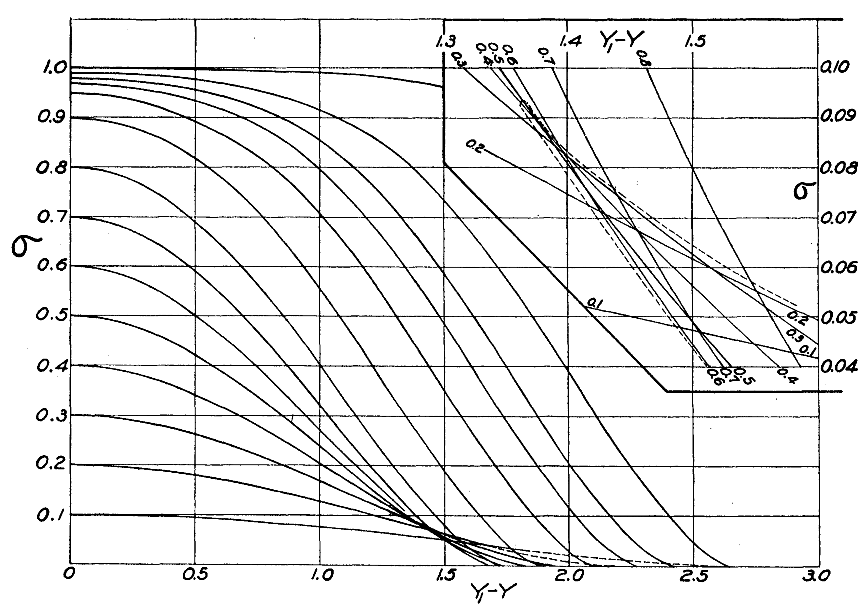

FIG. 1. σ as a function of $Y_1 - Y$ for various values of σ_1.

$$Y_1 - Y = 0.3491 \cosh^{-1}[(1-\sigma)/(1-\sigma_1)]. \quad (66)$$

Calculation of θ_1 as a function of φ_0 and β (Tables II to IV)

In most cases where it is desired to take into account the cooling effect of the leads upon the filament, it is not necessary to know the temperature distribution over the whole length of the filament, but a knowledge of the temperature at the center of the filament and the resistance of the filament suffices. Tables II to IV contain data on the values of θ_1 for selected integral values of β and a set of evenly spaced values of φ_0.

For low values of φ_0 the most convenient method of calculating θ_1 is by use of the series expansion furnished by Eqs. (53) and (54). Taking $\omega = 5.332$; $\rho = 1.23$; $\epsilon = 0.87$; $k = -0.30$, and $\beta_0 = 0$, we obtained in this way a series giving φ_0 in terms of θ_1/β. However, since we desired to tabulate θ_1 for specified values of φ_0 and β, this series, by reversion, was converted into the following:

$$\theta_1/\beta = \varphi_0^2 + (1.175\beta - 3.718)\varphi_0^4 + (1.552\beta^2 - 17.56\beta + 13.49)\varphi_0^6 + (2.206\beta^3 - 57.30\beta^2 + 144.3\beta - 48.84)\varphi_0^8 + \cdots. \quad (67)$$

Beyond the range of usefulness of this series we employed the Y function for which we had previously constructed tables giving Y as a function of σ for a set of 10 values of σ_1. For each of these σ_1 values we then interpolated by Newton's rule to find $Y_1 - Y_0$ for each of the values of σ_0 in Table I which correspond to integral values of β.

For each value of β we thus had a set of values of σ_1 for a number of definite values of $Y_1 - Y_0$. Taking then the selected values of φ_0 used in Tables II to IV and the values of σ_0 corresponding to β in Table I we calculated $Y_1 - Y_0$ by Eq. (64), and used these to obtain by interpolation the corresponding values of σ_1 from which θ_1 could be obtained by Eq. (44) and Eq. (48). This interpolation was carried out graphically. We found it was often advantageous for each value of β to plot $Y_1 - Y_0$ against colog $(1-\sigma_1)$, since for larger values of σ_1 a nearly straight line was obtained,

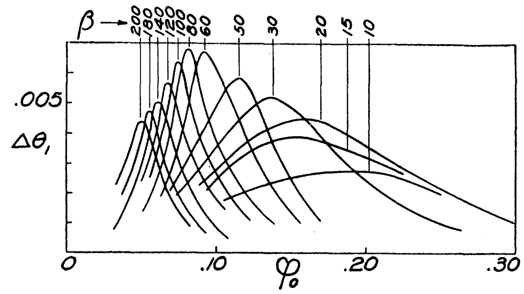

FIG. 2. Corrections $\Delta\theta_1$ applied in the calculation of θ_1 for the tables.

which by Eq. (60) approached a limiting slope of 0.8039.

Since the tables are to be used mainly for the case that $T_B = T_0$, ($\beta_0 = 0$), there is a slight error involved in the calculations of θ_1 by the foregoing method due to the replacement of ϵ by ρ in deriving Eq. (47). By using the series expansion of Eq. (54) again using $\epsilon = 1.23$ instead of 0.87 and comparing this with Eq. (67), it was possible to calculate the small correction, $\Delta\theta_1$, to apply to θ_1. As can be seen from Fig. 2, the correction was negligible for very large and very small values of both β and θ, and only rarely exceeded one percent. Many checks were made of the accuracy of these corrections by direct calculations of φ by Eq. (19), using numerical integration by Simpson's rule.

It is believed that with these corrections that were applied in calculating the data for Tables II to IV that the values of θ are accurate to within a couple of units in the last figure.

Calculations of $\Delta\Omega/\Omega_0$ in Tables V to VII

For sufficiently low values of θ_1 we obtain from Eq. (54) the following expansion:

$$\Delta\Omega/\Omega_0 = 0.8200\theta_1 + (0.01720 + 0.1220/\beta)\theta_1^2 + (0.0088 + 0.2510/\beta + 0.4148/\beta^2)\theta_1^3. \quad (68)$$

The lower values of θ_1 from Tables II to IV were used in this way to calculate $\Delta\Omega/\Omega_0$ for given values of φ_0 and β. For values of θ_1 too large for rapid convergence of this series, $\Delta\Omega/\Omega_0$ was calculated from Y and Z by Eq. (51). The necessary interpolations were facilitated by using the following relationships.

When θ_1 is small a very good approximation is given by

$$\Delta\Omega/\Omega_0=(1+\theta_1)^\rho[1-(\rho/3)\theta_1/(1+\theta_1)]. \quad (69)$$

By using the tables of Y and Z, values of $\Delta\Omega/\Omega_0$ were calculated and compared with those given by Eq. (69) placing $\rho=1.23$; the differences thus obtained were then plotted as a family of curves for various values of β and θ_1. These curves together with tables of the function involved in Eq. (69) were used for interpolation. This method was useful for values of θ_1 much larger than those for which Eq. (68) could be applied.

For still larger values of θ_1 we may calculate $\Delta\Omega/\Omega_0$ by

$$\Delta\Omega/\Omega_0=\sigma_0^{-\rho}-1-B/\varphi_0. \quad (70)$$

This equation, which is a generalization of Eq. (41), may be looked upon merely as a definition of B. It follows then from Eqs. (51) and (47) that

$$B=\sigma_0^{1.086}[(Y_1-Y_0)-(Z_1-Z_0)]. \quad (71)$$

As θ_1 approaches θ_M, so that $\sigma_1\rightarrow 1$, B approaches a limiting value B_∞ which may be obtained by combining Eqs. (71), (60), (61), (62), and (63):

$$B_\infty=\sigma_0^{1.086}(0.4294y_0+0.2173y_0^2+0.09815y_0^3 +0.04375y_0^4+0.02327y_0^5+\cdots). \quad (72)$$

This is sufficiently accurate for $\sigma_0>0.5$.

By calculating B by Eq. (71) for various values of β and σ_1 a family of curves was obtained giving $B-B_\infty$ as a function of σ_1 which was very convenient for accurate interpolation. Empirically it was found for large values of σ_1 and for values of β from 1 to 200 that

$$B-B_\infty=0.145\beta^{-0.21}(1-\sigma_1)^{1.17}. \quad (73)$$

By these various methods the values of $\Delta\Omega/\Omega_0$ were obtained which are given in Tables V, VI, and VII.

Let V be the voltage drop across the half filament. Then

$$V=\beta^{\frac{1}{2}}V_0(\Omega/\Omega_0).$$

If the leads did not cool the ends of the filament, the current A would heat the whole filament to T_M. Let V_M be the voltage drop that would then be required for the half-filament. We find readily that

$$V_M=\beta^{\frac{1}{2}}V_0\sigma_0^{-\beta}. \quad (74)$$

The effect of each lead in cooling the filament is thus to lower the voltage by an amount $\Delta V=V_M-V$. By Eqs. (70) and (26) we get

$$\Delta V=\beta^{\frac{1}{2}}h_0B, \quad (75)$$

where B is given by Eq. (73).

Effects due to the heating of the leads or springs

The tables in this paper are adapted primarily for calculations involving filaments whose ends are at the bulb temperature or some other definitely known temperature. The heat that is conducted from the filament into the leads must heat the junction, but if the diameter of the leads is 10 times that of the filament, the temperature drop in the leads is about that in a length of filament only one-hundredth of the lead length and so produces negligible effects.

In experimental work with tungsten filaments it is often desired to mount a filament in a definite position in a tube, such as at the axis of a cylindrical anode. To maintain the filament in this position even when it elongates upon heating, it is necessary to use a spring to hold the filament taut. The proper design[10] of springs for this purpose often requires that the spring shall contain a considerable length of wire of diameter ranging from 3 to 6 times the filament diameter. The heat conducted into the spring may then heat the spring to a degree that cannot be neglected in calculations of the temperature distribution and resistance of the filament.

Let T_2 be the temperature at the junction between the filament and spring. It is evident that with a given current A passing through the wire the temperature distribution over the whole filament will remain unchanged if we replace the spring by an additional section of filament whose length Δx is so chosen that the temperature drop in it is the same as that which occurs in the spring (T_2-T_0).

Thus, if we can calculate Δx for each lead and add these increments to the length of the filament before calculating φ_0 by Eq. (15), we can proceed

[10] K. B. Blodgett and I. Langmuir, Rev. Sci. Inst. 5, 321 (1934).

to determine θ_1 by the equations and tables already given on the assumption that the lead temperature is T_0. Let us then find methods of calculating Δx.

The temperature distribution along a filament near a lead whose temperature is T_0 may be calculated by Eq. (19). The value of F can be obtained by expanding Eq. (18) in a power series in θ. Placing $\beta_0=0$ and using the data of Eq. (55) we get

$$F=-\beta\theta+(2.231-0.465\beta)\theta^2 +(3.42+0.011\beta)\theta^3. \quad (76)$$

Numerical calculations for springs of practical sizes have shown that θ_2 is so small that only the first term of this expansion is needed. By Eq. (19), by integrating between 0 and θ_2, we obtain

$$\beta\Delta\varphi=(-F_1)^{\frac{1}{2}}-(-F_1-\beta\theta_2)^{\frac{1}{2}}. \quad (77)$$

Introducing the value of F into Eq. (17) and letting b denote the value of $d\theta/d\varphi$ at the point θ_2 we have

$$b=(d\theta/d\varphi)_{\theta=\theta_2}=2(-F_1-\beta\theta_2)^{\frac{1}{2}}. \quad (78)$$

By eliminating F_1 between Eqs. (77) and (78), we find

$$\Delta\varphi=(\theta_2/b)(1-\beta\theta_2/b^2+\cdots). \quad (79)$$

In Fig. 2 let the curve ABC represent the temperature distribution along a filament. Eq. (79) enables us to calculate $\Delta\varphi$ (the distance AD) from θ_2 (the distance BD) and the temperature gradient at B. The heat flow Q at B, by Eqs. (29), (30) and (78), is found to be

$$Q=2A_0h_0(1+\theta_2)^k b. \quad (80)$$

Now if we replace the section AB of the filament by a spring along which the temperature distribution is EFB, the curve BC is unchanged. If the spring is of very great length, it will be heated, except near its ends, to a uniform temperature T_S by the current A that flows through it. We may calculate this temperature from the data of Table II. Let A_S be the current that would be needed to maintain the spring at T_0 if its ends are at T_0 while the bulb is at 0°K. Since the current to heat a long filament to any temperature varies with $d^{\frac{3}{2}}$ where d is the diameter, we have

$$A_S=s^{\frac{3}{2}}A_0, \quad (81)$$

where s is the ratio of the diameter of spring wire to that of the filament. The value of β for the spring wire, by Eq. (6), is

$$\beta_S=(A/A_S)^2=(A/A_0)^2/s^3=\beta/s^3. \quad (82)$$

Since s^3 is large, β_S is small, and therefore in Table II we take $\beta=0$, $\varphi_0=\infty$, and so find

$$\theta_S=0.224\beta/s^3. \quad (83)$$

In general, the temperature distribution along a spring of finite length will be of the type illustrated by the curve EFB in Fig. 3. We may assume that one end is welded to a lead of such large diameter that $\theta=0$, while the other end, which joins the filament, is heated to θ_2 which is greater than θ_S. Thus to calculate the temperature distribution, we cannot use Eq. (17) for this was based on the assumption that $d\theta/d\varphi=0$ at θ_1, but must go back to Eq. (16). By a method like that used in the derivation of Eq. (33) we thus find that the temperature distribution along the spring is given by

$$C\varphi_S=\sinh^{-1}(C\theta_S/a)+\sinh^{-1}[C(\theta_2-\theta_S)/a], \quad (84)$$

where C is defined by Eq. (34) and a is an integration constant which is equal to the value of $d\theta/d\varphi$ at θ_S.

By analogy with Eq. (78), we let b_S be the value of $d\theta/d\varphi_S$, for the spring at its junction with the filament (θ_2).

Applying Eq. (80) to both the spring and filament and considering Eq. (81) we find

$$b_S=b/s^{\frac{3}{2}}. \quad (85)$$

By obtaining $d\theta/d\varphi$ at θ_S and at θ_2 from Eq. (84) and equating these derivatives to a and b_S, we obtain the relation

$$a^2=b_S{}^2-C^2(\theta_2-\theta_S)^2. \quad (86)$$

In Eqs. (78), (79), (83), (84), (85), and (86) we have six equations involving six unknowns, θ_2, θ_S, a, b, b_S, and $\Delta\varphi$ and so may solve for $\Delta\varphi$ in terms of the known parameters s, φ_S, β, and F_1 (or θ_1). A few numerical calculations involving springs of practicable design have shown that θ_S is negligible in its effect and that F_1, according to Eq. (76), can be replaced without appreciable loss of accuracy by $-\beta\theta_1$. The problem is thus greatly simplified so that the calculation of $\Delta\varphi$ can be reduced to the following procedure.

Calculate a quantity P (which is equal to θ_2/b) by the equation

$$P=(1/Cs^{\frac{1}{2}})\tanh C\varphi_S. \quad (87)$$

If $C\varphi_S \gg 1$ this reduces to

$$P=(1/Cs^{\frac{1}{2}})(1-2e^{-2C\varphi_S}\cdots). \quad (88)$$

If $C\varphi_S \ll 1$ it becomes

$$P=(\varphi_S/s^{\frac{1}{2}})[1-(2/3)C^2\varphi_S{}^2]. \quad (89)$$

Then $\Delta\varphi$ is given by

$$\Delta\varphi=P-P^2\beta/b, \quad (90)$$

where $$b/\beta=2(\theta_1/\beta)^{\frac{1}{2}}-2P. \quad (91)$$

This may be used to calculate Q by Eq. (80).

The value of θ_2, when desired, may be found from

$$\theta_2=2P\beta[(\theta_1/\beta)^{\frac{1}{2}}-P]. \quad (92)$$

According to Eqs. (34) and (55), the numerical value of C is 2.99. The value of φ_S used in these calculations can be obtained from the length x_S of the spring wire by Eq. (15) using a value of H_S which by Eq. (8) is sH where H is the value for the filament.

From Eqs. (90) and (15)

$$\Delta x=(2H)^{\frac{1}{2}}P(1-P\beta/b), \quad (93)$$

while for short spring wires or leads this becomes,

$$\Delta x=x_S/s^2. \quad (94)$$

If for short leads a material is used which has a heat conductivity λ_S, the value of Δx given by the above equation should be multiplied by (λ_0/λ_S).

The heating of the leads has also an effect on the change in resistance. As seen in Fig. 3 the observed resistance when a spring is attached is the sum of the resistance Ω_S of the spring and the resistance Ω_F of the filament, so that

$$\begin{aligned}\Omega&=\Omega_S+\Omega_F=\Omega_S+\Omega_2+\Omega_0\\&=\Omega_S-\Omega_{AD}+2\Omega_0\end{aligned} \quad (95)$$

since $\Omega_2=\Omega_0-\Omega_{AD}$.

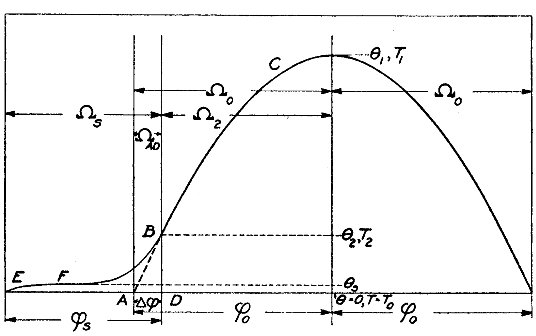

FIG. 3. The temperature distribution along a filament supported at one end by a spring.

On passing a current, the resulting changes in resistance are given by

$$\Delta\Omega_S=(b\rho\Omega_S/C^2s^{\frac{1}{2}}\varphi_S)(1-1/\cosh C\varphi_S), \quad (96)$$

(neglecting θ_S compared to θ_2) and

$$\Delta\Omega_{AD}=\tfrac{1}{2}\rho\theta_2\Omega_{AD}, \quad (97)$$

where Ω_S and Ω_{AD} are the cold resistances. The increase in Ω_0, *viz.*, $\Delta\Omega_0$ may be calculated from β and φ_0 by the tables.

IV. Experimental Determination of the Heat Conductivity λ

The relations given in Part II show that the heat conductivity λ may be calculated from a determination of the value of T or of Ω resulting from passage of a current through the filament. For filaments below 600°K it is more convenient to use the resistance Ω. The experimental procedure was as follows.

Three tubes were used. One (Tube No. 1) was the tube described in Paper I which by use of hollow copper leads allowed control of T_0 independently of T_B. Two other tubes (Nos. 2 and 3) had heavy leads of 120-mil molybdenum, arc-welded in hydrogen to the filament to insure good thermal contact. The filaments were of thoriated tungsten 0.00499 cm in diameter, and had total lengths in Tubes 1, 2, and 3 of 25.82 cm, 12.86 cm and 5.87 cm, respectively. The use of three filament lengths provided a check on the method of calculation and increased the range of values of β, for which Ω could be accurately measured. For the shortest filament, low values of β gave changes in resistance too low to be measured accurately, while for long filaments the higher values of β gave filament temperatures outside the range for which radiating properties had been determined.

The tubes were baked at 450°C and pumped during the experiments through liquid-air traps. The filaments were heated initially at 2000°K to produce a fine grained structure and then at 2400° and 2800°, as described in Paper I, to a condition where further aging caused no change in resistance or emissivity.[11]

In most of the experiments T_0 was made equal to T_B by immersing the tube and its leads in a constant temperature bath. The filament current A and resistance Ω were found from voltage measurements with a precision potentiometer.

An example is given below of the steps and quantities involved in the calculation of the heat conductivity from experimental data.

Tube No.	L (cm)	$T_0=T_B$ (°K)	A (amp.)	β	$A_0\times10^3$	$V_0\times10^3$	$R_0\times10^6$	Ω_0
2	12.86	244.1	0.01949	20	4.358	6.3871	4.4567	1.4665
3	5.87	244.1	0.01949	20	4.358	2.9154	4.4567	0.6675

Tube No.	$\Omega-\Omega_0$ $=\Delta\Omega$	$\frac{\Delta\Omega}{\Omega_0}$	φ_0	h_0	λ_0	λ (Eq. 104)	θ_1	T_1-T_0	T_1 (°K)
2	0.3335	0.2274	0.1050	0.06083	1.701	1.720	0.2765	67.5	311.6
3	0.02545	0.03813	0.0473	0.06177	1.745	1.720	0.0473	11.6	255.7

Ω_0, the resistance of the filament when entirely at the temperature T_0, was first determined as in Paper I, and then the change in resistance, $\Omega-\Omega_0=\Delta\Omega$, caused by passage of the current A. This current A was chosen by calculation from (Eq. (6) Part II)

$$\beta=(A/A_0)^2 \tag{98}$$

to correspond exactly to integral values of β. In the examples shown, β is 20. A_0, and the quantities V_0 and R_0 were obtained from Paper I, Table II, using the equations

$$V_0=V'x_0/d^{\frac{1}{2}}, \tag{99}$$

$$A_0=A'd^{\frac{3}{2}}, \tag{100}$$

$$R_0=R'\pi/4. \tag{101}$$

From the value of β and $\Delta\Omega/\Omega_0$, given by the experiments, the corresponding value of φ_0 was found from Tables V, VI, or VII of Part V.

Now from Eq. (26) of Part II,

$$h_0=V_0/\varphi_0, \tag{102}$$

and so the heat conductivity, λ_0, at the temperature, T_0, of the leads could be calculated from Eq. (27).

$$\lambda_0=h_0^2/2R_0T_0. \tag{103}$$

The values of θ_1, $=(T_1-T_0)/T_0$, and of T_1 were found as described later in Part V.

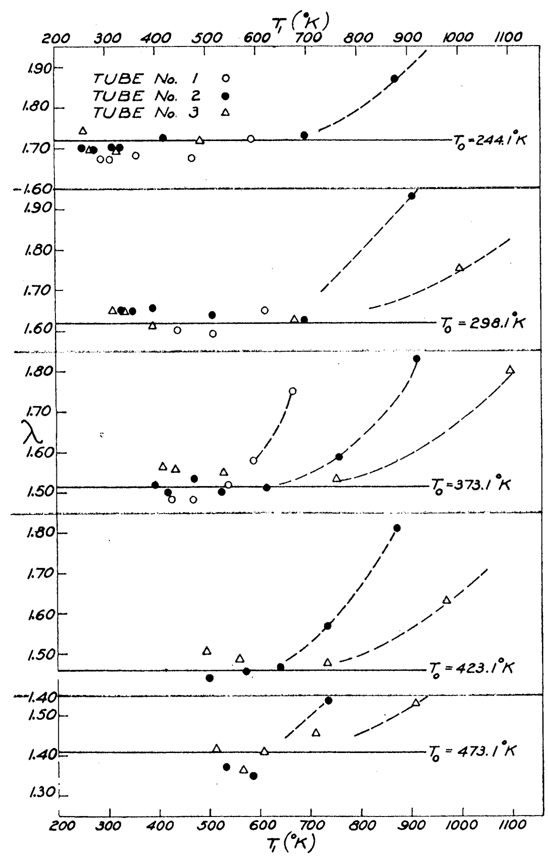

FIG. 4. Experimental data on the heat conductivity λ_0 as a function of the temperature T_1 at the middle of the filament as determined for five values of lead temperature T_0.

[11] It should be noted that brief aging (few minutes) at temperatures even as high as 2600°K will not produce a tungsten filament which is unchanged by further heating. For example, in the case of filaments used in these experiments, after the initial aging at 2000° for about 2 minutes, aging at 2400°K caused a decrease in cold (264°K) resistance of 2 percent after 1 minute, 5 percent after 40 minutes, and after 4 hours at 2400° and 30 seconds at 2800° the total decrease was about 7 percent. At the same time, in spite of this decrease in cold resistance, the voltage for a constant current giving a maximum filament temperature of about 600°K increased. Thus the emissivity had decreased, which indicates either a cleaning or a smoothing of the filament surface. (See also reference 6.) It is possible that filaments when only slightly aged are clean, but unless special tests are made, it is clear that the filament temperature is uncertain. Experiments, involving adsorbed films on tungsten for example, can be interpreted most easily when the tungsten by proper aging has been brought into a condition where its surface and temperature are reproducible.

By experiments at values of $T_0(=T_B)$ from 244° to 473°K, the dependence of λ_0 on T_0 was determined. For each value of T_0 a series of values of β (i.e. of A) was used giving temperatures, T_1, at the middle of the filament up to 1100°K. However, in each case, the calculation as described above gave λ_0, i.e., λ at $T=T_0$.

Preliminary experiments (described below) under simplifying conditions had indicated that the variation of λ with T was given by

$$\log \lambda = 0.9518 + k \log T, \qquad (104)$$

where $k=-0.30$. This value of k was used in the construction (see Part III) of the tables in Part V. That the present much more extensive experiments confirm this choice of k is shown by examination of the collected data in Fig. 4.

λ_0 is plotted as a function of T_1, for the five values of T_0 from 244 to 473°K. Considering only values of T_1 below 600°K,[12] it is seen that λ_0, at each T_0, is independent of T_1 and shows no significant change in the three different tubes, i.e., no change with filament length. This fact that λ_0 is independent of T_1 indicates that the value of k equal to -0.30 used in Eq. (104) is correct. Otherwise there would have been a progressive change in λ_0 as T_1 was varied.

The values of λ at the five values of $T=T_0$, calculated from Eq. (104) are given as solid horizontal lines in Fig. 4. The evident agreement (below $T_1=600$°K) of the experimentally determined points with these calculated values of λ is an independent confirmation of the correctness of k.

Since we at first had no knowledge of k, i.e., of the variation of λ with T, the preliminary experiments mentioned above were carried out. Using Tube No. 1, the bulb was immersed in liquid nitrogen and the leads were held at temperatures, T_0, between 240 and 450°K. At each value of T_0 the resistance, Ω, was measured for several values of current, A, chosen to cause only small deviations of the filament temperature from T_0. Since for each value of T_0 the maximum temperature rise was small ($\sim 10°$), λ could be taken as independent of temperature ($k=0$), and Eq. (36) of Part II, Case I (low values of θ_1), could be applied. With the bulb in liquid nitrogen T_B is effectively equal to zero and this equation takes the form,

$$\Delta\Omega/\Omega_0 = 0.300(\beta-1)\left[1-\frac{\tanh 2.864\varphi_0}{2.864\varphi_0}\right], \qquad (105)$$

where β and Ω_0 are found as already described.

Since the resistance changes, $\Delta\Omega$, are small, the measurements tend to be less accurate than in the experiments where $T_1 - T_0$ was large. However, it is seen from Eq. (105) that for the small resistance changes involved $\Delta\Omega$ is a linear function of β (or of A^2). So by plotting $\Delta\Omega$ *vs.* A^2, the slope of a line through the individual points allowed a satisfactorily accurate calculation of φ_0. From Eqs. (102) and (103), λ_0 was then calculated.

Thus at several temperatures $T(=T_0)$, $\lambda(=\lambda_0)$ was determined and found to depend on temperature as given by Eq. (104).

The values of λ determined in the present work may be extrapolated to join the data of Forsythe and Worthing,[2] giving a curve of λ *vs.* T which has a minimum at about 1300°K and joins $F-W$ at 1500°K. Values of λ from this curve are as follows.

T (°K)	λ	T (°K)	λ
600	1.313	1200	1.00
800	1.18	1300	0.98
1000	1.08	1500	0.99

These values justify the corrections to the $F-W$ equation found necessary by Langmuir, Mac-

[12] In experiments where T_1 exceeded 600°K, λ_0 increased with T_1. This behavior, observed in all tubes, was not caused by changes in emissivity due to an attack of the filament surface by residual gases, since on returning to temperatures below 600° the normal values of λ_0 were obtained. Also the effect was independent of bulb temperature. It was most marked in Tube No. 1, which had the longest filament and least in Tube No. 3 with the shortest filament. It is believed that the explanation lies in the fact that the relations of Paper I between energy radiated and filament temperature are not applicable much above the maximum filament temperature (580°K) for which they were determined. Between 225 and 580°K, the energy radiated is given by $W_R = KT^{5.332}$. However, as pointed out on p. 325 of Paper I, the exponent of T must increase above 600°K in order to reach the values of Jones and Langmuir and Forsythe and Watson which are accepted as accurate above 1000°K. Calculations show that this can account for both the magnitude and direction of the observed deviations of λ_0. The effect should be greatest for the longest filament, since a greater proportion of its length is at temperatures near T_1.

Incidentally, it was observed that very slight traces of water vapor from bulbs insufficiently baked cause appreciable changes in emissivity which increase with bulb temperature, and are immediately recognized by the lag effects produced when the filament temperature is raised or lowered.

Lane and Blodgett[3] for temperatures below 1500°K (see Introduction).

Since the variation of λ with T is established by the experiments, an equation for h as a function of T can be derived. From Eqs. (27a), (101), and Eq. (6) of Paper I,

$$\log h = 96.4826 - 100 + 0.965 \log T. \qquad (106)$$

Likewise $(2H)^{\frac{1}{2}}$ for use in Eq. (93) can be expressed as a function of T. Since H depends not only on T but also on the filament radius r, it is convenient to use the quantity $(r/2H)^{\frac{1}{2}}$. From Eqs. (15) and (104) and Eqs. (6) and (7) of Paper I,

$$\log (r/2H)^{\frac{1}{2}} = 91.3793 - 100 + 2.316 \log T. \qquad (107)$$

Table VIII in Part V gives h, λ, and $(r/2H)^{\frac{1}{2}}$ for a series of values of T, as calculated from the above equations.

V. Calculation of Filament Temperature and Resistance from the Tables

Tables I to VIII, prepared as described in Part III, are given to facilitate calculations of filament temperature or resistance from a knowledge of the filament dimensions, the filament current, and the lead and bulb temperature. Directions and examples are given for the several usual cases. The theory and equations in Parts II and III need not be referred to.

In general there will be known:

r = radius of filament (cm),
x_0 = half length of filament (cm),
T_0 = lead temperature (°K),
T_B = bulb temperature (°K),
A = filament current (amp.).

Case 1. $(T_B = T_0)$

This is the experimental condition existing, for example, when the bulb and the filament leads are immersed in a bath at constant temperature. The filament leads are to be chosen of such a diameter (at least 10 times filament diameter) and as short as possible so that they are not appreciably heated by the filament currents to be used, or by conduction of heat from the filament.

It is ordinarily convenient to construct a table or curve giving filament temperature as a function of current, from which temperatures can be found for any value of current used in experiments. This is done in the following way.

From Table VIII or Eq. (107) find the value of $(r/2H)^{\frac{1}{2}}$ corresponding to T_0. Then using the known value of r, calculate φ_0 from

$$\varphi_0 = x_0/(2H)^{\frac{1}{2}}. \qquad (108)$$

Now for integral values of β as given in the Tables (II, III, IV) find the values of θ_1 corresponding to this value of φ_0. T_1, the temperature at the middle of the filament, is given by

$$T_1 = (1+\theta_1) T_0. \qquad (109)$$

The currents, A, required to produce each temperature T_1, are found from the values of β using the equation

$$\beta = (A/A_0)^2, \qquad (110)$$

where A_0 for the filament in question is calculated from

$$A_0 = A' d^{\frac{3}{2}}, \qquad (111)$$

where d is the filament diameter and A' is taken from Table II in Paper I, or calculated from the equation

$$\log A' = 96.1952 - 100 + 2.051 \log T_0. \qquad (112)$$

$\Delta\Omega/\Omega_0$ is found in a similar way by using φ_0,

Table I. *Limiting values of various functions.*

1 β $(\beta_0=0)$	2 σ_0	3 θ_M	4 B_∞	5 $(\Delta\Omega/\Omega_0)_\infty$	6 β $(\beta_0=1)$ $(\sigma_0^{-4.102})$	7 $\sigma_0^{0.36}$
0	1.0000	0.0000	0.00000	0.0000	1.000	1.000
1	0.8504	.1759	.0577	.2205	1.943	0.943
2	.7708	.2974	.0839	.3775	2.910	.910
3	.7182	.3923	.0983	.5024	3.888	.888
4	.6798	.4709	.1075	.6074	4.87	.870
6	.6257	.5981	.1186	.7801	6.84	.845
8	.5881	.7004	.1248	.9212	8.83	.826
10	.5594	.7877	.1287	1.043	10.81	.811
15	.5104	.9594	.1336	1.287	15.79	.785
20	.4774	1.095	.1355	1.483	20.77	.766
25	.4529	1.208	.1361	1.649	25.75	.752
30	.4338	1.305	.1361	1.793	30.74	.740
40	.4051	1.468	.1354	2.038	40.72	.722
50	.3840	1.604	.1343	2.245	50.71	.709
60	.3676	1.720	.1330	2.425	60.70	.697
80	.3429	1.916	.1305	2.730	80.68	.680
100	.3249	2.078	.1281	2.986	100.7	.667
120	.3109	2.217	.1260	3.209	120.7	.657
140	.2994	2.340	.1241	3.407	140.6	.648
160	.2899	2.450	.1223	3.586	160.6	.640
180	.2817	2.550	.1207	3.750	180.6	.634
200	.2746	2.641	.1192	3.902	200.6	.628

β, and Tables V, VI, and VII.[13] This value of $\Delta\Omega/\Omega_0$ is usually not required, but if compared with a measured value of $\Delta\Omega/\Omega_0$ serves as a check on the calculation of T_1. Also if the filament length and hence φ_0 are unknown, then a determination of Ω_0 and of $\Delta\Omega$ caused by the current A enables one to find φ_0 and then θ_1 (and T_1) from the tables.

Case 2. ($T_B < T_0$)

In some types of experiments the bulb may be at a lower temperature than the leads. The tables have been prepared primarily for use when $T_B = T_0$. However, when $T_B < T_0$, a current temperature scale may be obtained by the following procedure:

Calculate φ_0 for the lead temperature T_0 which is to be used and find θ_1 and T_1 for the various values of β in the tables exactly as in Case 1. These values of β, however, must not be used to calculate the currents A. Instead, a new value of β must be calculated using Eq. (65) of Part III,

$$\beta = \sigma_0^{-4.102} - (1-\beta_0)\sigma_0^{0.36}, \tag{113}$$

where from Eq. (11), Part II,

$$\beta_0 = 1 - (T_B/T_0)^{4.462}, \tag{114}$$

and σ_0 is assigned the values given in the tables (just below the values of β which are applicable only to Case 1). σ_0, $\sigma_0^{0.36}$, and $\sigma_0^{-4.102}$ are given in Table I.

[13] It should be noted that for convenience in tabulation, θ_1/β and $\Delta\Omega/\beta\Omega_0$ are given in Tables II and V instead of θ_1 and $\Delta\Omega/\Omega_0$ as in the remainder of the tables.

TABLE II. θ_1/β *as function of* φ_0 *and* β.

φ_0	$\beta=0$ $\sigma_0=$ 1.0000	$\beta=1$ $\sigma_0=$ 0.8504	$\beta=2$ $\sigma_0=$ 0.7708	$\beta=3$ $\sigma_0=$ 0.7182	$\beta=4$ $\sigma_0=$ 0.6798	$\beta=6$ $\sigma_0=$ 0.6257	$\beta=8$ $\sigma_0=$ 0.5881	$\beta=10$ $\sigma_0=$ 0.5594
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.02	.00040	.00040	.00040	.00040	.00040	.00040	.00040	.00040
.04	.00159	.00159	.00160	.00160	.00160	.00161	.00162	.00162
.06	.00355	.00357	.00358	.00359	.00361	.00364	.00367	.00370
.08	.00625	.00630	.00634	.00638	.00643	.00653	.00662	.00672
.10	.00964	.00974	.00984	.00993	.01006	.01029	.01053	.01078
.12	.01367	.01385	.01405	.01424	.01450	.01495	.01543	.01595
.14	.01827	.01857	.01891	.01922	.01970	.02072	.02162	.02234
.16	.02337	.02382	.02434	.02488	.02560	.02660	.02800	.0294
.18	.02891	.02951	.03024	.03105	.03208	.03318	.03514	.0369
.20	.03481	.03554	.03646	.03754	.03890	.04078	.0429	.0442
.22	.04099	.04179	.04285	.04412	.04575	.04850	.0505	.0507
.24	.04740	.04857	.0499	.0514	.0530	.0554	.0566	.0563
.26	.05396	.05506	.0565	.0585	.0604	.0616	.0619	.0611
.28	.06061	.0618	.0633	.0656	.0674	.0674	.0668	.0647
.30	.06730	.0686	.0700	.0720	.0733	.0727	.0706	.0676
.32	.07398	.0753	.0765	.0781	.0788	.0772	.0737	.0698
.34	.08061	.0820	.0829	.0838	.0836	.0810	.0763	.0717
.36	.08714	.0881	.0885	.0889	.0879	.0838	.0784	.0731
.38	.09355	.0941	.0940	.0937	.0919	.0863	.0801	.0743
.40	.09981	.0996	.0988	.0978	.0952	.0885	.0814	.0751
.42	.10591	.1049	.1035	.1017	.0982	.0902	.0826	.0759
.44	.11181	.1102	.1080	.1051	.1008	.0918	.0835	.0765
.46	.11751	.1149	.1120	.1083	.1031	.0931	.0842	.0769
.48	.12301	.1197	.1157	.1110	.1050	.0942	.0848	.0773
.50	.12829	.1239	.1190	.1134	.1067	.0950	.0852	.0776
.55	.14052	.1336	.1260	.1181	.1100	.0966	.0862	.0781
.60	.15146	.1413	.1313	.1214	.1124	.0977	.0867	.0784
.65	.16110	.1476	.1354	.1240	.1139	.0984	.0870	.0785
.70	.16956	.1530	.1384	.1258	.1150	.0988	.0872	.0786
.75	.1771	.1573	.1408	.1272	.1158	.0991	.0873	.0787
.80	.1834	.1608	.1426	.1280	.1163	.0993	.0874	
.85	.1890	.1637	.1440	.1288	.1167	.0995	.0875	
.90	.1938	.1660	.1451	.1293	.1170	.0995		
1.00	.2016	.1687	.1466	.1300	.1174	.0996		
1.10	.2074	.1712	.1474	.1303	.1175	.0996		
1.20	.2116	.1728	.1479	.1305	.1176			
1.30	.2148	.1738	.1482	.1306	.1177			
1.40	.2172	.1746	.1484	.1307				
∞	.2241	.1759	.1487	.1308	.1177	.0997	.0876	.0788

TABLE III. θ_1 *as function of* φ_0 *and* β.

φ_0	$\beta=15$ $\sigma_0=$ 0.5104	$\beta=20$ $\sigma_0=$ 0.4774	$\beta=25$ $\sigma_0=$ 0.4529	$\beta=30$ $\sigma_0=$ 0.4338	$\beta=40$ $\sigma_0=$ 0.4051	$\beta=50$ $\sigma_0=$ 0.3840	$\beta=60$ $\sigma_0=$ 0.3676	$\beta=80$ $\sigma_0=$ 0.3429
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.01	.0015	.0020	.0025	.0030	.0040	.0050	.0060	.0081
.02	.0060	.0081	.0101	.0121	.0163	.0204	.0247	.0332
.03	.0136	.0183	.0229	.0278	.0375	.0473	.0575	.0784
.04	.0245	.0330	.0417	.0505	.0687	.0877	.1075	.1497
.05	.0388	.0526	.0666	.0813	.1120	.1448	.1800	.2582
.06	.0568	.0774	.0991	.1216	.1705	.2239	.2886	.429
.07	.0786	.1081	.1401	.1730	.2473	.3328	.429	.684
.08	.1048	.1456	.1909	.2384	.3494	.4839	.641	1.027
.09	.1358	.1908	.2527	.3206	.4800	.674	.895	1.321
.10	.1718	.2447	.3259	.4170	.6330	.879	1.121	1.521
.11	.2135	.310	.414	.532	.794	1.065	1.296	1.650
.12	.2605	.377	.509	.651	.945	1.209	1.418	1.734
.13	.3137	.452	.608	.770	1.068	1.312	1.502	1.788
.14	.3703	.533	.704	.873	1.164	1.387	1.561	1.827
.15	.428	.609	.789	.961	1.233	1.440	1.604	1.854
.16	.486	.681	.864	1.031	1.286	1.478	1.634	1.871
.17	.543	.745	.926	1.084	1.326	1.509	1.656	1.884
.18	.597	.803	.977	1.126	1.357	1.531	1.672	1.894
.19	.643	.850	1.019	1.161	1.380	1.548	1.685	1.900
.20	.684	.889	1.053	1.188	1.399	1.561	1.694	1.905
.21	.721	.921	1.081	1.210	1.414	1.571	1.701	1.908
.22	.754	.948	1.1027	1.227	1.425	1.579	1.706	1.911
.24	.806	.990	1.1354	1.253	1.441	1.589	1.712	1.914
.26	.844	1.020	1.1580	1.271	1.451	1.595	1.716	1.915
.28	.873	1.0409	1.1733	1.282	1.458	1.599	1.718	1.916
.30	.8945	1.0557	1.1837	1.289	1.462	1.601	1.719	
.35	.9271	1.0772	1.1980	1.299	1.466	1.603	1.720	
.40	.9430	1.0868	1.2038	1.303	1.468	1.604		
∞	.9594	1.0946	1.2078	1.305	1.468	1.604	1.7205	1.916

When T_B is so low, e.g. when the bulb is immersed in liquid air, that back-radiation is negligible; then T_B may be taken as effectively equal to zero. For this condition, then from Eq. (114),

$$\beta_0=1, \quad \text{and} \quad \beta=\sigma_0^{-4.102}.$$

In Table I, column 6, there are tabulated the values of β for this special case ($\beta_0=1$), corresponding to the values in column 1 of β for Case 1 ($T_B=T_0$), for which, from Eq. (114), $\beta_0=0$. In column 2 are the values of σ_0. Columns 1 and 2 list the same β's and σ_0's as head the Tables II to VII.

Case 3. Filament joined to a spring

As discussed in Part III it is often necessary to hold a filament taut by means of a spring between one end of the filament and its lead. The spring, heated by conduction of heat from the filament, may increase the maximum temperature T_1 of the filament. It was shown in III that the effect is as if the filament length were increased by an amount Δx.

TABLE IV. θ_1 *as function of* φ_0 *and* β.

φ_0	$\beta=100$ $\sigma_0=$ 0.3249	$\beta=120$ $\sigma_0=$ 0.3109	$\beta=140$ $\sigma_0=$ 0.2994	$\beta=160$ $\sigma_0=$ 0.2899	$\beta=180$ $\sigma_0=$ 0.2817	$\beta=200$ $\sigma_0=$ 0.2746
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.005	.0025	.0030	.0035	.0040	.0045	.0050
.010	.0101	.0122	.0142	.0163	.0184	.0205
.015	.0231	.0279	.0327	.0376	.0425	.0475
.020	.0419	.0508	.0599	.0691	.0786	.0882
.025	.0673	.0821	.0974	.1132	.1295	.1465
.030	.1003	.1230	.1476	.1732	.2002	.2288
.035	.1425	.1766	.2145	.2544	.2987	.3465
.040	.196	.2458	.3040	.3673	.4390	.5203
.045	.264	.3386	.4258	.526	.646	.789
.050	.350	.461	.594	.762	.957	1.197
.055	.463	.630	.829	1.081	1.371	1.642
.060	.612	.846	1.141	1.446	1.719	1.956
.065	.794	1.112	1.440	1.723	1.963	2.161
.070	1.013	1.364	1.669	1.920	2.128	2.297
.075	1.228	1.578	1.838	2.058	2.239	2.391
.080	1.407	1.713	1.960	2.157	2.320	2.458
.085	1.544	1.828	2.050	2.226	2.377	2.506
.090	1.656	1.917	2.114	2.279	2.419	2.541
.095	1.742	1.977	2.164	2.319	2.451	2.566
.100	1.809	2.027	2.202	2.349	2.475	2.585
.105	1.863	2.066	2.232	2.373	2.493	2.600
.110	1.902	2.096	2.256	2.390	2.506	2.610
.115	1.934	2.121	2.274	2.403	2.516	2.617
.120	1.960	2.139	2.288	2.414	2.524	2.623
.130	1.999	2.166	2.307	2.428	2.535	2.631
.140	2.024	2.184	2.319	2.436	2.541	2.636
.150	2.041	2.196	2.327	2.442	2.544	2.638
∞	2.078	2.217	2.339	2.449	2.549	2.641

Δx is found from either Eq. (93),

$$\Delta x = (2H)^{\frac{1}{2}} P(1 - P\beta/b)$$

or from Eq. (94),

$$\Delta x = x_S/s^2.$$

In Eq. (93), $2H$ is obtained from the value of $(r/2H)^{\frac{1}{2}}$ for the filament just as in Case 1; P has the values given by Eqs. (88) or (89); and β/b is given by Eq. (91).

In Eq. (94), x_S is the total length of the spring in cm, and s is the ratio of the diameter of the spring to that of the filament.

For springs of practical design,[10] the simpler Eq. (94) is usually sufficiently accurate. If for some reason the spring must be longer than required by usual design, it may become necessary to use Eq. (93).

Having obtained Δx, φ_0 is calculated from Eq. (108), in which x_0 is now not the actual half-length of the filament as before, but the half-length plus $\Delta x/2$. θ_1, T_1, and A are then found as in Cases 1 or 2.

The above method must be used, of course, even in the absence of a spring, when the filament leads are of such small diameter or so long as to be heated by conduction from the filament. As can be seen from Eq. (94), this is avoided if the leads are only a few cm long and have a diameter at least 10 times that of the filament.

TABLE V. $\Delta\Omega/\beta\Omega_0$ *as function of* φ_0 *and* β.

φ_0	$\beta=0$ $\sigma_0=$ 1.0000	$\beta=1$ $\sigma_0=$ 0.8504	$\beta=2$ $\sigma_0=$ 0.7708	$\beta=3$ $\sigma_0=$ 0.7182	$\beta=4$ $\sigma_0=$ 0.6798	$\beta=6$ $\sigma_0=$ 0.6257	$\beta=8$ $\sigma_0=$ 0.5881	$\beta=10$ $\sigma_0=$ 0.5594
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
.02	.00033	.00033	.00033	.00033	.00033	.00033	.00033	.00033
.04	.00130	.00131	.00131	.00131	.00131	.00132	.00132	.00134
.06	.00291	.00293	.00294	.00295	.00296	.00299	.00302	.00307
.08	.00513	.00517	.00520	.00524	.00528	.00536	.00544	.00553
.10	.00792	.00799	.00808	.00818	.00827	.00846	.00866	.00888
.12	.01123	.01139	.01156	.01174	.01194	.01232	.01273	.01318
.14	.01502	.01528	.01557	.01588	.01624	.01711	.01788	.01851
.16	.01924	.01962	.02007	.02060	.02114	.02201	.02326	.02446
.18	.02382	.02434	.02497	.02577	.02655	.02754	.02933	.0309
.20	.02871	.02934	.03015	.03120	.03228	.03394	.03604	.0371
.22	.03385	.03443	.0353	.0366	.03806	.04056	.0428	.0433
.24	.0392	.04022	.0414	.0429	.0444	.0466	.0483	.0484
.26	.0447	.04579	.0472	.0491	.0509	.0522	.0534	.0534
.28	.0503	.0515	.0531	.0553	.0571	.0577	.0580	.0573
.30	.0559	.0573	.0589	.0609	.0624	.0629	.0620	.0605
.32	.0616	.0629	.0644	.0661	.0675	.0673	.0654	.0634
.34	.0672	.0687	.0700	.0712	.0719	.0707	.0684	.0660
.36	.0728	.0740	.0750	.0758	.0760	.0741	.0712	.0682
.38	.0783	.0792	.0798	.0802	.0800	.0771	.0736	.0702
.40	.0837	.0841	.0842	.0842	.0835	.0798	.0757	.0719
.42	.0890	.0889	.0885	.0878	.0867	.0823	.0777	.0735
.44	.0942	.0936	.0927	.0914	.0897	.0846	.0794	.0750
.46	.0992	.0979	.0963	.0946	.0925	.0866	.0810	.0763
.48	.1041	.1023	.1002	.0978	.0951	.0885	.0825	.0774
.50	.1088	.1063	.1036	.1006	.0974	.0902	.0838	.0785
.55	.1200	.1157	.1114	.1070	.1026	.0939	.0867	.0809
.60	.1302	.1236	.1179	.1123	.1068	.0970	.0891	.0829
.65	.1394	.1306	.1235	.1167	.1103	.0995	.0911	.0845
.70	.1478	.1368	.1283	.1204	.1133	.1017	.0928	.0859
.75	.1554	.1426	.1324	.1236	.1159	.1037	.0944	.0872
.80	.1622	.1476	.1360	.1264	.1182	.1053	.0956	.0882
.85	.1684	.1520	.1392	.1288	.1202	.1068	.0968	.0892
.90	.1741	.1559	.1420	.1310	.1220	.1081	.0978	.0900
1.00	.1838	.1625	.1467	.1347	.1250	.1102	.0996	.0915
1.10	.1920	.1679	.1506	.1377	.1274	.1120	.1010	.0926
1.20	.1989	.1723	.1538	.1402	.1294	.1135	.1022	.0936
1.30	.2047	.1761	.1565	.1423	.1312	.1148	.1032	.0944
0.140	.2098	.1793	.1588	.1441	.1326	.1159	.1040	.0951
∞	.27566	.2205	.18875	.16746	.15185	.13002	.11515	.10433

TABLE VI. $\Delta\Omega/\Omega_0$ *as function of* φ_0 *and* β.

φ_0	$\beta=15$ $\sigma_0=$ 0.5104	$\beta=20$ $\sigma_0=$ 0.4774	$\beta=25$ $\sigma_0=$ 0.4529	$\beta=30$ $\sigma_0=$ 0.4338	$\beta=40$ $\sigma_0=$ 0.4051	$\beta=50$ $\sigma_0=$ 0.3840	$\beta=60$ $\sigma_0=$ 0.3676	$\beta=80$ $\sigma_0=$ 0.3429
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.01	.0012	.0016	.0020	.0025	.0033	.0041	.0050	.0066
.02	.0049	.0066	.0083	.0100	.0134	.0168	.0202	.0273
.03	.0112	.0150	.0188	.0228	.0308	.0389	.0472	.0644
.04	.0201	.0271	.0342	.0414	.0564	.0721	.0884	.1231
.05	.0318	.0432	.0547	.0669	.0921	.1192	.1483	.2129
.06	.0466	.0636	.0815	.1000	.1405	.1848	.2386	.356
.07	.0646	.0889	.1186	.1426	.2043	.2759	.3563	.574
.08	.0862	.1200	.1575	.1969	.2900	.4046	.539	.879
.09	.1118	.1575	.1804	.2657	.4005	.569	.763	1.152
.10	.1418	.2010	.2200	.3480	.5343	.748	.970	1.363
.11	.1766	.257	.318	.446	.677	.916	1.142	1.511
.12	.2161	.314	.429	.550	.812	1.061	1.278	1.624
.13	.2610	.378	.517	.655	.929	1.174	1.378	1.715
.14	.3091	.448	.597	.749	1.030	1.262	1.459	1.791
.15	.359	.515	.676	.835	1.109	1.333	1.527	1.856
.16	.409	.578	.749	.909	1.174	1.394	1.586	1.912
.17	.458	.637	.812	.968	1.230	1.447	1.638	1.961
.18	.506	.694	.867	1.019	1.277	1.494	1.682	2.004
.19	.550	.742	.913	1.063	1.319	1.534	1.722	2.044
.20	.591	.784	.953	1.102	1.357	1.571	1.758	2.078
.21	.629	.821	.989	1.137	1.390	1.604	1.790	2.109
.22	.664	.854	1.021	1.168	1.420	1.633	1.819	2.137
.24	.720	.910	1.076	1.222	1.472	1.685	1.870	2.187
.26	.764	.956	1.122	1.267	1.517	1.728	1.913	2.228
.28	.803	.995	1.161	1.306	1.554	1.765	1.950	2.264
.30	.837	1.029	1.194	1.338	1.586	1.797	1.981	2.295
.35	.906	1.096	1.260	1.404	1.652	1.861	2.045	2.358
.40	.952	1.144	1.309	1.453	1.700	1.909	2.092	2.404
∞	1.2872	1.4830	1.6489	1.7930	2.0385	2.2449	2.4247	2.7304

TABLE VII. $\Delta\Omega/\Omega_0$ *as function of* φ_0 *and* β.

φ_0	$\beta=100$ $\sigma_0=$ 0.3249	$\beta=120$ $\sigma_0=$ 0.3109	$\beta=140$ $\sigma_0=$ 0.2994	$\beta=160$ $\sigma_0=$ 0.2899	$\beta=180$ $\sigma_0=$ 0.2817	$\beta=200$ $\sigma_0=$ 0.2746
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
.005	.0021	.0025	.0029	.0033	.0037	.0041
.010	.0083	.0100	.0117	.0134	.0151	.0168
.015	.0190	.0229	.0268	.0308	.0349	.0390
.020	.0344	.0417	.0492	.0568	.0646	.0725
.025	.0553	.0675	.0800	.0930	.1066	.1205
.030	.0824	.1013	.1215	.1426	.1650	.1887
.035	.1172	.1457	.1768	.2100	.2469	.2867
.040	.162	.2037	.2512	.3042	.3646	.4329
.045	.218	.2801	.3534	.438	.540	.663
.050	.290	.383	.496	.640	.810	1.023
.055	.385	.527	.699	.921	1.182	1.435
.060	.511	.714	.977	1.247	1.512	1.746
.065	.670	.951	1.252	1.522	1.764	1.980
.070	.864	1.184	1.473	1.734	1.956	2.150
.075	1.061	1.389	1.647	1.892	2.098	2.283
.080	1.228	1.549	1.796	2.018	2.215	2.393
.085	1.364	1.665	1.904	2.120	2.312	2.487
.090	1.486	1.768	2.002	2.209	2.397	2.569
.095	1.588	1.854	2.082	2.286	2.472	2.641
.100	1.670	1.928	2.153	2.354	2.537	2.706
.105	1.741	1.994	2.216	2.415	2.597	2.764
.110	1.803	2.052	2.272	2.470	2.650	2.815
.115	1.858	2.105	2.323	2.519	2.698	2.863
.120	1.908	2.153	2.369	2.564	2.743	2.908
.130	1.994	2.236	2.450	2.644	2.821	2.985
.140	2.067	2.307	2.519	2.712	2.888	3.050
.150	2.132	2.368	2.579	2.770	2.945	3.107
∞	2.9861	3.2087	3.4069	3.5860	3.7501	3.9020

TABLE VIII. *Values of* h, λ *and* $(r/2H)^{\frac{1}{2}}$.

T_0(°K)	h	λ	$(r/2H)^{\frac{1}{2}}\times 10^3$
240	0.0602	1.729	0.780
250	.0626	1.708	0.857
260	.0650	1.688	0.938
270	.0674	1.669	1.024
273	.0682	1.663	1.051
280	.0698	1.651	1.114
290	.0722	1.633	1.209
298	.0742	1.620	1.287
300	.0746	1.617	1.307
310	.0770	1.601	1.410
350	.0866	1.544	1.868
400	.0985	1.483	2.545
450	.1104	1.432	3.343
500	.1222	1.387	4.267
550	.1340	1.348	5.321
600	.1457	1.313	6.510

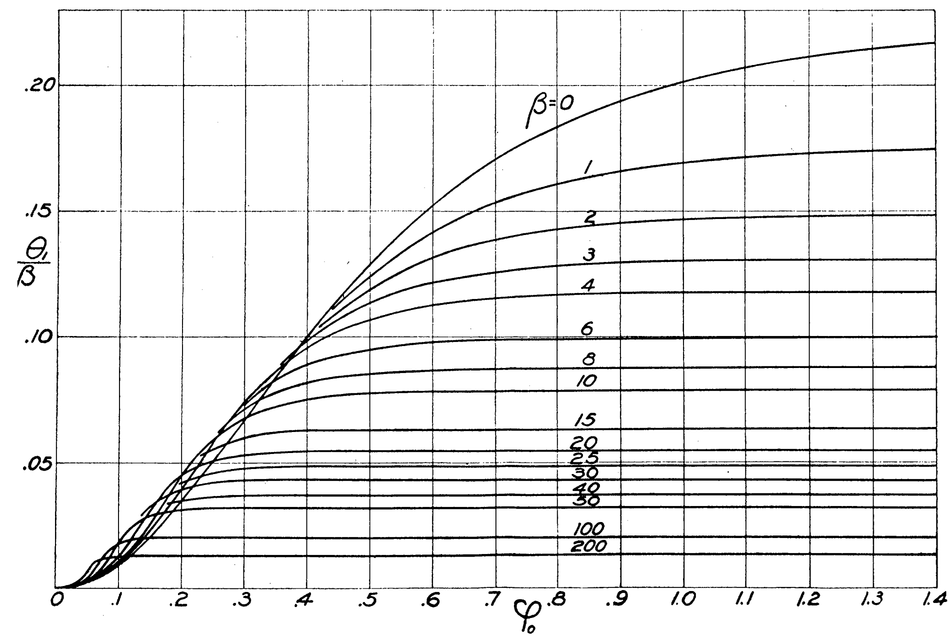

FIG. 5. θ_1/β as a function of φ_0 for various values of β as in Tables II, III, and IV. For clearness some of the curves are not completed to the origin.

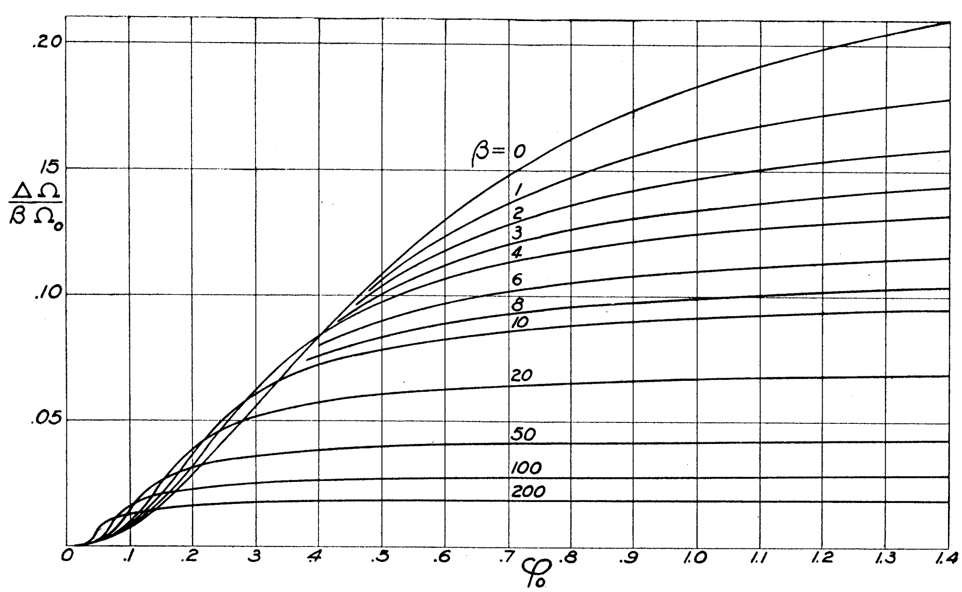

FIG. 6. $\Delta\Omega/\beta\Omega_0$ as a function of φ_0 for various values of β as in Tables V, VI, VII.

Examples of the calculation

Consider the filament in tube No. 2, used in the experiments on heat conductivity, which had a length of 12.86 cm and a diameter of 0.00499 cm ($\sim$2 mil).

	T_0 (°K)	T_B (°K)	β_0	Δx (cm)	x_0 (cm)	$(r/2H)^{\frac{1}{2}}$ $\times 10^3$	φ_0
(1)	300	300	0	0	6.43	1.3071	0.1683
(2)	300	0	1	0	6.43	1.3071	0.1683
(3)	300	300	0	0.40	6.63	1.3071	0.1735

	θ_1	T_1 (°K)	β	A' amp. cm$^{-3/2}$	$A_0 \times 10^3$ (amp.)	A (amp.)
(1)	0.740	522.0	20	18.869	6.651	0.02974
(2)	0.740	522.0	20.77	18.869	6.651	0.03031
(3)	0.768	530.4	20	18.869	6.651	0.02974

(1) Given $T_B = T_0 = 300$°K, the filament mounted directly on heavy leads. Calculate T_1 and the corresponding current A, for $\beta = 20$. This is an example of Case 1. As shown in the accompanying table x_0, the half-length, is $12.86/2 = 6.43$ cm. $(r/2H)^{\frac{1}{2}}$ at 300° from Table VIII is 1.3071×10^{-3}. From x_0, $(r/2H)^{\frac{1}{2}}$, the known value of the filament radius r, and Eq. (108), φ_0 is then calculated to be 0.1683. From Table III for $\beta = 20$ and this value of φ_0, θ_1 is found to be 0.740. From Eq. (109) T_1 is 522°K. From Table II, Paper I, A' at 300° is 18.869, so that by Eq. (111) A_0 is 6.651×10^{-3} amp. Finally, using Eq. (110) with $\beta = 20$ and this value of A_0, we find A to be 0.02974 amp.

(2) Given $T_0 = 300$°K as in Example (1), but the bulb is now immersed in liquid air so that $T_B = 0$. Calculate T_1 and A for $\beta = 20$. This is an example of Case 2, $\beta_0 = 1$.

One proceeds exactly as in Example (1) up to the calculation of A. Here instead of using $\beta = 20$, the corresponding value of β in column 6 of Table I, equal to 20.77, is used with Eq. (110) to calculate A. As to be expected, this value is slightly larger than the current required in Example (1).

(3) $T_0 = T_B = 300$°K as in Example (1), but one end of the filament is joined to a *spring*. Calculate T_1 and A. This is an example of Case 3.

Making use of the relations given in reference 10 on spring design, one finds that a spring suitable for a filament of the present length and diameter can have the following dimensions: a 10-cm length of 10-mil spring wire wound into a spring of 12-mm diameter ($\sim$3 turns).

Δx equal to 0.40 cm is found from Eq. (94), in which x_S is the spring length (10 cm) and s is the ratio $(10/2 = 5)$ of the diameters of spring and filament. φ_0, equal to 0.1735, is calculated as before from Eq. (108) in which x_0 is now 6.43 $+(\Delta x/2)$ or 6.63 cm. The remaining steps in the calculation of T_1 and A are unchanged. One notes that although the current is the same as in Example (1), the presence of the spring has increased T_1 about 8°.

Table I

Table I gives limiting values of various functions, and has in part been referred to under case 2. In addition, column 3 lists values of θ_M. $(1+\theta_M)T_0 = T_M$, the temperature at the middle of the filament when it is infinitely long, or so long that the central part is not cooled by the leads. For a filament of given diameter, θ_1 approaches θ_M as a limit as the filament is made longer and as β is increased. $(\Delta\Omega/\Omega_0)_\infty$ in column 5 is the corresponding limiting value of $\Delta\Omega/\Omega_0$. This behavior is shown in Figs. 5 and 6 which are plots of Tables II to VII. It will be noted in Tables II, III, and IV that θ_1 as a function of φ_0 extends practically to the value of θ_M. However, in Tables V, VI, and VII for the same values of φ_0, $\Delta\Omega/\Omega_0$ has not closely approached its limiting value. These tabulated values of $\Delta\Omega/\Omega_0$ may be extended to $(\Delta\Omega/\Omega_0)_\infty$ by calculation from the equation

$$\Delta\Omega/\Omega_0 = (\Delta\Omega/\Omega_0)_\infty - B_\infty/\varphi_0.$$

where B_∞ has the values given in column 4 of Table I.

Column 7 gives values of $\sigma_0^{0.36}$ for use with Eq. (113). In this same equation $\sigma_0^{-4.102}$ is given by the values of β in column 6.