The Townsend Coefficients and Spark Discharge

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A quantitative study has been made of the prespark current between plane parallel electrodes in purified N_2. The results yield the relationship between (X/p) (X in volts/cm, p pressure in mm of Hg) and (α/p) (α is the number of new pairs of ions created by one electron in advancing one cm in the direction of the field). The range in values of X/p covered is from 20 to 1000. The portion of the data between $X/p=20$ and $X/p=38$ may be represented at 22°C by

$$\alpha/p=(5.76\pm1.56)\times10^{-7}e^{(0.245\pm0.003)X/p}.$$

In this region the Townsend equation $i=i_0e^{\alpha d}$ for electron ionization alone was found to hold up to the passage of a spark. Between $X/p=44$ and 176 the curve $\alpha/p=F(X/p)$ is represented by

$$\alpha/p=(1.166\pm0.022)\times10^{-4}(X/p-32.1\pm1.4)^2.$$

The portion of the curve between $X/p=200$ and 1000 may be represented by $(\alpha/p+3.65)^2=0.21X/p$. These results are in excellent agreement with those of Masch but differ by an order of magnitude and more from those of Ayres below $X/p=70$. The deviations from $i=i_0e^{\alpha d}$ (due to a secondary process of ionization) were first observed at $X/p=100$. These deviations yield values of a coefficient β due to the secondary ionization mechanism in terms of Townsend's equation

$$i=i_0(\alpha-\beta)e^{(\alpha-\beta)d}/(\alpha-\beta e^{(\alpha-\beta)d})$$

which extend from $X/p=100$ to $X/p=1000$. To obtain consistent values of β it was found necessary to reduce the initial photoelectric current density from the cathode to less than 10^{-13} amp./cm². Current densities of magnitude greater than 10^{-12} amp./cm² give rise to space charge field distortion and consequently falsify values of the secondary coefficient. The values of α/p and β/p as functions of X/p are applied to the Townsend criterion for spark breakdown between plane parallel electrodes and yield a curve for sparking potential plotted against the product of pressure and electrode separation in agreement with the experimental data of Strutt and Hurst. Since the quantity β cannot be due to ionization by positive ions in the gas it must be ascribed to electron liberation at the cathode either by positive ion impact or by a photoelectric effect. The character of the variation of α/p and β/p as functions of X/p lends support to the hypothesis that it is the photoelectric effect at the cathode which is the effective secondary mechanism under these conditions.

INTRODUCTION

RECENT investigations involving the phenomena of spark breakdown in gases have indicated a need for adequate data on certain fundamental constants involved in the theory of these phenomena; the investigations have also revealed a lack of agreement between the results of several investigators. The constants referred to describe the primary and secondary mechanisms which play a role in spark discharge. These are the Townsend coefficients α and β, ascribed[1] by him to impact ionization of molecules by electrons and by positive ions in the gas. Lately,[2–8] the recognition of the fact that the secondary ionization (measured by β) could not be due to positive ion impact in the gas has accentuated the interest in the problem as a whole. A number of proposed alternate mechanisms have been shown[9] to give an equation of *type* similar to that holding for α and β.

The majority of the studies undertaken in recent times to fulfill the need have been made on air, inasmuch as this is the gas of most immediate practical importance in the field of applied electrical engineering. The earlier investigations made on other gases have been confined to rather narrow limits of the field strength and pressure.

The purpose of the present work was to make a detailed study of the ionization processes in a pure diatomic gas. Such a study should yield data for computing sparking characteristics for a gas which does not offer complications due to the presence of molecules of various ionizing potentials. It was also desirable to use a gas in which electrons remain relatively free (i.e., in which negative ion formation is rare). Furthermore, it was necessary to use a gas which did not react with the electrodes, as it was the intention

[1] Townsend, *Electricity in Gases*, Chaps. 8, 9.
[2] A. Von Hippel, Ann. d. Physik **81**, 1053 (1926).
[3] W. Rogowski, Archiv. f. Elek. **16**, 761 (1926).
[4] L. B. Loeb, Science **66**, 627 (1927).
[5] R. M. Sutton, Phys. Rev. **34**, 547 (1929).
[6] Mouzon, Phys. Rev. **35**, 695 (1930).
[7] O. Beeck, Phys. Rev. **38**, 967 (1931).
[8] R. N. Varney, Phys. Rev. **47**, 483 (1935).
[9] L. B. Loeb, Rev. Mod. Phys. **8**, 267 (1936).

to investigate the dependence of the secondary mechanism upon the cathode material. Purified N_2 fulfills the above conditions and was therefore selected for the present investigations.

Method and Experimental Procedure

The apparatus used was essentially that set up by Sanders.[10, 11] The condensers employed in the rectification of the high potential were of much greater capacity than those used by him. Each bank of condensers had a value of 0.25 mf. whereas those of Sanders were of value 0.02 mf. The general technique was that of Sanders and will not be described here.

A photoelectric emission of about 10^{-12} amp./cm^2 was attained by focusing a small, very bright section of a quartz mercury arc on the cathode. During the course of the experiments it became desirable to control the current density thus liberated from the cathode. (Sanders had not considered this as essential.) To obtain a smaller current density a less intense section of the arc was employed. The initial current density thereby obtained was about 10^{-14} amp./cm^2.

The N_2, taken from a tank, was passed through a purifying train of heated copper filings, $CaCl_2$, and P_2O_5. Gas temperature and pressure were reduced to a standard temperature (22°C) in admitting a given *amount* of N_2 to the chamber. Thus all data here given refer to 22°C.

Experimental Results

At given field strengths (X volts/cm) and pressures (mm of Hg), with ultraviolet light liberating an initial electron current i_0 from the cathode, the current i reaching the anode was studied as a function of the electrode separation d. Fig. 1, in which $(1/p) \log i/i_0$ is plotted against d, illustrates the fact that for the range in values of X/p from 20 to 46 the equation

$$i = i_0 e^{\alpha d} \qquad (1)$$

for electron impacts alone holds very well. For this work pressures between 100 and 700 mm were used, requiring the high tension set. Fig. 2, also drawn from data taken with the high tension set and depicting data analogous to those in Fig. 1, but at higher X/p, illustrates the deviation of the current growth from the simple relation $i = i_0 e^{\alpha d}$. This figure shows that the deviations begin at $X/p = 100$. The fact that at $X/p = 88$ the simple equation appeared to be accurately obeyed up to a spark whereas at $X/p = 100$ a very great deviation from this law was observed may be explained by assuming that at $X/p = 88$ and lower, small surges of potential sufficed to lead to accidental breakdown before the deviations could become manifest. At higher X/p, on the contrary, the small surges though occurring did not lead to breakdown owing to a greater range of values in d between the appearance of a deviation and breakdown.

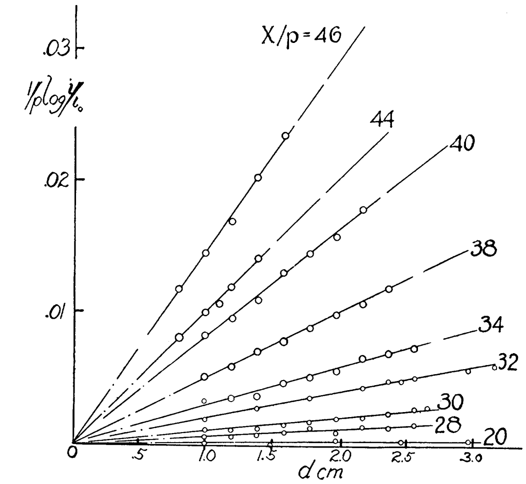

FIG. 1. Plots showing that for values of X/p from 20 to 46 the relation $i = i_0 e^{\alpha d}$ is valid.

The slopes of the straight portions of these curves are the values of α/p. These were evaluated from the data by least squares reduction. Fig. 3 shows the entire α/p, X/p curve obtained by the writer indicated as circled points. The data of Townsend[1] are represented by a dashed curve, while Ayres'[12] data are given by the triangles and Masch's[13] results by the crosses. For values of X/p above 70 there is a fair agreement in values of α/p among all the investigators. The differences between the values of Ayres and the writer at X/p above 600 are of the order of a few percent. However, for values of X/p in the neighborhood of breakdown at atmospheric pressure (about 40 and lower) there is excellent

[10] F. H. Sanders, Phys. Rev. **41**, 667 (1932).
[11] F. H. Sanders, Phys. Rev. **44**, 1020 (1933).
[12] T. L. R. Ayres, Phil. Mag. **45**, 353 (1923).
[13] K. Masch, Archiv. f. Elek. **26**, 589 (1932).

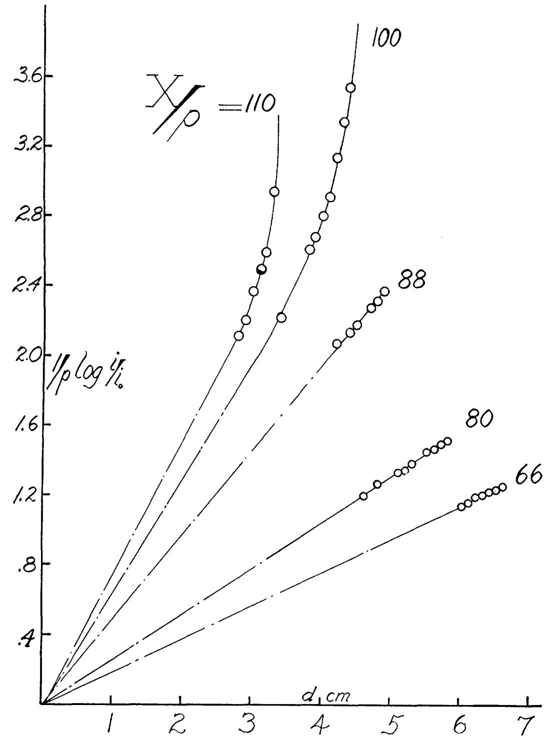

FIG. 2. Plots showing that at high values of X/p the simple exponential relation is not valid.

agreement between the results of Masch and the writer, whereas the results of Ayres are of the order of 100 times greater. The writer has great confidence in his own low X/p values in view of the very great resolving power of his apparatus with the large values of d. (See Sanders' papers for details.) Since the scale of Fig. 3 is such that an $\alpha/p \sim 10^{-2}$ cannot be distinguished from an $\alpha/p \sim 10^{-4}$ comparison must be made by a study of Table I in which the writer's least-squares valuation of α/p as $f(X/p)$ and the probable error in α/p are listed together with the values of Ayres[12] and Masch.[13]

It was found possible to fit almost the entire α/p, X/p curve (Fig. 3) by three simple equations. Fig. 4 shows that in the lower X/p range (20 to 38) the writer's data yield $\alpha/p = Ae^{BX/p}$ since $\log_e \alpha/p$ plotted against X/p fits a fairly straight line. The actual relation is

$$\alpha/p = (5.76 \pm 1.56) \times 10^{-7} e^{(0.245 \pm 0.003)X/p}$$

at 22°C. These data are of value in computations involving the development of the spark in N_2 at

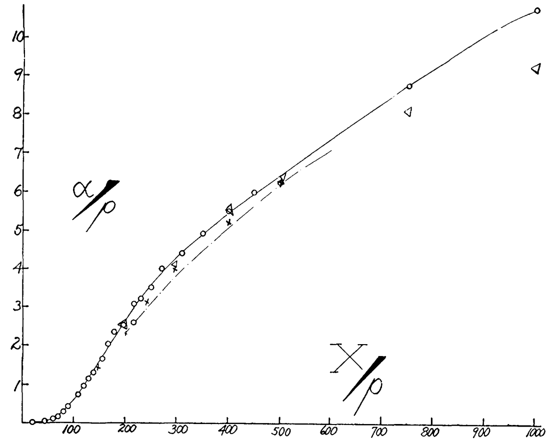

FIG. 3. Variation of α/p with X/p. Circled points—writer; dot-dash curve—Townsend; crosses—Masch; and triangles—Ayres.

low X/p, as will be seen later in this paper when sparking equations are derived. For air in a similar region Sanders[10] obtained

$$\alpha/p = (2.67 \pm 0.26) \times 10^{-8} e^{(0.350 \pm 0.002)X/p}.$$

In the present investigations in N_2 it was found possible to fit the curve of Fig. 3 from $X/p = 44$ to 176 by the equation

$$\alpha/p = (1.166 \pm 0.022) \times 10^{-4} (X/p - 32.1 \pm 1.4)^2$$

at 22°C. (From $X/p = 40$ to 120 Jodlbauer found the equation $\alpha/p = 1.35 \times 10^{-4}(X/p - 28.8)^2$ to fit Sanders'[11] results in air.) The fit of the experimental points to this curve is shown in the plot of $(\alpha/p)^{\frac{1}{2}}$ against X/p in Fig. 5. From $X/p = 200$ to 1000 it was found that the following relationship could be established:

$$(\alpha/p + 3.65)^2 = 0.21 X/p.$$

The fit of the inverted parabolic curve to the experimental data is shown in Fig. 6 where α/p is plotted against $(X/p)^{\frac{1}{2}}$. From $X/p = 176$ to 200 it was not possible to find a simple function conveniently representing α/p as an $f(X/p)$.

As indicated previously the deviations of the current from $i = i_0 e^{\alpha d}$ were first observed at $X/p = 100$. An extensive study of the deviations was made at $X/p = 120$ where current-distance curves were obtained for a series of pressures. As was to be expected from Paschen's law, the higher the pressure the smaller the value of d at which deviation set in. In order to analyze these deviations an attempt was made to fit them to Townsend's equation

$$i = i_0(\alpha - \beta)e^{(\alpha-\beta)d}/(\alpha - \beta e^{(\alpha-\beta)d}), \qquad (2)$$

taking into account the primary *and* secondary ionization mechanisms. It was found that the equation could not be made to fit a given curve if β were to be constant as required by Eq. (2). A study of the deviations and their behavior eventually led to the suspicion that the relatively large photoelectric current densities $\sim 10^{-12}$ amp./cm^2 used were creating space charges which distorted the field between the plates at sufficiently large d. This proved to be correct. The curves obtained with the large ($\sim 10^{-12}$ amp./cm^2) initial current densities are shown in Fig. 7 as *full* lines. Curves obtained with lower current densities ($\sim 10^{-14}$ amp./cm^2) were found to fit Townsend's Eq. (2) satisfactorily. These curves are shown in Fig. 7 as *broken* lines. It is seen that the rate of current rise after the turning point depends on the initial current density; the curve with the higher initial current density leads more readily to the spark.

It was in fact this discovery that led the writer and others to the theoretical interpretation of the appearance of space charges in these experiments. This interpretation has already been published.[14] The study[14] shows that space charges can cause a more rapid current increase than the equation $i = i_0 e^{\alpha d}$ warrants, by distorting the effective α to higher values when the current density (i_0 per cm^2) is sufficiently great. That is, an α distorted by space charges may be mistaken for a secondary ionization mechanism. This, however, can occur only in the first two regions of variation of α/p with X/p discussed above, not in the third. Moreover, the deviation of α from a constant value occurs very sharply as d increases; this is especially true in the first region. Thus, in this region, for large initial current density the space charge effect may possibly be the primary cause of the spark breakdown.[9, 14] In the second region the effect of space charge is separable only with difficulty from the true *secondary* ionization mechanism. In the third region α is not increased by a space charge effect. With sufficient ultraviolet inten-

[14] R. N. Varney, H. J. White, L. B. Loeb and D. Q. Posin, Phys. Rev. **48**, 818 (1935).

TABLE I. *Values of α/p for nitrogen (α is the number of new pairs of ions created by one electron in advancing 1 cm in the direction of the field).*

	α/p			Probable Error in α/p	Pressure Used
X/p	Ayres	Masch	Posin	Posin	
10	0.0110				
15	.0170				
20	.0239		0.000087	0.000008	700
25	.0305	0.00009			
26		.00022	.000258	.00002	500
27		.00041			
28		.00060	.000456	.000036	500
29		.00081			
30	.0382	.00112	.000911	.000040	250
31		.00150			
32		.00190	.00199	.000048	250
33		.00245			
34		.00315	.00281	.000108	250
35		.00385	.00305	.000076	250
36		.00475	.00441	.000160	250
38		.0071	.00521	.000120	150
40	.0550	.0100	.00734	.000330	100
44			.01070	.001100	100
45		.0208	.01353	.000900	100
50	.0820	.0373			
59			.09340	.007110	0.985
60	.121	.087			
68			.14500	.007110	.985
70		.162			
78			.28300	.009450	.635
80	.257	.260			
88			.41200	.025200	.635
90		.375			
100	.471	.505	.70000		
108			.72900	.053500	
110		.65	.73000		
120		.80	.95000	.007000	1.000
125	.850				
127			1.13500	.102000	0.490
130		.98			
137			1.272	.0160	.425
140		1.15	1.400	.0140	.720
142			1.451	.0145	.345
150	1.32	1.32			
156			1.635	.0170	.420
160		1.50			
166			2.020	.0241	.290
175	1.84				
176			2.350	.1500	.425
180		1.95			
196			2.522	.0304	.230
200	2.50	2.25			
215			3.07		.330
230			3.20		.25
250		3.15	3.50		.25
270			4.00		.25
300	4.10	3.90			
310			4.40		.18
350		4.55	4.93		.18
400	5.43	5.2	5.50		.14
450		5.7	6.00		.125
500	6.29	6.1	6.23		.125
750	7.91		8.78		.075
1000	9.02		10.8		.075
2000	11.24				
3000	12.10				

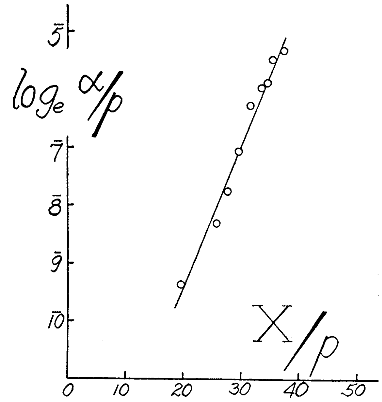

FIG. 4. Plot showing that in the range X/p from 20 to 38 $\alpha/p = ae^{bX/p}$.

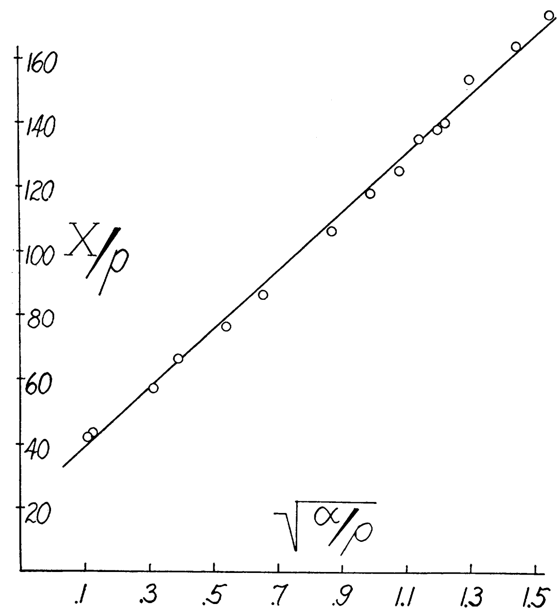

FIG. 5. Plot showing that in the range from $X/p = 44$ to 176 the values of α/p satisfy the relation $\alpha/p = A(X/p - B)^2$.

sity, therefore, field distortion can and does occur and has in the past affected some of the results of the investigations of Townsend coefficients (e.g., the results of Sanders in air and apparently those of Paavola[15] in air at low X/p).

When sufficiently low current densities were used, the currents fitted the Townsend Eq. (2) for the entire X/p range. Fig. 8 shows the relation between β/p obtained from these curves and X/p for a range in X/p of 100 to 1000. The data obtained by the writer (circled points) are shown together with those of Townsend (full curve no points) and Ayres (dashed curve). The β/p values of Ayres are considerably higher than those of the writer in the region of low $X/p(\sim 100)$ as was also the case with Ayres' values of α/p. The disagreement is not serious at the higher values. (This was likewise the case for α/p.) The comparison of the data can be made with greater advantage by referring to Table II in which are listed the β/p values in N_2 of both Ayres and the writer.

Inasmuch as the energy gained by a positive ion between impacts is hardly more than five or six volts in N_2 even at $X/p = 1000$, it is certain that the additional ionization observed in the region under discussion ($X/p = 100$ to 1000) *is not due to positive ion impact with the gas molecules*, i.e., to a β as interpreted by Townsend. The alternate Townsend mechanism which accounts for the ionization, *viz.*, the electron liberation due to positive ion bombardment of the cathode, may possibly be the active mechanism leading to values of β. Inasmuch as very little is known about the efficiency of positive ions of energy ~ 10 volts and less in liberating electrons from metal as a function of energy, little can be concluded from the data on β/p in this connection except that such a liberation is possible. As has been pointed out in this laboratory by Cravath, there is evidence that one may find some help in explaining the Townsend coefficient β by considering photoelectric effects. That is, one may look to photoelectric ionization of the gas itself for mixed gases and electron liberation from the cathode by the excitation radiation produced in the gas by primary electron impact. The existence of appreciable photoelectric effects of this character in corona discharge has been recently demonstrated by Cravath. His work was confined to atmospheric pressure, where he found the existence of two components of excitation radiation, one easily absorbed by the gas and having an absorption coefficient of 10 cm^{-1}, the other penetrating to the cathode and having an absorption coefficient of 2 cm^{-1}. At higher $X/p \sim 500$, where lower pressures ~ 0.5 mm are used, a minimum of such radiation would be absorbed by the gas; the rest would liberate photoelectrons from the cathode. As a consequence of the above findings a new prespark

[15] M. Paavola, Archiv. f. Elek. **22**, 443, 450 (1929).

current equation may be derived[9] which turns out to be of the Townsend *type* (see Eq. (2)). The approximate form of this new equation (i.e., when absorption of radiation by the gas is neglected) is

$$i = i_0 \alpha e^{\alpha d}/(\alpha - \theta\eta g(e^{\alpha d} - 1)).$$

Here β is replaced by $\theta\eta g$ where θ is the number of photons produced by one electron in one cm of path in the field direction, η is the number (fraction) of photoelectrons liberated from the cathode by one photon, g is a geometrical factor (fraction of photons, created in the gas, that can reach the cathode). It is of interest now to consider how β, or its equivalent, $\theta\eta g$, compares with α as X/p increases. In Fig. 9 β/α is plotted against X/p. It is seen that β/α *flattens out for* $X/p > 500$ (the same characteristics are revealed by an examination of Ayres' values of α and β). If β were truly the Townsend coefficient giving the production of new ions by positive ion impact with gas molecules (which is quite impossible, as shown by Varney), then it is difficult to understand why the rate of increase of β with respect to α should fall off as X/p increases. There is also at present no reason for believing that the liberation of electrons from the cathode by positive ions, having 0 to 10 volts energy varies in the fashion indicated by $\gamma \cong \beta/\alpha$ as a function of X/p. On the other hand, ascribing the secondary emission to a photoelectric effect at the cathode caused by electron impact in the gas would lead one to expect that $\beta/\alpha = \phi(X/p) = \theta\eta g/\alpha$ would vary in the manner represented in Fig. 9, as pointed out by Loeb.[9] For, assuming that in the region of X/p used (100 to 1000) η and g are nearly constant (η may increase at high X/p due to higher energy photons), the efficiency of photon production by electrons as a function of velocity (excitation function for electron impact) relative to the ionization function for electrons is such that photon production begins at lower electron energies than ionization; it then rises more steeply and reaches a maximum of efficiency before the ionization curve has done so. Hence, since the curves for probabilities of excitation and ionization in a gas as a function of electron energy have the form described above it is clear that in the region of terminal electron energies of 10 to 100 volts the ratio of $\theta\eta g/\alpha$ or β/α will have just about the form observed.

The *values* of β or $\theta\eta g$ together with α now permit one to test the simple criterion for a spark early laid down by Townsend. If in the Townsend Eq. (2) the denominator vanishes, that is,

$$\alpha - \beta e^{(\alpha-\beta)d} = 0 \quad (3)$$

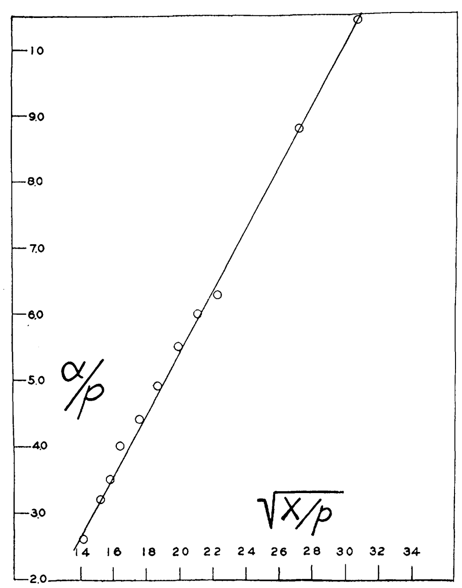

FIG. 6. Plot showing that in the range from $X/p = 200$ to 1000 the values of α/p satisfy the relation $(\alpha/p + C)^2 = DX/p$.

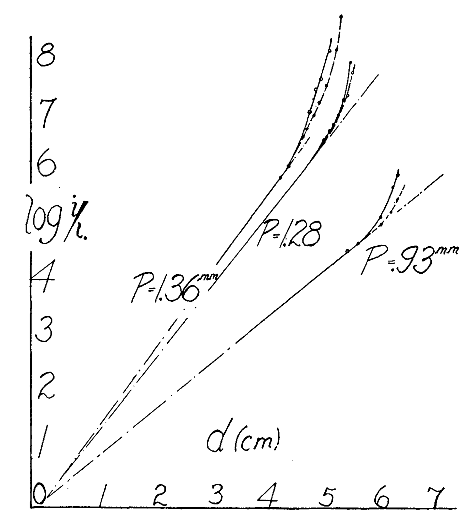

FIG. 7. Values of log i/i_0 as a function of d for 3 different pressures. Full curves are for current density approximately 10^{-12} amp./cm^2; dotted curves for current density approximately 10^{-14} amp./cm^2.

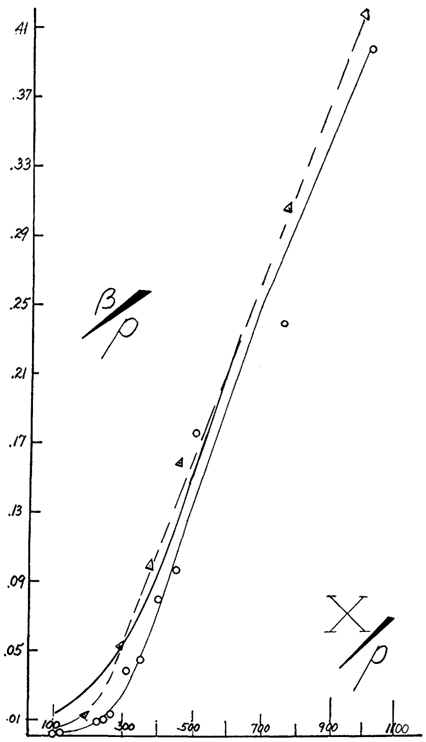

FIG. 8. Values of β/p as a function of X/p. Full curve, circled points—writer; full curve—Townsend; dashed curve, triangles—Ayres.

then i will become indefinitely large, regardless of the value of i_0. One can then interpret this as the condition leading to a spark. Such an unstable condition takes place if one increases d at a constant α and β (constant X/p and p) until the critical value δ for which the spark passes. If β be neglected in the exponent of (3) relative to α (β/α at $X/p=100$ is 5×10^{-4}, β/α at $X/p=1000$ is 3.9×10^{-2}) then $\beta/\alpha=e^{\alpha\delta}$. Thus $\log(\beta/\alpha)=\alpha\delta$, or $\log F(X/p)=p\delta f(X/p)$, and $\log F(X\delta/p\delta)=p\delta f(X\delta/p\delta)$. Here $X\delta$ is the sparking potential V_s, assuming uniform fields. As a result one can write $V_s=F(p\delta)$. This is merely the well-known Paschen law long ago deduced in this form by Townsend and tested by him over a limited range of values. As a result of the present work α/p as $f(X/p)$ and β/p as $\phi(X/p)$ are now known for N_2 over a wide range of values: $X/p=20$ to 1000. It is thus possible to insert the proper functions expressing α/p and β/p in terms of X/p into the relation (3) above and determine $V_s=F(p\delta)$. These values of V_s may be compared with those obtainable from the well-known sparking curves in which V_s is plotted against $p\delta$. Strictly, one should insert the values of β/α as $\phi(X/p)$ (empirical equations) into Eq. (3). (For α in the exponent this *is* done.) However, β/α as $\phi(X/p)$ would be a very involved formulation and thus in Eq. (3) the value of the important parameter V_s could not be obtained explicitly. Furthermore it happens that Eq. (3) is quite insensitive to changes in β/α of 100 percent or more. This follows since one or more logarithms are taken of the relatively small β/α in order to obtain V_s explicitly. In the following study, therefore, use will be made of an average β/α for a substantial range in X/p. The critical feature in formulations of V_s as $F(p\delta)$ is the nature of the relationship α/p as an $f(X/p)$. Since this relationship depends on the region of X/p used, the formulation of $V_s=F(p\delta)$ must be given in at least three parts [corresponding to at least three laws of $\alpha/p=f(X/p)$].

Region I:

For $X/p=20$ to 38, $\alpha/p=5.76\times10^{-7}e^{0.245X/p}$. The value of β/α may be taken as 10^{-5} by extrapolating from the data of Table II. Substitution into $\alpha/\beta=e^{\alpha\delta}$ yields $V_s=69p\delta-4.08\, p\delta\log p\delta$, which is to hold so long as the ratio of $V_s/p\delta\equiv X/p$ ranges between 20 and 38. This corresponds to the limits of $p\delta$ (mm×cm) of $\sim$800 to $>$10,000 and V_s (volts) 33,400 to 314,000.

Region II:

For $X/p=44$ to 176, $\alpha/p=1.17\times10^{-4}(X/p-32.2)^2$. Here β/α is taken as 10^{-3}; these yield $V_s=32.2p\delta+244(p\delta)^{\frac{1}{2}}$ which holds for the stated X/p region, the range in $p\delta$ being 3 to 600 (mm×cm) and in volts 520 to 25,320.

Region III:

For $X/p=200$ to 1000, $(\alpha/p+3.65)^2=0.21X/p$. For the range $X/p=320$ to 1000, β/α is chosen

TABLE II. *Values of β/p for nitrogen.*

X/p	β/p Ayres	β/p Posin	X/p	β/p Ayres	β/p Posin
100	0.0002	0.000016	250		.0105
110		.000016	270		.0138
120		.00054	300	.058	
125	.0013		310		.0389
150	.0048		350		.0455
175	.0084		400	.103	.0792
200	.013		450		.0963
210			500	.157	.176
215		.008	750	.301	.24
230		.0094	1000	.422	.40

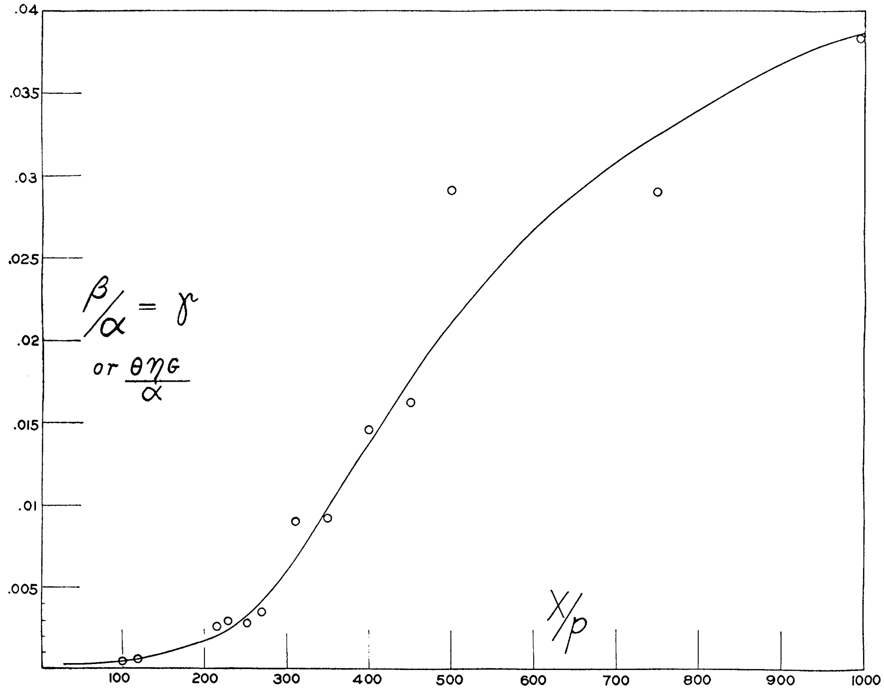

FIG. 9. β/α as a function of X/p.

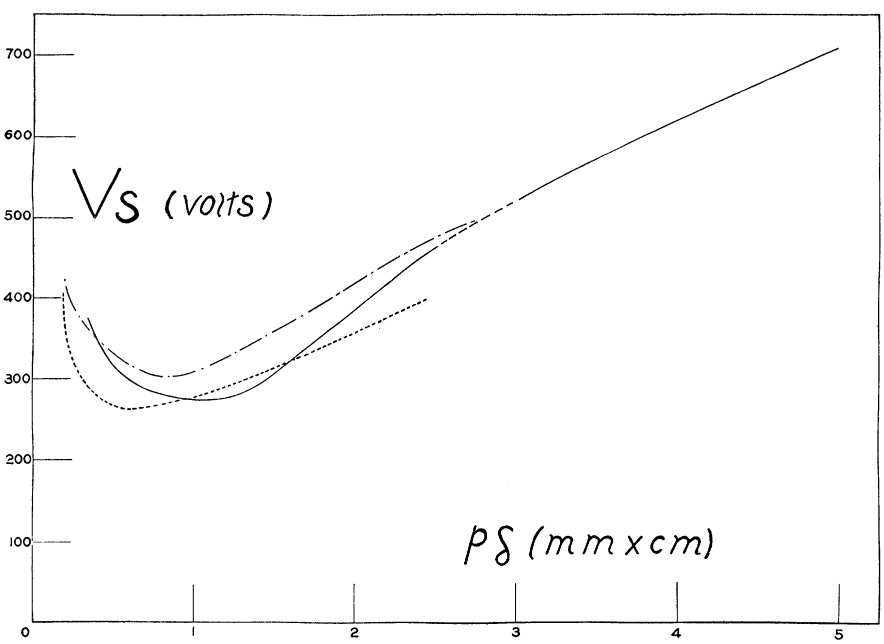

FIG. 10. Values of the sparking potential as a function of $p\delta$ for values of $p\delta$ up to 5 mm cm. The dot-dashed curve is from Strutt, experimental; the dashed curve, Hurst, experimental; full curve is calculated from the writer's equations.

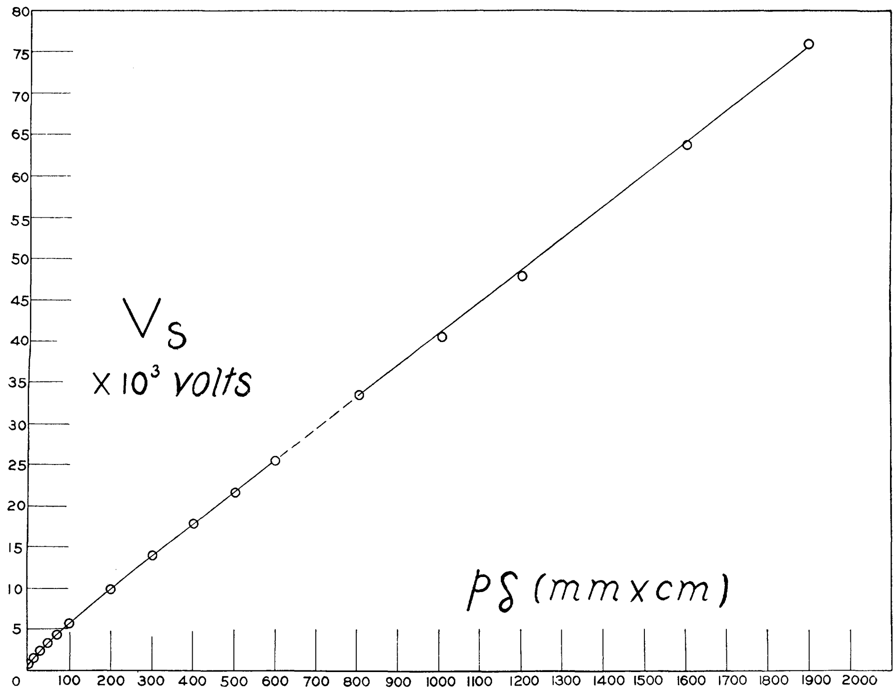

FIG. 11. Values of sparking potential as a function of $p\delta$ for values of $p\delta$ up to 1900 mm cm calculated from the writer's equations.

as 0.02; these give $V_s=63.5p\delta+73.5/p\delta+137$ for $p\delta=0.4$ to 1.2. The minimum sparking potential, obtained by differentiating the above equation, is found to be 274 volts. The corresponding value of $p\delta$ is 1.07. The average minimum sparking potential in N_2, as obtainable from the Paschen law curves of Hurst and Strutt, is about 270 volts, the $p\delta$ (average) is about 0.8. For $X/p\sim200$, β/α is about 0.0015. Using this value instead of 0.02, the equation obtained is $V_s=63.4p\delta+203/p\delta+226$, which holds from $X/p=200$ to 280, $p\delta=1.6$ to 2.5.

Fig. 10 shows the relation between V_s and $p\delta$ as found many years ago by Hurst and Strutt.[16] This figure also shows the plot of the equation for V_s against $p\delta$ found by the writer. It is seen that the latter curve falls between those of the investigators named where the ranges are common. No reliable sparking data in N_2 are available for ranges of $p\delta$ greater than 2.5 (mm×cm). Thus, although Fig. 11 shows the computed relation between V_s and $p\delta$ from $p\delta=2.5$ to $p\delta>10{,}000$, the relation cannot be at present verified beyond the limits already indicated. It is hoped that a full curve will be obtained for N_2 in this laboratory in the near future.

The hope of ultimately using the present apparatus[10] to study the effect of cathode material on the values of β was abandoned owing to the contamination of the large chamber with mercury vapor.

In conclusion, the writer wishes to express his gratitude to Dr. R. N. Varney and Professor R. T. Birge for a number of valuable suggestions. The writer is especially appreciative of the help of Professor L. B. Loeb, who proposed the investigations and supervised them throughout.

[16] See W. O. Schumann, *Elektrische Durchbruchsfeldstärke von Gasen.*