

A Double-Focusing Mass Spectrograph and the Masses of N^{15} and O^{18}

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After a short summary of the theory of double focusing, a mass spectrograph which works as an "achromatic lens" for all masses is described in some detail. It is found to work according to the theoretical expectations. The differences of "packing fraction" (i.e., $\Delta M/M \times 10^4$) of doublets containing the rare isotopes N^{15} and O^{18} have been determined. It is found that for the doublet at mass number 15 ($C^{12}H_3^1 - N^{15}$): $\Delta M/M \times 10^4 = 15.8 \pm 0.05$, $\Delta M = 0.02382$ and for the 18-doublet ($O^{16}H_2^1 - O^{18}$): $\Delta M/M \times 10^4 = 6.9 \pm 0.10$, $\Delta M = 0.01257$. With Aston's masses of C^{12} and H^1 the isotopic weights referred to $O^{16} = 16$ are: $N^{15} = 15.0040 \pm 0.0008$ and $O^{18} = 18.0037 \pm 0.0007$. The measurement of N^{15} gives the rare chance to compare the energy release of a nuclear reaction *viz.*: $N^{14} + H^2 = N^{15} + H^1$ with its value derived entirely by mass-spectrographic determinations.

TWO years ago I published together with R. Herzog¹ the complete theory of all arrangements of a radial electric field and a homogeneous magnetic field for which direction focusing as well as velocity focusing is obtained. Allowing velocity to play the part of wave-length these combinations are truly analogous to achromatic lenses, that is, they give for each pencil of positive rays of given mass an image of the limiting slit. For the masses of course they act as spectrographs. Since that time two different double-focusing mass spectrographs have been set up by A. J. Dempster² and by K. T. Bainbridge and E. B. Jordan³ which as far as can be seen from the data given are special cases of our theory. The exceedingly fine lines which Dempster was able to get are proof of the advantage of double-focusing. During the last year I succeeded with R. Herzog in constructing a mass spectrograph which corresponds to one of the special cases we had computed in our paper¹ and which could be built with the limited means of the Institute shop. It was completed at the end of last year. It differs from the two mass spectrographs mentioned in that it gives, theoretically, double focusing along the whole length of the plate. Since neither Dempster nor Bainbridge and Jordan have stated yet the conditions under which their double focusing should occur I may be permitted to give a short summary of a specialized theory here.

THEORY OF DOUBLE FOCUSING

After it was shown⁴ that Hughes and Robjansky's case of direction focusing in a radial electric field could be generalized to a certain extent R. Herzog, upon my request, undertook a similar generalization for magnetic fields. He was able to go far beyond this task by disclosing a complete parallel to geometric optics.⁵ According to this theory each radial electric or homogeneous magnetic field or any simultaneous combination of the two works like a combination of a prism and a cylindrical lens to which definite focal and principal planes and a focal length f can be ascribed. In Fig. 1 the field-free spaces I (every coordinate ') and II (every coordinate '') are the conjugate spaces of objects and images or *vice versa*. In space III a radial electric field $E = A/r$ and a homogeneous magnetic field H perpendicular to the plane of drawing is set up so that a charged particle of velocity v_0 and mass M_0 entering along x' travels along a circle of radius $r = a$ and, after being deflected through the angle ϕ , leaves along x'' . A bundle of parallel rays of definite velocity $v = v_0(1 + \beta)$ and mass $M = M_0(1 + \gamma)$ slightly differing from v_0 and M_0 is united at the focus F with the coordinates g and y_F . A pencil of rays diverging from an object point $P'(l', b')$ is focused at the image point $P''(l'', b'')$. If we denote with h the distance

⁴ W. R. Smythe, *Phys. Rev.* **45**, 299 (1934); R. Herzog and J. Mattauch, *Ann. d. Physik* **19**, 345 (1934).

⁵ R. Herzog, *Zeits. f. Physik* **89**, 447 (1934). At the same time W. E. Stephens, *Phys. Rev.* **45**, 513 (1934) has published a generalization for magnetic fields with radial boundaries without however establishing the optical analogy.

¹ J. Mattauch and R. Herzog, *Zeits. f. Physik* **89**, 786 (1934).

² A. J. Dempster, *Proc. Am. Phil. Soc.* **75**, 755 (1935).

³ K. T. Bainbridge and E. B. Jordan, *Phys. Rev.* **49**, 421A (1936).

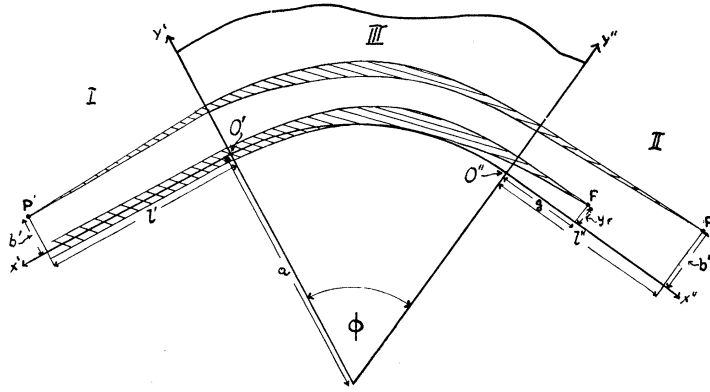


FIG. 1. A radial electric field or a homogeneous magnetic field behaves like a combination of a prism and a cylindrical lens.

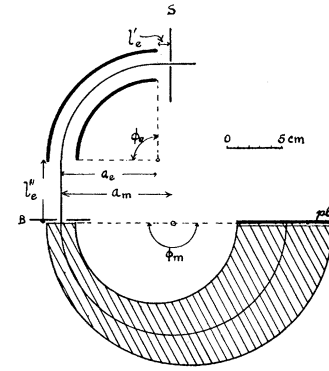


FIG. 2. Dempster's arrangement of electric and magnetic fields.

of the principal plane from O'' (or O'),⁶ the results are:

$$f = g - h = (a/\kappa)(1/\sin \kappa\phi), \quad (1)$$

$$g = f \cdot \cos \kappa\phi \quad \text{and} \quad h = f(\cos \kappa\phi - 1), \quad (2a, b)$$

$$y_F = a \cdot \delta, \quad (3)$$

for which as in optics the well-known equations hold:

$$(l' - g) \cdot (l'' - g) = f^2$$

$$\text{or} \quad 1/(l' - h) + 1/(l'' - h) = 1/f. \quad (4a, b)$$

$$\text{and} \quad \frac{(b'' - y_F)}{(b' - y_F)} = -\frac{f}{(l' - g)} = -\frac{(l'' - g)}{f}. \quad (5a, b)$$

Here κ and δ are constants which depend on the fields and on β and γ . In purely electric (subscript e) or purely magnetic (subscript m) fields they become, respectively:

$$\kappa_e = \sqrt{2}, \quad \delta_e = \beta + \frac{1}{2}\gamma \quad \text{or} \quad \kappa_m = 1, \quad \delta_m = \beta + \gamma. \quad (6)$$

The well-known special cases where O' and O'' are conjugate are analogous to telescopic image formation where:

$$g = -f, \quad \phi = n\pi/\kappa, \quad \phi_e = n\pi/\sqrt{2}, \quad \phi_m = n\pi, \quad n = 1, 2, 3 \dots \quad (7)$$

We can now proceed¹ to combine a radial

⁶ Here we confine ourselves to fields with radial boundaries where $g' = g'' = g$ and $h' = h'' = h$.

⁷ W. Bartky and A. J. Dempster, Phys. Rev. **33**, 1019 (1929); W. Henneberg, Ann. d. Physik **19**, 335 (1934).

⁸ A. L. Hughes and V. Rojansky, Phys. Rev. **34**, 284 (1929).

⁹ A. J. Dempster, Phys. Rev. **20**, 631 (1922) and others.

electric field and a homogeneous magnetic field to form an "achromatic" lens. We wish to construe the image b_m'' of a slit S of width s ($b_e' \leq \frac{1}{2}s$) so that b_m'' is independent of β at least for one particular mass. If the image of the first lens (subscript e) is the object of the second (subscript m) and if D denotes the distance between the end (O'') of the electric and the beginning (O') of the magnetic field, then:

$$l_e'' = D - l_m' \quad \text{and} \quad b_e'' = \pm b_m'. \quad (8a, b)$$

The upper sign holds when the deflections in the two fields are in the same direction and the lower sign when they are in opposite directions.¹⁰ Introducing the abbreviations:

$$K_e = a_e(1 + f_e/(l_e' - g_e)) = a_e(1 - \cos \sqrt{2}\phi_e) + l_e'' \cdot \sqrt{2} \sin \sqrt{2}\phi_e, \quad (9)$$

$$K_m = a_m(1 + (l_m' - g_m)/f_m) = a_m(1 - \cos \phi_m) + (D - l_e'') \cdot \sin \phi_m, \quad (10)$$

and using Eqs. (8b) and (5a) we get for the ordinates of the final image b_m'' in terms of the ordinates b_e' of the points of the slit S :

$$\pm b_m'' = \{(K_e/a_e - 1) \cdot b_e' - \beta \cdot (K_e \mp K_m) - \gamma \cdot (\frac{1}{2}K_e \mp K_m)\} \cdot \frac{1}{(K_m/a_m - 1)}, \quad (11)$$

from which the condition for double focusing can

¹⁰ In our paper we omitted the upper sign because we were interested in cases where double focusing holds for all masses.

be read off immediately:

$$K_e = \pm K_m. \tag{12}$$

The resolution of a double-focusing mass spectrograph is given by the ratio of the image width of the slit for one mass, $2b_m''$, to the coefficient of γ in Eq. (11):

$$\frac{\Delta M}{M} = \frac{2s}{a_e} \left(1 - \frac{a_e}{K_e}\right) = \frac{2s}{a_e} \left(1 \mp \frac{a_e}{K_m}\right). \tag{13}$$

The resolution therefore depends on the data of the electric field only. The energy of the rays passing a diaphragm B at l_e'' is constant, therefore:

$$M = k \cdot a_m^2, \tag{14}$$

where $k = eH^2/Ac^2$. Thence we get the difference of packing fraction of the lines of a doublet:

$$\Delta M/M = 2\Delta a_m/a_m \tag{15}$$

as well as the mass scale on the plate by computing the relation between a_m and the distance x from a fiducial point.

DESIGN OF APPARATUS

Perhaps an obvious way to proceed is to use one of the well-known special cases of (7) for one of the lenses and to adjust the other lens to fit Eq. (12). This apparently was done by Dempster^{2,9} and by Bainbridge and Jordan.³ Since the way by which they arrived at their respective mass spectrographs was probably a different one it may be interesting to see how far their data can be derived from our equations.

Dempster (Fig. 2) chooses $\phi_m = \pi$ [i.e., $l_m' = l_m'' = 0$, $f_m = -g_m$, $D = l_e''$, $K_m = 2 \cdot a_m$], $\phi_e = \frac{1}{2}\pi$ [i.e., $f_e = a_e/(\sqrt{2} \cdot \sin \sqrt{2}(\pi/2))$], $g_e = (a_e/\sqrt{2}) \cot \sqrt{2}(\pi/2)$, $K_e = a_e(1 - \cos \sqrt{2}(\pi/2)) + l_e'' \cdot \sqrt{2} \sin \sqrt{2}(\pi/2)$ and $l_e' = 1$ cm. Other data are not given. From a photograph to scale of his apparatus however we take a_e to be equal to 8.3 cm, l_e'' to lie between 5 and 6 cm and $2a_m$ to lie between 19 and 20 cm. With this value of a_e we get from Eq. (4a) $l_e'' = 5.48$ cm and from Eq. (12) $2 \cdot a_m = 19.50$ cm in good agreement with Dempster's photograph.¹¹ With the comparatively wide slit width used by Dempster ($s = 0.01$ cm) we get from Eq. (13) a theoretical resolution of 1 in 700 or slightly better, which agrees quite well with Dempster's Fig. 1 in 1000.¹²

¹¹ Cf. A. J. Dempster, reference 9, Fig. 5. In answer to a letter Professor Dempster kindly communicated to me his data: " a_e about 7.55 cm, $l_e'' = 8.6$ cm, $a_m = 10$ cm." There must be a mistake, however, at least about the first two items. The max. value of l_e'' ($l_e' = 0$) is 0.94. $a_e \cdot l_e''$ must therefore be smaller than a_e . Besides these values of a_e and l_e'' do not fit Dempster's Fig. 5. If we take 8.6 cm to be the correct value of a_e , then we compute $l_e'' = 5.75$ and $a_m = 10.14$ cm.

¹² In his letter to the author Professor Dempster says: "As to the resolution, the figure, 1 in 1000, is only approx-

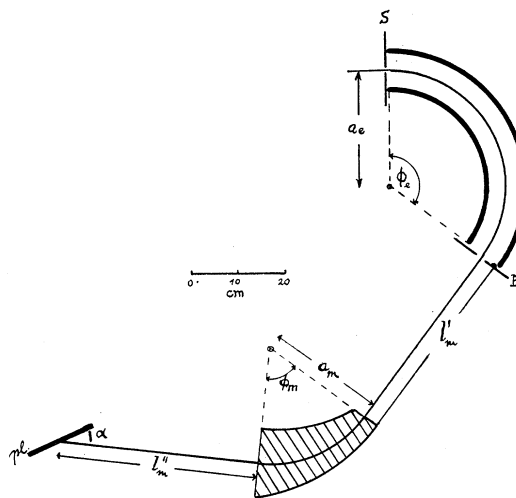


FIG. 3. Bainbridge and Jordan's arrangements of the electric and magnetic fields.

Bainbridge and Jordan choose $\phi_e = (\pi/2)\sqrt{2}$ [i.e., $l_e' = l_e'' = 0$, $f_e = -g_e$, $D = l_m'$, $K_e = 2 \cdot a_e$] and $\phi_m = (\pi/3)$ [i.e., $f_m = \frac{2}{3}\sqrt{3} \cdot a_m$, $g_m = \frac{1}{3}\sqrt{3} \cdot a_m$, $K_m = \frac{1}{2}a_m + \frac{1}{2}\sqrt{3} \cdot l_m'$]. According to Eqs. (12) and (4a) double focusing is obtained if

$$2a_e = \frac{1}{2}a_m + \frac{1}{2}\sqrt{3} \cdot l_m' \quad \text{and} \quad l_m' \cdot l_m'' - \frac{1}{3}\sqrt{3}a_m \cdot (l_m' + l_m'') = a_m,$$

from which l_m' and l_m'' may be computed if a_e and a_m are known. All we can gather from the short note of B. and J. is: "The mean dispersion is 5 mm for one percent mass difference, the mean radius is 25.4 cm, and a resolving power of 12,000 has been attained. Over 140 mm on the plate the maximum divergences from linearity are $\pm 1/7000$." Hence we can only make a guess at their apparatus since it is not stated whether "the mean radius" means a_e or a_m or both. However the smallest slit width ever used by Aston or Bainbridge in this kind of work is 0.002 cm and from Eq. (13) we see that one has to make a_e at least 24 cm to get the high resolving power attained by B. and J.; so we may take the figure 25.4 cm to mean a_e . If it should mean $a_e = a_m$, we get from the equations above $l_m' = l_m'' = a_m \cdot \sqrt{3} = 44.0$ cm (Fig. 3). Best approximation to linearity could be obtained if the plate is inclined by an angle α about 30° to the median ray. As it happens the same value of α also gives direction focusing as in Dempster's case. Again double focusing is obtained for one mass only. The dispersion of a mass spectrograph of so big dimensions, however, would be about three times as large as is given by B. and J. Probably a_m is much smaller than is assumed for the sketch of Fig. 3.

We wish to adjust the magnitudes which are not determined by Eqs. (12) and (13) in such a

imate, judged from the widths of the images as seen under a microscope. Actually the image should be wider than the slit, but in the photographs, due perhaps to less intensity at the edge, the images appear about the same width as the slit. But I have only approximate data at present."

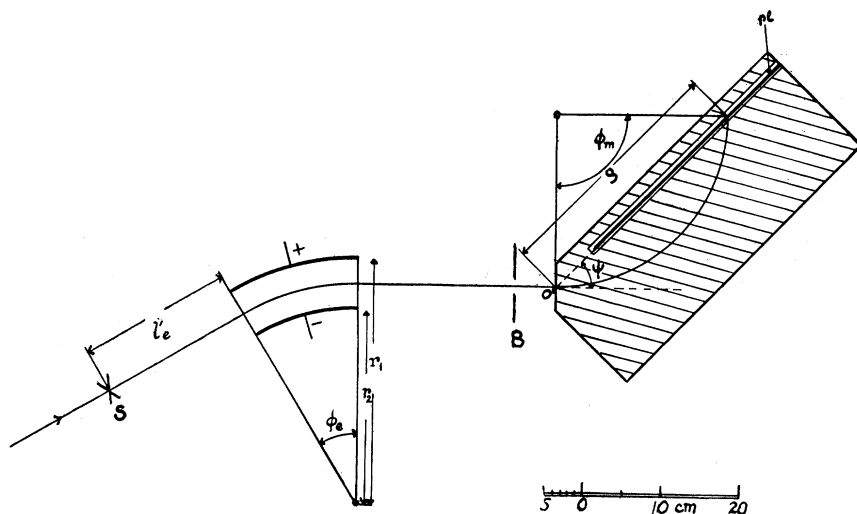


FIG. 4. Adjustment of electric and magnetic fields to give double focusing for all values of M .

way as to get double focusing for all values of M . That means, we have to look for a special case for which Eq. (12) becomes independent of a_m . This we can easily achieve by dividing Eq. (12) by l_e'' and computing the limit $l_e'' \rightarrow \infty$ (that means $l_e' = g_e$). Taking the lower sign we get:

$$\sqrt{2} \cdot \sin \sqrt{2} \phi_e = \sin \phi_m. \quad (16)$$

Since bundles of parallel rays are entering the magnetic field $l_m'' = g_m$ and D becomes indeterminate. Since ϕ_e is a constant we see from Eq. (16) that ϕ_m has to be constant too though M varies. In order not to get into contradiction with our assumption that the boundary of the field be radial we simply choose $l_m'' = 0$; then

from Eqs. (2a) and (16) we get $\phi_m = \pi/2$, $\phi_e = \pi/4\sqrt{2}$ and $l_e' = a_e/\sqrt{2}$ (see Fig. 4). Introducing new polar coordinates $\rho = 2a_m \cdot \sin \psi$ and $\psi = \frac{1}{2}\phi_m$ we see that all images lie on a straight line which goes through O' and is inclined to the median ray entering the field by an angle of $\frac{1}{4}\pi$. To get the best resolution a_e was made as large as was possible in the Institute shop. The data are: $a_e = 28.0$ cm, $r_1 = 28.4$ cm, $r_2 = 27.6$ cm. The mass scale on the plate becomes:

$$M = k \cdot \rho^2, \quad (17)$$

measuring M in units of isotopic weight ($O^{16} = 16$) and denoting by F Faraday's constant corrected for the physical scale the constant of Eq. (17) becomes:

$$k = \frac{F \cdot \ln(r_1/r_2)}{c^2 \cdot 4 \sin^2 \psi} \frac{300H^2}{X} = 1.41 \cdot 10^{-6} \cdot \frac{H^2}{X}$$

if H is measured in oersteds and X in volts. Difference of packing fraction and resolution are

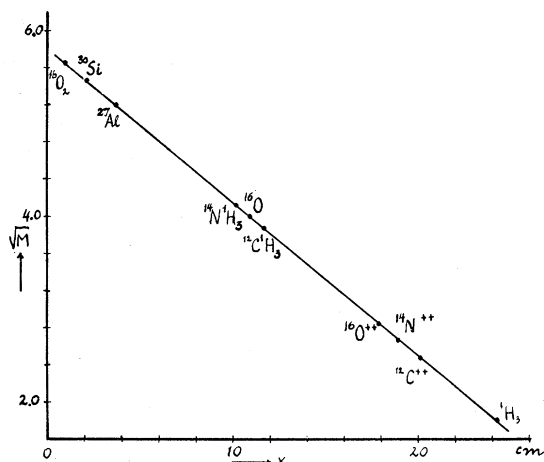


FIG. 5. Showing the accuracy with which the M^3-x law is fulfilled.

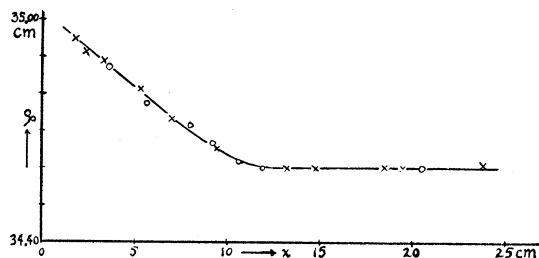


FIG. 6. Values of ρ_0 as a function of position on the plate.

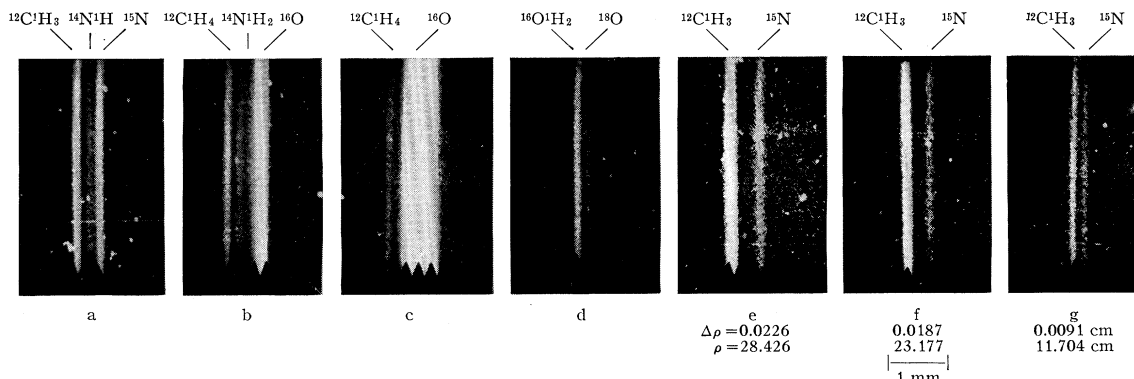


FIG. 7. Samples of the lines obtained with the present mass spectrograph as they appear in the Fraunhofer-microscope.

given by:

$$\Delta M/M = 2\Delta\rho/\rho = 2s/a_e \quad (18)$$

with $s=0.008$ cm in the present experiments. Therefore the resolution with the present slit should be 1 in 1750. The plate which is 298 mm long reaches from about $\rho=6.5$ to 36.3 cm. The distances x of the lines are measured from a fiducial point about 2 cm from the farther end of the plate and $\rho=\rho_0-x$, where ρ_0 denotes the distance of the fiducial point from O' . As far as can be seen from the design of the apparatus ρ_0 should be 34.6 cm. The distance D between the fields may be chosen arbitrarily. It was made large enough to place a diaphragm B the width of which can be set from outside the vacuum to any value between 0 and 2 mm.

EXPERIMENTAL DETAILS

Actually the plates were calibrated by means of well-known lines the masses of which were all remeasured recently by F. W. Aston.¹³ Fig. 5 shows how accurately the $\sqrt{M}-x$ law is fulfilled. To comprehend the deviations ρ_0 was determined for a number of exposures with different fields from groups of two (x) or more (0) neighboring lines (in this case by means of least squares) and plotted against the mean value of x of the group used (see Fig. 6). For the first 16 cm of the plate ρ_0 proved to be constant and equal to 34.6 cm; then its value rises slowly to 35.0 cm for lines closer to the fiducial point. This can be accounted for by a slight inhomogeneity of the magnetic field, for these rays have to pass closer to the edge of the field. The values of k range from 1.32 to $1.41 \cdot 10^{-6} \cdot H^2/X$, the main error being

¹³ F. W. Aston, Nature **137**, 357, 613 (1936).

due to the error of II and of (r_1-r_2) . Fig. 7 shows samples of the lines as they show up in the Fraunhofer-microscope. Darkfield illumination is used since weak lines are seen then much better. Fig. 7a and b are triplets at mass number 15 and 16 of one and the same 4.5-min. exposure in undried room air. The same exposure shows a weak line at 17 and none at 18. That means that N is accompanied by weak lines NH, NH₂ and NH₃ in a way similar to that which is well known of C(CH, CH₂, CH₃, CH₄) or O(OH, OH₂). Fig. 7c and d are the doublets at mass number 16 and 18 of a 5-min. exposure in dried oxygen. On the original plate CH₄ is decidedly weaker than O¹⁸. That means that O¹⁶ is at least 500 times overexposed. Nevertheless the doublet remains clearly resolved. Though doublets of such unequal intensity are unsatisfactory for precise work I measured two of them and the triplet Fig. 7b and got the values 20.5, 20.6 and 23.2 for $(\Delta M/M) \cdot 10^4$ which is to be compared with Aston's precision value 22.4₈. This comparatively close agreement means that even strong overexposure does not make the lines unsymmetrical. The closest doublet hitherto obtained in air or oxygen is that at 18. It shows that a resolution of at least 1 in 1430 has been attained. From the first part of Eq. (18) it follows that the dispersion for one percent difference of

TABLE I.

Doublet	Number of doublets measured	$(\Delta M/M) \cdot 10^4$	ΔM
(C ¹² H ₃ ¹ -N ¹⁵)	12	15.8 ₆ ±0.05	0.02382
(O ¹⁶ H ₂ ¹ -O ¹⁸)	9	6.9 ₈ ±0.10	0.01257

mass decreases linearly with ρ from 1.8 mm for lines near the fiducial point to 0.3₅ mm at the other end of the plate. This is demonstrated by Fig. 7e, f, g, showing the 15-doublet at different values of ρ . This comparatively small dispersion is a consequence of the small size of the magnet and the steep angle (45°) which was chosen for the incidence of the rays on the plate. The resolution however has according to the second part of Eq. (18) to remain constant which means that the lines become sharper with decreasing ρ . One therefore can safely say that in every respect the mass spectrograph is working according to theory.

RESULTS

There were now taken (mostly in oxygen with a small trace of air) altogether 12 doublets at mass number 15 and 9 doublets at mass 18. As a rule these were on the same exposure. The value of ρ for the middle of each doublet can easily be determined to a fraction of 1 per mille, ρ being corrected according to Fig. 6. For each doublet $\Delta\rho$ was measured independently by three observers (Dr. R. Herzog, Dr. H. Hintenberger and the author). The conditions of experiment have been widely varied. The time of exposure though on the average 5-min. range from 1 min. 40 sec. to 9 min.; the width of the diaphragm B was set at 0.025 and 0.015 cm; ρ for the 15 doublet was changed from 10.465 to 30.463 cm, $\Delta\rho$ correspondingly from 0.0083 to 0.0237 cm. The values of $(\Delta M/M) \cdot 10^4$ (difference of packing fraction) lie all between 15.54 and 16.15 with a probable error of ± 0.05 . The corresponding changes for the 18-doublet were much smaller. The results are given in Table I. It has to be emphasized that these results have been obtained with a slit width four times as wide as is commonly used in this kind of work. By narrowing the slit we hope to get still finer lines and an increase in accuracy. Using Aston's masses of C^{12} and H^1 the isotopic weights referred to $O^{16}=16$ are:

$$N^{15} = 15.0040 \pm 0.0008 \text{ and } O^{18} = 18.0037 \pm 0.0007.$$

These may be compared with the band-spectroscopic values: $N^{15} = 15.0027 \pm ?$ measured by G. Herzberg¹⁴ and recalculated by R. T. Birge,¹⁵

¹⁴ G. Herzberg, Zeits. f. physik. Chemie **B9**, 43 (1930).

¹⁵ R. T. Birge, Phys. Rev. **37**, 841 (1931).

$O^{18} = 17.991 \pm 0.010$ (R. Mecke and K. Wurm¹⁶) and $O^{18} = 18.0065 \pm 0.00018$ (H. D. Babcock and R. T. Birge¹⁷) of which the last seems by far the most trustworthy.

DISCUSSION

The mass of N^{15} is of some interest in atomic disintegration work. By bombarding nitrogen atoms with deuterons J. D. Cockcroft and W. B. Lewis¹⁸ measured a proton group of 85.0 ± 1.0 cm corresponding to 8.53 ± 0.1 MEV.¹⁹ If in this reaction the N^{15} nucleus is left in the ground state there should be:

$$N^{14} + H^2 - N^{15} - H^1 = (91.6 \pm 1.1) \cdot 10^{-4} \quad (19)$$

mass units on the physical scale. If we simply insert Aston's values for N^{14} , H^2 , H^1 and our value for N^{15} we get: $(98.5 \pm 9.5) \cdot 10^{-4}$. The agreement might be considered as satisfactory within the rather large limits of error due to the reference of the masses to O^{16} . However, the test can be made much sharper by going back to the originally measured doublets. If we extend the left side of Eq. (19) by + and $-CH_3$ we get:

$$\begin{aligned} &-(C^{12}H_2^1 - N^{14}) + (C^{12}H_3^1 - N^{15}) \\ &-(H_2^1 - H^2) = (98.5 \pm 1.1) \cdot 10^{-4} \quad (19^*) \end{aligned}$$

if we use for the 14 and the 2-doublet Aston's values: $(8.89 \pm 0.05) \cdot 14.01$ and $(7.54 \pm 0.2) \cdot 2.015$ and for the 15-doublet the value given above. As Professor G. Stetter kindly pointed out to me neither an error in the measurement of the range of the proton group nor its reduction to MEV may be blamed for the whole discrepancy.

Of the three doublets used the one at 14 is one of the most accurately measured by Aston. $(\Delta M/M) \cdot 10^4$ of the 15-doublet would have to be 15.40 to fit Eq. (19) which is below the lowest value we measured. The 2-doublet seemed to be one of the hardest to measure since Aston needed 53 doublets to get a probable error of ± 0.2 . If we take instead of this the difference between the doublets $(H_2^1 - He^{4++})$ and $\frac{1}{2}(H_2^2 - He^4)$

¹⁶ R. Mecke and K. Wurm, Zeits. f. Physik **61**, 37 (1930).

¹⁷ H. D. Babcock and R. T. Birge, Phys. Rev. **37**, 233 (1931).

¹⁸ J. D. Cockcroft and W. B. Lewis, Proc. Roy. Soc. **A154**, 261 (1936).

¹⁹ This group was discovered by E. O. Lawrence, E. McMillan and M. C. Henderson, Phys. Rev. **47**, 273 (1935) who give for the kinetic energy release 8.0 MEV. The corresponding value of the proton range may presumably not be compared with that of C. and L. because Al instead of mica was used for absorption.

measured by Bainbridge²⁰ we get for ΔM of the ($H_2^1-H^2$) doublet $(19.2 \pm 0.5) \cdot 10^{-4}$ and for the mass equivalent of the energy release the value $(94.5 \pm 1.2) \cdot 10^{-4}$, which is in better agreement with the disintegration value. This, however, would involve a slight change of all recently measured masses.

On the other hand we can use the disintegration value of Eq. (19) to compute ΔM of the 2-doublet. We get then ΔM of ($H_2^1-H^2$) $= (22.2 \pm 1.6) \cdot 10^{-4}$. J. Chadwick and M. Goldhaber²¹ found that $H^2 = H^1 + n^1 - 23 \cdot 10^{-4}$ mass units or:

$$(n^1 - H^1) = 23 \cdot 10^{-4} - (H_2^1 - H^2).$$

With the above value of ($H_2^1 - H^2$) we find that the difference between the masses of neutron and proton is zero within the limits of error as is known of other stable isobars of adjacent elements.

In the measurements and calculations I had the valuable help of Messrs. V. Hauk and H. Lichtblau. The author desires further to thank the Rockefeller Foundation and the Akademie der Wissenschaften in Wien for grants which have made these investigations possible.

Note added in proof: There were only two other nuclear reactions observed by Cockcroft and Lewis which can be checked up entirely by the masses measured by Aston. These are the disintegrations of N^{14} and of O^{16} both by H^2 giving α -rays. The reaction energies given by C. and L. are $(13.22 \pm 0.1) \cdot 10^6$ ev and $(2.95 \pm 0.04) \cdot 10^6$ ev so that in units of isotopic weight:

$$N^{14} + H^2 - C^{12} - He^4 = (142.0 \pm 1.1) \cdot 10^4 \quad (20)$$

$$\text{and} \quad O^{16} + H^2 - N^{14} - He^4 = (31.7 \pm 0.4) \cdot 10^4. \quad (21)$$

The corresponding mass-spectrographic equations with the doublets measured by Aston (see Table II) are:

$$-(C^{12}H_2^1 - N^{14}) + (H_2^1 - H^2) + (H_2^2 - He^4) = (145.7_5 \pm 1.1) \cdot 10^4 \quad (20^*)$$

and

$$-(C^{12}H_4^1 - O^{16}) + (C^{12}H_2^1 - N^{14}) + (H_2^1 - H^2) + (H_2^2 - He^4) = (34.7_5 \pm 2.7) \cdot 10^4. \quad (21^*)$$

The fact that the difference of the two:

$$(C^{12}H_4^1 - O^{16}) - 2 \cdot (C^{12}H_2^1 - N^{14}) = (111.0 \pm 2.5) \cdot 10^4 \quad (20^*) - (21^*)$$

agrees well with the difference (20)-(21) of the reaction energies: $(110.3 \pm 1.2) \cdot 10^4$ was another reason for not

seeking the cause of the discrepancies between the disintegration and mass-spectrographic values of the reaction energies at the 14-doublet.

Recently, however, the situation has changed since K. T. Bainbridge and E. B. Jordan²² have remeasured the 2-, 14- and 16-doublet. With their values (19^*), (20^*) and (21^*) become: $(92.9 \pm 2.2) \cdot 10^4$, $(140.4 \pm 2.2) \cdot 10^4$ and $(31.4 \pm 3.0) \cdot 10^4$ in excellent agreement with the disintegration values. The agreement of the difference (20^*) - (21^*) giving $(109.0 \pm 4.5) \cdot 10^4$ happens to be preserved also. That means that Aston's values for the 14- and 16-doublet must be abandoned in favor of the new ones. In addition B. and J. have measured the doublet $C^{12}H^1 - C^{13}$ which together with the 2-doublet permits us to check up on the disintegration of C^{12} by H^2 giving H^1 -rays. The kinetic energy release for this reaction is given by C. and L. as $(2.66 \pm 0.06) \cdot 10^6$ ev so that:

$$C^{12} + H^2 - C^{13} - H^1 = (28.6 \pm 0.6) \cdot 10^4, \quad (22)$$

or with B. and J.'s values from Table II,

$$(C^{12}H^1 - C^{13}) - (H_2^1 - H^2) = (29.7 \pm 1.1) \cdot 10^4. \quad (22^*)$$

TABLE II.

Mass-Number	Doublet	M · 10 ⁴	Sign	Measured by:
2	H ¹ - H ²	15.2 ± 0.4*		Aston
2	H ¹ - H ²	15.3 ± 0.4*	a	Bainbridge and Jordan
4	H ² - He ⁴	255.1 ± 0.8*	b	Aston
6	H ₂ ² - C ¹²⁺¹³	423.6 ± 1.8	c	Aston
13	C ¹² H ¹ - C ¹³	45 ± 1*	d	Bainbridge and Jordan
14	C ¹² H ₂ ¹ - N ¹⁴	124.5 ± 0.7		Aston
14	C ¹² H ₂ ¹ - N ¹⁴	130 ± 2*	e	Jordan and Bainbridge
15	C ¹² H ₃ ¹ - N ¹⁵	238.2 ± 0.7 ₅ *	f	Mattauch
16	C ¹² H ₄ ¹ - O ¹⁶	360.1 ± 2.4		Aston
16	C ¹² H ₄ ¹ - O ¹⁶	369 ± 2*	g	Jordan and Bainbridge
18	O ¹⁶ H ₂ ¹ - O ¹⁸	125 ₇ ± 1.8	h	Mattauch

Of the recently measured doublets those denoted by * are therefore checked up by kinetic energy releases observed by atomic disintegration.

In order to refer the masses to O¹⁶ = 16 we have to make use of Aston's keydoublet c which could not be checked up by disintegration values. All masses are influenced by this doublet, however, as well as by the doublets a and g.

$$\begin{aligned} H^1 &= 1 + \frac{1}{4}g + \frac{1}{8}c + \frac{3}{8}a = 1.00818 \pm 0.00003 \\ H^2 &= 2 + \frac{2}{4}g + \frac{2}{8}c - \frac{2}{8}a = 2.01482 \pm 0.00005 \\ He^4 &= 4 + \frac{4}{4}g + \frac{4}{8}c - \frac{4}{8}a - b = 4.0041_3 \pm 0.00013 \\ C^{12} &= 12 + 1\frac{2}{4}g - \frac{4}{8}c - 1\frac{2}{8}a = 12.0042 \pm 0.0002 \\ C^{13} &= 13 + 1\frac{3}{4}g - \frac{3}{8}c - \frac{3}{8}a - d = 13.0079 \pm 0.0002 \\ N^{14} &= 14 + 1\frac{4}{4}g - \frac{2}{8}c - \frac{6}{8}a - e = 14.0076 \pm 0.0003 \\ N^{15} &= 15 + 1\frac{5}{4}g - \frac{1}{8}c - \frac{3}{8}a - f = 15.0049 \pm 0.0002 \\ O^{16} &= 16 + \frac{2}{4}g + \frac{2}{8}c + \frac{9}{8}a - h = 18.0038 \pm 0.0002 \end{aligned}$$

At present these are the best mass-spectrographic determinations checked up as far as possible by nuclear reaction energies.

²⁰ K. T. Bainbridge, Phys. Rev. **43**, 103; **44**, 57 (1933).

²¹ J. Chadwick and M. Goldhaber, Proc. Roy. Soc. **A151**, 479 (1935).

²² K. T. Bainbridge and E. B. Jordan, Phys. Rev. **49**, 883 (1936).

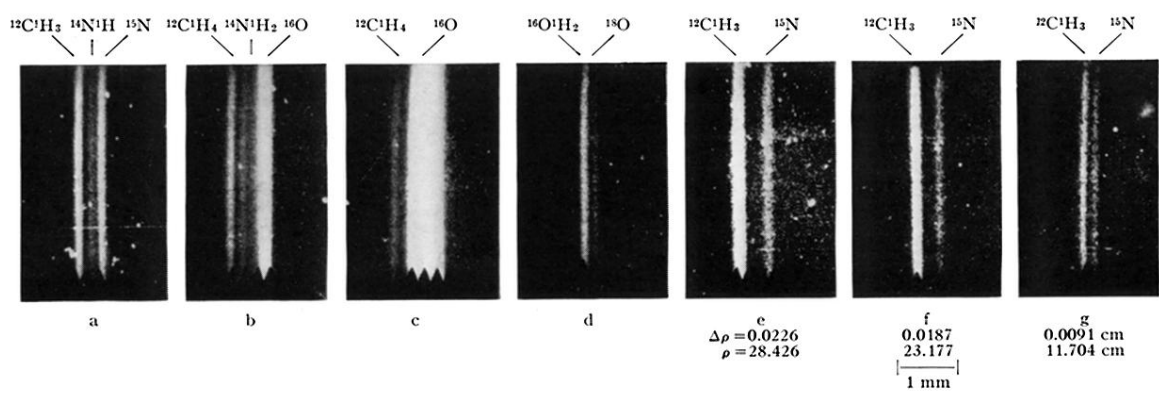


FIG. 7. Samples of the lines obtained with the present mass spectrograph as they appear in the Fraunhofer-microscope.