The Scattering of Fast Neutrons by Heavy Nuclei

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The experimental work of Dunning has shown that for fast neutrons the capture cross sections of heavy nuclei are small compared to their cross sections for elastic scattering. While it cannot be concluded from this that polarization effects (i.e., contributions to the elastic scattering from quasi-stationary states of the nucleus-neutron system) are negligible, it seemed worth while to investigate the results of a treatment neglecting polarization. We use the effective potential for a neutron in the field of a heavy nucleus developed by Van Vleck. The values of the constants of the inter-particle interactions are adjusted slightly from those found by Feenberg from binding energy calculations. The problem reduces to that of a neutron in the field of a potential well whose depth is a slowly varying function of the neutron velocity. The scattering problem is solved by the method of partial cross sections. Anomalous resonances due to the attractive potential are found in the higher partial cross sections. Although these resonances have anomalously high peaks it is shown that on account of their sharpness they contribute little to the cross section for a neutron beam having a broad velocity distribution. The cross section is found to vary markedly with both velocity and atomic number. Even for a broad velocity distribution corresponding to Dunning's experimental source oscillations remain in the cross section as a function of atomic number. However, the existing experimental points fit the curve within the experimental error. That the experimental results are in no case much larger than those given by the theory would perhaps indicate that contributions due to polarization effects are small. Possible experimental work to test the theory is mentioned.

INTRODUCTION

EXPERIMENTAL work on the scattering and absorption of fast neutrons has been carried out by several investigators.[1, 2, 3] Perhaps the most comprehensive investigation of the scattering of fast neutrons has been made by J. R. Dunning and his co-workers. Using as a source the neutrons emitted by the nuclear reaction between beryllium and alpha-particles from radon and its equilibrium products, Dunning has obtained values for the collision cross section of several heavy nuclei. He has shown further that even for nuclei known experimentally to capture fast neutrons, as evidenced by the production of gamma-rays or of artificial radioactivity, the capture cross section is of the order of less than ten percent of the cross section for elastic scattering. The experimental collision cross sections fit a $Z^{\frac{2}{3}}$ law within the experimental error, Z being the atomic number.

It is to be emphasized that we are here interested in fast neutrons, of kinetic energy greater than one-half million electron volts. The experimental results for slow neutrons,[3, 4] or neutrons having approximately thermal velocities, are much more complicated. For these slow neutrons abnormally large collision cross sections are found for certain nuclei. In these cases the cross section for elastic scattering is negligible in comparison with that for neutron capture.

Bohr[5] has shown since the inception of this work that these results can be explained only by consideration of the numerous and closely spaced energy levels of the nucleus. The chance of the neutron giving up some of its kinetic energy to other degrees of freedom of the many-particle system is very good; robbed of most of its velocity, the neutron stays long enough in the nucleus to make the probability of capture large. Breit and Wigner[6] have independently dealt with the considerations discussed by Bohr in more explicit mathematical detail. They consider the effect of a single quasi-stationary level of the combined system of neutron and nucleus which has an energy in the thermal region, and show that it is reasonable that the breadth of the level due to radiation damping is large in comparison to that due to re-emission of a neutron. The ratio of the absorption to the extra scattering due to the level is correspondingly large. Of course the effect is only important near resonance, i.e., when the

[1] Chadwick, Proc. Roy. Soc. **A136**, 702 (1932); **A142**, 1 (1933).
[2] Curie-Joliot, J. de phys. et rad. **4**, 21 (1933); **4**, 278 (1933).
[3] Dunning, Phys. Rev. **45**, 586 (1934); Dunning, Pegram, Fink, and Mitchell, Phys. Rev. **48**, 265 (1935).
[4] Amaldi, d'Agostino, Fermi, Pontecorvo, Rasetti and Segrè, Proc. Roy. Soc. **A149**, 522 (1935).
[5] N. Bohr, Nature **137**, 344 (1936).
[6] Breit and Wigner, Phys. Rev. **49**, 519 (1936).

energy of the system is near that of the quasi-stationary level.

According to Bohr the nucleus plus neutron system has such quasi-stationary levels, more closely spaced than those at thermal energies, in the energy range corresponding to fast neutrons. Here the absorption due to such levels may be small in comparison to the scattering, and iti s impossible to conclude from the experimental absence of large absorption that the extra scattering due to the existence of these levels, or, in other language, to polarization effects, may be neglected.

In the light of the above remarks the simple treatment of the problem dealt with in this paper is not to be taken too seriously. The work was well begun, however, before Bohr's paper made clear the possibility of large polarization effects, and it was thought of some interest to complete it. At the present stage of nuclear theory a more refined treatment appears impractical. It should be said that the vices of our treatment are inherent in the use of the statistical model. Its results may be worse than those for binding energy calculations with the statistical model, but only because scattering calculations are generally more sensitive to approximations than energy calculations.

Extension of Van Vleck's Treatment of the Interaction Between a Neutron and a Heavy Nucleus

J. H. Van Vleck[7] has considered the interaction of a neutron and a heavy nucleus on the basis of a statistical treatment of the particles of the nucleus. He employs the types of inter-particle interaction found necessary to the theory of the binding energy of nuclei. Binding energy considerations lead to neutron-proton interaction of the Majorana[8] (exchange) type. The Wigner[9] (ordinary) potential makes the binding energy of heavy nuclei increase too rapidly with atomic weight. Similarly neutron-neutron interaction of the spin coupling type[7] is required; in other language than that of the vector model this corresponds to one part Wigner to two parts Majorana interaction.[10]

[7] Van Vleck, Phys. Rev. **48**, 367 (1935).
[8] E. Majorana, Zeits. f. Physik **82**, 137 (1933).
[9] Wigner, Phys. Rev. **43**, 252 (1933).
[10] Feenberg and Knipp, Phys. Rev. **48**, 906 (1935).

The complete Hamiltonian is thus of the form

$$H=H_0-(h^2/8\pi^2M)\nabla_n{}^2+\sum_{j=1}^{Z}J(r_{np_j})P^M(n,\,p_j)+\sum_{i=1}^{N}N(r_{nn_i})\mathbf{s}_n\cdot\mathbf{s}_{n_i},\quad(1)$$

where H_0 is the nuclear Hamiltonian, M is the mass of the neutron (or more properly the reduced mass of neutron and nucleus if the origin of coordinates is chosen as the center of gravity of the nucleus), $J(r_{np_j})P^M(n,\,p_j)$ is the Majorana interaction between the neutron and the jth nuclear proton, and $N(r_{nn_i})\mathbf{s}_n\cdot\mathbf{s}_{n_i}$ is the interaction between the neutron and the ith nuclear neutron. $P^M(n,\,p_j)$ is the Majorana exchange operator permuting the coordinates of the neutron and the jth proton, $\mathbf{s}_{n_i}$ is the vector spin matrix for the ith nuclear neutron, Z is the atomic number, and N the number of neutrons in the nucleus. H_0 of course contains terms similar to the last two in Eq. (1), so that the Hamiltonian is symmetric in the neutrons.

We shall neglect nuclear neutrons not in closed shells, so that the last term of Eq. (1) will not contribute directly to a self-consistent field effective potential. However, when exchange is taken into account it does enter. From the Dirac vector model,[11] which shows that the exchange effect for identical particles a and b is formally equivalent to insertion of a spin coupling factor $-(\frac{1}{2})(1+4\mathbf{s}_a\cdot\mathbf{s}_b)$, the exchange potential due to neutron exchange is here

$$-\sum_{i=1}^{N}(1/2)P^M(n,\,n_i)(1+4\mathbf{s}_n\cdot\mathbf{s}_{n_i})N(r_{nn_i})\mathbf{s}_n\cdot\mathbf{s}_{n_i},$$

or

$$-\sum_{i=1}^{N}(3/8-(1/2)\mathbf{s}_n\cdot\mathbf{s}_{n_i})N(r_{nn_i})P^M(n,\,n_i),$$

in virtue of the matrix identity $16(\mathbf{s}_a\cdot\mathbf{s}_b)^2+8\mathbf{s}_a\cdot\mathbf{s}_b-3=0$. Thus the Hartree-Fock equation satisfied by an approximate one-particle wave function for the neutron is

$$\begin{aligned}-(h^2/8\pi^2M)\nabla_n{}^2\psi(\mathbf{x}_n)+\sum_{j=1}^{Z}\int\!\!\int\!\!\int\varphi_j{}^*(\mathbf{x}_p)&\\ \times J(r_{np})\varphi_j(\mathbf{x}_n)\psi(\mathbf{x}_p)d\tau_p&\\ -(3/8)\sum_{i=1}^{N}\int\!\!\int\!\!\int\psi_i{}^*(\mathbf{x}_{n_n})N(r_{nn_n})&\\ \times\psi_i(\mathbf{x}_n)\psi(\mathbf{x}_{n_n})d\tau_{n_n}=W\psi(\mathbf{x}_n),&\end{aligned}\quad(2)$$

[11] Cf. Van Vleck, Phys. Rev. **45**, 405 (1934).

where $\varphi_j(\mathbf{x})$, $\psi_i(\mathbf{x})$ are one-particle wave functions for the jth proton and ith nuclear neutron respectively, and W is the kinetic energy of the free neutron.

We need an approximate expression for

$$\sum_{j=1}^{Z} \varphi_j^*(\mathbf{x}_p)\varphi_j(\mathbf{x}_n),$$

the Dirac density matrix[12] for the protons in the nucleus. This may be obtained by supposing that phase space is as full as permitted by the Pauli principle within the volume $r<R$, $p<P$, and empty elsewhere. Necessarily $(4\pi/3)^2P^3R^3 = Zh^3/2$. We assume further that the proton wave functions are adequately represented locally by plane waves, that is, that the local momentum does not vary too rapidly with position within the nucleus, and that all momenta $p<P$ are equally probable. Then

$$\sum_{j=1}^{Z} \varphi_j^*(\mathbf{x}_p)\varphi_j(\mathbf{x}_n) \doteq (2/h^3)\int\int\int_P \exp\left[(2\pi i/h)(\mathbf{x}_p-\mathbf{x}_n)\cdot\mathbf{p}\right]d\mathbf{p}, \quad |\mathbf{x}_p+\mathbf{x}_n|/2<R;$$
$$\doteq 0, \quad |\mathbf{x}_p+\mathbf{x}_n|/2>R,$$

the integration being over a sphere of radius P. This gives

$$\sum_{j=1}^{Z} \varphi_j^*(\mathbf{x}_p)\varphi_j(\mathbf{x}_n) \doteq (\sin k_pr_{np} - k_pr_{np}\cos k_pr_{np})/\pi^2 r_{np}^3, \quad |\mathbf{x}_p+\mathbf{x}_n|/2<R;$$
$$\doteq 0, \quad |\mathbf{x}_p+\mathbf{x}_n|/2>R, \tag{3}$$

where $\qquad k_p = 2\pi P/h = (9\pi Z/4R^3)^{\frac{1}{3}}$.

A similar expression holds for

$$\sum_{i=1}^{N} \psi_i^*(\mathbf{x}_{n_n})\psi_i(\mathbf{x}_n),$$

the Dirac density matrix for the nuclear neutrons, with k_p replaced by $k_n = (9\pi N/4R^3)^{\frac{1}{3}}$. It is convenient to assume R the same for both expressions; since N is roughly equal to Z and k_n involves N in a low power a further simplification may be attained by replacing k_n by k_p. With these simplifications our result is what we would have obtained had we considered only neutron-proton interaction with a $J(r)$ equal to $J(r) - 3N(r)/8$; Eq. (2) becomes

$$\nabla_n^2\psi(\mathbf{x}_n) + k^2\psi(\mathbf{x}_n) = (8\pi^2M/h^2)\iiint[J(r') - (3/8)N(r')][(\sin k_pr' - k_pr'\cos k_pr')/\pi^2r'^3] \times\psi(\mathbf{x}_n+\mathbf{x}')d\tau', \quad r_n<R; \quad =0, \quad r_n>R, \tag{4}$$

where we have written $k^2 = 8\pi^2MW/h^2$. In obtaining Eq. (4) we have replaced the conditions on Eq. (3) by the condition that the Dirac density is zero when the neutron is outside the nucleus and has the given value inside, a procedure justified by the small range of the neutron-proton and neutron-neutron interactions in comparison with R. Consideration of edge effects would prove a useless refinement in view of the fact that we have neglected them in obtaining Eq. (3).

We proceed to the problem of solving Eq. (4) for $r_n<R$. It is of the type

$$\nabla^2u(\mathbf{x}) + k^2u(\mathbf{x}) = \iiint f(r')u(\mathbf{x}+\mathbf{x}')d\tau'.$$

Now this apparently troublesome integro-differential equation may be reduced to an ordinary homogeneous differential equation, as Feenberg[13] has very kindly shown. Namely, if we try a plane wave solution, $u = \exp i\mathbf{k}'\cdot\mathbf{x}$, we find it a solution provided

$$-k'^2 + k^2 = \iiint f(r')\exp i\mathbf{k}'\cdot\mathbf{x}'d\tau'.$$

Now on account of the spherical symmetry of $f(r')$ the integral is independent of the direction of the vector $\mathbf{k}'$. Since any solution of $\nabla^2u + k'^2u = 0$ can be constructed from plane wave solutions (having of course the same magnitude of $\mathbf{k}'$), this solution is also a solution of the integro-differential equation. Thus we may solve Eq. (4) by solving

$$\nabla^2\psi + k'^2\psi = 0, \quad r<R;$$
$$\nabla^2\psi + k^2\psi = 0, \quad r>R, \tag{5}$$

with

$$k'^2 = k^2 - (8\pi^2M/h^2)\iiint(J(r) - 3N(r)/8) \times[(\sin k_pr - k_pr\cos k_pr)/\pi^2r^3] \times\exp i\mathbf{k}'\cdot\mathbf{x}d\tau. \tag{6}$$

[12] Dirac, Proc. Camb. Phil. Soc. **26**, 376 (1930).

[13] Private communication to J. H. Van Vleck. Van Vleck had previously obtained the result for an s wave function by explicit calculation, which we extended to p and d functions.

The problem is thus reduced to that of a neutron in the field of a potential well, the depth of the well being a function of the velocity of the neutron on account of the appearance of k' in the integral in Eq. (6).

Feenberg[10] has succeeded in fitting the observed binding energies of light nuclei by taking $J(r)$ and $N(r)$ of the same functional form, $J(r)=A\exp(-\alpha r^2)$, $N(r)=B\exp(-\alpha r^2)$. Integration of Eq. (6) with these gives

$$\begin{aligned}k'^2&=k^2-(8\pi^2M/h^2)(A-3B/8)[E(w_+)-E(w_-)\\&\quad+(4\alpha/\pi k'^2)^{\frac{1}{2}}(\exp(-w_+^2)-\exp(-w_-^2))],\\w_\pm&=(k'\pm k_p)/2\alpha^{\frac{1}{2}},\\E(w)&=(2/\pi^{\frac{1}{2}})\int_0^w\exp(-x^2)dx.\end{aligned}\qquad(7)$$

Calculation with (7) shows that the potential well becomes gradually shallower as the neutron velocity is increased, as would be expected from the nature of exchange forces.

Before considering the scattering problem it will be well to examine the numerical magnitudes involved. Feenberg[10] gives as values of A, B, and α best fitting the binding energies of light nuclei

$$A=-74mc^2,\qquad 3B/4=26mc^2,\qquad \alpha^{-\frac{1}{2}}=2.17\cdot10^{-13}\text{ cm.}\qquad(8)$$

We assume the Gamow value $R=7.8\cdot10^{-13}$ cm for Pb and that R is proportional to $Z^{\frac{1}{3}}$. The latter assumption has an extremely important effect; namely, since Z and R enter the expression for k' only in $k_p=(9\pi Z/4R^3)^{\frac{1}{3}}$ the depth of the well is independent of Z, in line with the fact that the binding energy per particle is approximately constant for heavy nuclei. (We neglect the slight variation of reduced mass with Z.) Now from $M=1.670$ 10^{-24} g we find $k=0.220\cdot10^{13}\,(W(\text{MEV}))^{\frac{1}{2}}$ cm^{-1}. Also $k_p=1.069\cdot10^{13}$ cm^{-1}. With these values k' has the value $1.060\cdot10^{13}$ cm^{-1} for $k=0$. We see that even for W as high as 7 MEV (million electron volts) k is considerably less than k', that is, the depth of the potential well is very large (of the order of 24 MEV) compared to the kinetic energy of the neutron. Since 7 MEV is about the upper limit of kinetic energy for the fast neutrons from a beryllium-radon source, these neutrons are not fast in the sense in which one speaks of fast electrons in discussing their scattering by atoms. The Born approximation,[14] so useful in many scattering problems, is inapplicable here. The problem may be solved in terms of known functions by the method of partial cross sections, which though tedious in its computational application leads in our case to resonances no simpler treatment could well be expected to fit.

The Method of Partial Cross Sections

The method of partial cross sections[15] depends on the fact that the wave equation for the motion of a particle in a central field is separable in polar coordinates, possessing particular solutions which are products of functions of the radius and spherical harmonics, in which more complicated solutions may conveniently be expanded. We wish to express a function representing an incoming monochromatic plane wave plus an outgoing radial wave, a reasonable idealization of the usual experimental arrangement for the study of scattering, in terms of these particular solutions. On account of the axial symmetry we shall need only the particular solutions of the form $f_l(r)P_l(\cos\theta)$. Our wave equation is of the form

$$\nabla^2\psi+(k^2-U(r))\psi=0.\qquad(9)$$

If $U(r)$ vanishes not less rapidly than $1/r^2$ for large r and behaves no worse than $1/r$ at the origin $f_l(r)$ satisfying the boundary condition at the origin exists and has the asymptotic form $(1/kr)\sin(kr-l\pi/2+\varphi_l)$ for large r. The plane wave $\exp(ikz)=\exp(ikr\cos\theta)$ is an asymptotic solution of Eq. (9) and has the asymptotic expansion

$$\exp(ikr\cos\theta)\doteq(1/kr)\sum_{l=0}^{\infty}(i)^l(2l+1)\sin(kr-l\pi/2)P_l(\cos\theta)$$

for large r. We write therefore

[14] See, e.g., Mott and Massey, *The Theory of Atomic Collisions*, Chapter VII.

[15] Cf. Mott and Massey, reference 14, Chapter II.

$$\exp(ikr\cos\theta)+f(\theta)\exp(ikr)/r$$

$$\doteq (1/kr)\sum_{l=0}^{\infty}(i)^l(2l+1)\sin(kr-l\pi/2)P_l(\cos\theta)$$

$$+\sum_{l=0}^{\infty}A_l(2l+1)P_l(\cos\theta)\exp(ikr)/kr \quad (10)$$

$$=(1/kr)\sum_{l=0}^{\infty}B_l(2l+1)\sin(kr-l\pi/2+\varphi_l)P_l(\cos\theta).$$

Equating the coefficients of $\exp(ikr)$ and of $\exp(-ikr)$ gives

$$A_l=\exp(2i\varphi_l)-1,$$

so that

$$f(\theta)=(1/k)\sum_{l=0}^{\infty}(2l+1)(\exp(2i\varphi_l)-1)P_l(\cos\theta). \quad (11)$$

From (11) we have

$$I(\theta)=|f(\theta)|^2=(1/k^2)\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}(2l+1)(2l'+1)$$
$$\times\sin\varphi_l\sin\varphi_{l'}\cos(\varphi_l-\varphi_{l'})P_l(\cos\theta)P_{l'}(\cos\theta), \quad (12)$$

the scattered current per unit solid angle when the incoming wave is normalized to unit current density. It is seen to be determined by the φ_l's, the so-called phase shifts of the radial functions.

The total cross section Q is obtained by integrating $I(\theta)$ over all directions, and is

$$Q=\sum_{l=0}^{\infty}Q_l=(4\pi/k^2)\sum_{l=0}^{\infty}(2l+1)\sin^2\varphi_l. \quad (13)$$

Physically Q_l may be regarded as the partial cross section due to particles of angular momentum about $(l+\frac{1}{2})h/2\pi$. On this basis one would expect Q_l to be important only when the classical distance of closest approach for particles of the corresponding angular momentum is such that the particle classically traverses regions where $U(r)$ is appreciable. This is not quite true, however; in the case of an attractive potential, such as we must consider, there may be three classical turning points (radii), with virtual states similar to those met with in the theory of radioactive decay and corresponding sharp high resonances which we may refer to as anomalous for want of a better term. These anomalous resonances complicate our problem, but on account of their sharpness contribute little to the cross section for a neutron beam having a broad distribution of velocities.

Solution of Problem of Scattering by a Potential Well

We now proceed to obtain an expression for the phase shifts of the radial solutions of (5). Inside the well the lth radial function is

$$f_l(r)=c_1r^{-\frac{1}{2}}J_{l+\frac{1}{2}}(k'r);$$

outside we have

$$f_l(r)=c_2r^{-\frac{1}{2}}[\cos\varphi_lJ_{l+\frac{1}{2}}(kr)+(-1)^l\sin\varphi_lJ_{-l-\frac{1}{2}}(kr)].$$

From the continuity of $f_l(r)$ and of its first derivative at $r=R$ we find

$$\tan\varphi_l=-(-1)^l\frac{k'J'_{l+\frac{1}{2}}(k'R)J_{l+\frac{1}{2}}(kR)-kJ'_{l+\frac{1}{2}}(kR)J_{l+\frac{1}{2}}(k'R)}{k'J'_{l+\frac{1}{2}}(k'R)J_{-l-\frac{1}{2}}(kR)-kJ'_{-l-\frac{1}{2}}(kR)J_{l+\frac{1}{2}}(k'R)}. \quad (14)$$

This may be reduced to a more convenient form by eliminating the derivatives by means of the recurrence formulas satisfied by the Bessel functions, with the final result

$$\tan\varphi_l=\frac{J_{l-\frac{1}{2}}(kR)J_{l+3/2}(k'R)-J_{l+3/2}(kR)J_{l-\frac{1}{2}}(k'R)}{(-1)^l[J_{-l+\frac{1}{2}}(kR)J_{l+3/2}(k'R)-J_{-l-3/2}(kR)J_{l-\frac{1}{2}}(k'R)]}, \quad (15)$$

a formula convenient for computation. From Eqs. (13) and (15) it is evident that here Q/R^2 is a function of but two parameters kR and $k'R$. We have computed tables for the evaluation of phase shifts and Q/R^2 for a range of values of these parameters sufficient to cover the values met with in our problem. These tables will be loaned to anyone who desires to use them.

We wish to make an estimate of the area under an anomalous resonance peak of a Q_l/R^2 *versus*

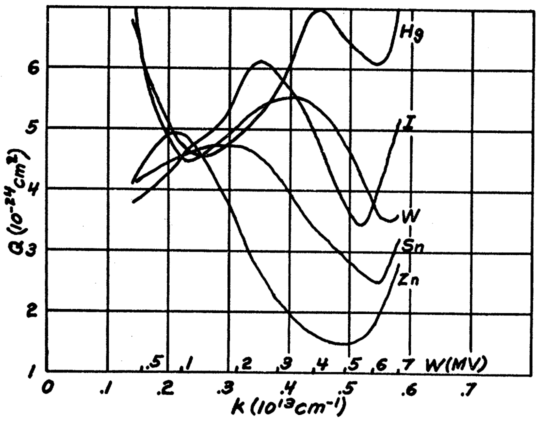

FIG. 1.

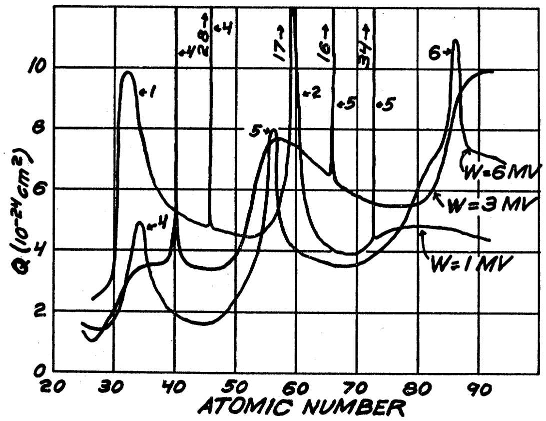

FIG. 2.

kR curve (we do not need to consider the manner of variation of $k'R$ with kR, assuming it constant over the small range involved); this we may do by using the first term of the power series expansions for $J_{l+\frac{1}{2}}(kR)$ and $J_{-l-\frac{1}{2}}(kR)$ in Eq. (15), since kR is small in comparison to l for anomalous resonance. It can be shown that

$$\tan\varphi_l \doteq (\mathfrak{k}\mathfrak{R})^{2l+1}/[(2l+1)(1\cdot3\cdot5\cdots(2l-3))^2 \times((\mathfrak{k}\mathfrak{R})^2-(kR)^2)], \quad (16)$$

where $\mathfrak{k}\mathfrak{R}$ is the value of kR for resonance. Thus for $\tan\varphi_l=\pm1$, the points at which the curve has half its peak value,

$$(kR)^2 \doteq (\mathfrak{k}\mathfrak{R})^2 \mp (\mathfrak{k}\mathfrak{R})^{2l+1}/[(2l+1) \times(1\cdot3\cdot5\cdots(2l-3))^2],$$

and the peak breadth at half value is

$$\Delta(kR) \doteq (\mathfrak{k}\mathfrak{R})^{2l}/[(2l+1)(1\cdot3\cdot5\cdots(2l-3))^2], \quad l\geq1.$$

Since the curve is like a $\sin^2$ curve the area under it is approximately this half value breadth times the peak height, or

$$(Q_l/R^2)_{\max}\Delta(kR) \doteq 4\pi(\mathfrak{k}\mathfrak{R})^{2l-2}/[1\cdot3\cdot5\cdots(2l-3)]^2, \quad (17)$$

a result which should be reasonably accurate for $\mathfrak{k}\mathfrak{R}$ small in comparison with l. We see that in spite of the increase of peak height with increasing anomaly (decreasing $\mathfrak{k}\mathfrak{R}$) the breadth decreases more rapidly, so that the area under the peak, that is, the effective contribution to the cross section for a broad distribution of velocities, decreases. It is evident that experimental detection of these anomalous resonances will require a source of neutrons of quite homogeneous velocity.

RESULTS

It was found that the best fit with the experimental data was obtained by increasing $|A-3B/8|$ of Eq. (8) by five percent; a suitable adjustment in R would have served about as well. k' is raised from $1.060\cdot10^{13}$ cm^{-1} to $1.074\cdot10^{13}$ cm^{-1} for $k=0$, or less than two percent, by this change.

Behavior of cross section with velocity. Fig. 1 illustrates the variation of cross section with neutron velocity for several elements. Although the constants are such for these elements that no highly anomalous peaks appear, the curves do show important resonances. The cross section varies so much with velocity that the effect ought to be experimentally detectable by the use of some other source than beryllium-radon, having a sufficiently different velocity distribution to lead to a measurably different cross section. It is hoped that such work will soon be undertaken by those having the necessary facilities. Of course a homogeneous source would be ideal, but its experimental realization involves difficulties not readily surmounted. It is possible that the neutron beam from beryllium bombarded with deuterons or from deuterium bombarded with deuterons may prove sufficiently homogeneous to allow the detection of narrow resonances.

Variation of cross section with atomic number. Fig. 2 indicates the behavior of the cross section

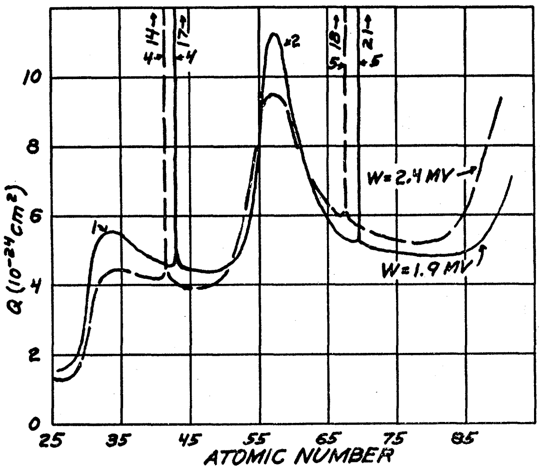

Fig. 3.

as a function of atomic number for neutron energies of 1, 3, and 6 MEV. Here of course anomalous resonances appear. The partial cross sections giving rise to the more prominent resonances are indicated, as are the heights of those peaks lying off the scale.

The most promising sources of homogeneous neutron beams give neutrons of kinetic energy around 2 MEV. Fig. 3 shows cross section as a function of atomic number for 1.86 MEV and for 2.39 MEV neutrons. It indicates what is to be expected on the basis of this theory for experimentally obtainable sources of homogeneous neutron beams, such as deuterium bombarded by deuterons.

It is evident from Figs. 2 and 3 that the cross section does not even begin to approximate a $Z^{\frac{2}{3}}$ law for any single value of neutron velocity. It would be desirable to attempt to locate some of the more anomalous peaks with a highly homogeneous neutron beam, as from their relative locations one could infer much concerning the importance of edge effects. Consideration of these in the theory would not change the nature of the resonances, but would alter their relative positions.

Results for Dunning's experimental velocity distribution. Dunning, by measuring the ranges of protons projected by neutrons from his source, has given us a fair idea of the velocity distribution of berylliium-radon neutrons. He suggests[16] a distribution of the general nature shown in Fig. 4. With this distribution we find a behavior of cross section with atomic number shown by Fig. 5. It is seen that the broader resonances indicated in Figs. 2 and 3 persist in the averaging process. The peak around $Z=34$ can be identified with resonance in the first partial cross section. The low at $Z=45$ is near the vanishing of the first partial cross section. Resonances in the first and second partial cross sections give the peak at $Z=60$. On the high velocity end the fourth partial cross section contributes somewhat to the peak at $Z=34$ and the fifth to the peak at $Z=60$.

The crosses on Fig. 5 indicate Dunning's experimental results for Cu, Zn, Sn, I, W, Hg, and Pb, the only elements in the range given for which he has made measurements with fast neutrons. Except fôr Cu, the agreement is within the possible experimental error, which he estimates to be about ten percent. However, his relative values are perhaps better than this, so that we are disappointed not to find the decline from Hg to Pb indicated by his results. As may be seen from the 6 MEV curve of Fig. 2, however, the rise we get comes from the high energy end of the distribution, and changes in the assumed distribution curve might reduce or eliminate the discrepancy. We cannot expect good agreement at low Z, so that the bad agreement for Cu should not be taken too seriously. It is unfortunate that Dunning has measured cross sections for slow neutrons for many elements lying at the interesting portions of our curve without investigating their cross sections for fast neutrons.

Comparison with constant depth well. It is of some interest to inquire how the scattering by the well of depth a function of the velocity com-

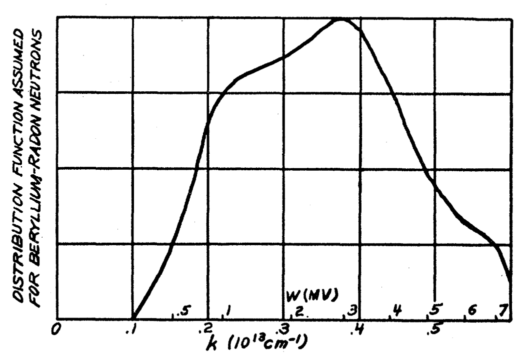

Fig. 4.

[16] I am indebted to Professor Dunning for an interesting conversation on this point.

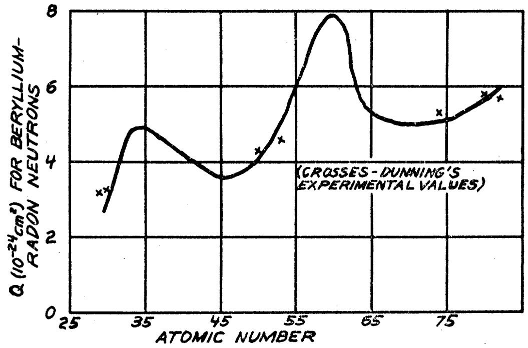

FIG. 5.

pares with a constant depth potential well. The essential difference between the two lies in the fact that k' varies more slowly with k for our well than for the constant depth well. This has the effect of making the cross section vary more slowly with velocity; resonances are broader for the well of varying depth. For this reason the results given by the constant depth well for our velocity distribution will show less oscillation with atomic number than our curve. At present there is no experimental reason for choosing one as preferable to the other, as the experimental points can be fitted equally well with the constant depth hole.

Importance of small angle scattering. The experimental work of Dunning does not measure scattering through angles of less than about 7°. The question suggested itself that possibly the theory might give such a large small angle scattering, particularly near resonances in the higher partial cross sections, that it would be necessary to take this into account in making comparisons with the experimental results. Calculation in a special case revealed that while the small angle scattering was of the order of four times the average, the contribution to the total cross section could be neglected on account of the small solid angle of the 7° cone, about 0.3 percent of the total solid angle.

Nature of polarization corrections. The experimental cross sections for fast neutron scattering are in no case much larger than those given by our theory. We may be justified in concluding from this that polarization effects are small. Contributions to the cross section from polarization may be expected simply to add to the total cross section, although it is possible that interference (nonorthogonality) is important.

It is possible to see in a general way the effect of quasi-stationary levels of the nucleus-neutron system. We should expect sharp resonances in the cross section corresponding to these levels, so that the behavior of the cross section as a function of velocity or as a function of atomic number for a homogeneous beam would be considerably more complicated. On the other hand, it is possible that the cross section as a function of atomic number for a broad velocity distribution might vary more smoothly than our result indicates, on account of the great number of resonances to be averaged over. In any event we believe that further experimental work on fast neutron scattering, a field somewhat neglected since the discovery of the exciting properties of slow neutrons, should yield results both interesting and important.

In conclusion I wish to thank Professor Van Vleck for his helpful interest in the problem.