The Precise Measurement of Three Radium B Beta-Particle Energies

F. T. ROGERS, JR., *The Rice Institute*

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The $H\rho$ of the three most intense Ra B β-particle lines have been measured by a precise magnetic spectrograph method and found to be 1406.0, 1671.1, and 1931.5 gauss cm. An explicit analysis has been made of the Cotton balance equations for H in this experiment and on the basis of this a complete analysis was made of the maximum relative error to be expected in the accepted values of $H\rho$. It is shown that the maximum relative error to be expected in the least $H\rho$ is 1 part in 3000 and that within this possible error the greatest $H\rho$ agrees with the value obtained by Scott. It is suggested that the accuracy of 1 part in 10,000 hoped for by Scott is rather too optimistic. The energies of these three β-particles are 1.512, 2.044, and 2.610 in units of 10^5 electron volts. It is shown that the least of these cannot be expected to be any more accurate than 1 part in 1000. The conclusion is that except for systematic errors which might have been present and undetected throughout this experiment and that of Scott, these experiments have established the values of these $H\rho$'s to within 1 part in 3000.

INTRODUCTION

IN 1924 Ellis and Skinner[1] measured $H\rho$ (H is the magnetic field necessary to produce a radius ρ of curvature in the path of a β-particle) for many of the β-particle energies of Ra B; for the three most intense lines of Ra B they got, correctly to within what they thought was 1 part in 500,

$$H\rho = 1410,\ 1677,\ 1938 \text{ gauss cm.}$$

In 1934 Ellis[2] repeated this work with more precision and got

$$H\rho = 1400.4,\ 1665.9,\ 1925.5 \text{ gauss cm}$$

(in which the last figures are not entirely significant) for these same lines. In 1934 also, Scott[3] performed a precise experiment on the most intense of these lines using the Cotton balance method of measuring the magnetic field rather than an inductive method; he got for this most intense line

$$H\rho = 1931.8 \text{ gauss cm}$$

and hoped it to be correct to 1 part in 10,000.

The present experiment was undertaken to check Scott's work and method and to extend the measurements to other lines in the Ra B β-particle spectrum. The values of

$$H\rho = 1406.0,\ 1671.1,\ 1931.5 \text{ gauss cm}$$

were obtained, the last of which is in good agreement with Scott's value for the most intense line. The $H\rho$'s for the other two lines as found by this experiment yield slightly different relative values of $H\rho$ from those previously obtained.

A more thorough investigation of the theory of the Cotton balance method of measuring H was carried out for this experiment. On the basis of it an analysis was made of the errors which might be present in the finally accepted values of $H\rho$. The conclusion was reached that in this experiment an accuracy of 1 part in 3000 is attainable if the value of H used in the finally accepted $H\rho$'s is determined from the mean of at least forty separate, equally weighted determinations of the mass used with the Cotton balance and if the values of ρ used finally are the means of seven separate, equally weighted determinations.

The good agreement (within 1 part in 6400 in $H\rho$) of the present experiment with that of Scott would seem to indicate that this type of measurement has at last established the absolute values of the $H\rho$'s for the three most intense lines in the Ra B β-particle spectrum to within 1 part in 3000 except for possible systematic errors which may have been present and undetected in both experiments.

The energies corresponding to the three $H\rho$'s we have determined are found to be

$$V = 1.512\times 10^5,\ 2.044\times 10^5,\ 2.610\times 10^5 \text{ electron volts}$$

and can be considered as accurate to not better than 1 part in 1000.

APPARATUS

The apparatus used in this experiment was, except for a few small changes, that used by

[1] Ellis and Skinner, Proc. Roy. Soc. **A105**, 165 (1924).
[2] Ellis, Proc. Roy. Soc. **A143**, 352 (1934).
[3] Scott, Phys. Rev. **46**, 633 (1934).

Scott in his work and described by him in his paper.[3] It consists of a large permanent magnet with cobalt steel poles and with a 1.52 cm air gap faced on each side by soft iron disks 15 cm in diameter and 7.5 cm thick; the faces are quite nearly parallel and vertical. The flux density in the air gap can easily be adjusted to the desired value to within 1.5 gauss by means of a controlled current in two solenoids around the pole pieces. An investigation of the direction of the field in the gap showed that it deviates from the normal to the pole faces by less than 0° 05′. Computation by the series for cos 0° 05′ shows that the horizontal component of H differs from H by less than 1 part in 500,000. Thus the force which the Cotton balance measures, i.e., the horizontal component of H, is sensibly H.

The field in the air gap could be explored with search coils and galvanometers calibrated as fluxmeters. H as a function of the distance from the center of the field was determined with a coil and galvanometer designed to give an accuracy of one percent. Small variations of H near the center of the field were obtained with a 1000 turn coil of diameter 0.9 cm and an 18 ohm, high sensitivity galvanometer, the combination having a sensitivity of 0.0241 gauss per cm.

Supported above the air gap was a new Becker precision balance of which one pan had been removed. In place of the pan was hung a Pyrex glass plate about 30 cm long and 3 cm wide with its longer edges optically worked to be parallel to within 1/500 mm for 15 cm from one end. This plate hung from the lower ends of two parallel aluminum strips which hung in turn from a support-assembly attached to a knife edge of the balance. The plate-assembly was free to swing in two dimensions and was protected from external air currents by a surrounding brass case. Two straight silver strips about 1/10 mm thick were cemented with very dilute white shellac around the lower end and the two long edges of the plate; they were joined by a flat fold at the middle of the lower edge and formed the conductor of current in the magnetic field. This lower edge was placed along the x axis of Fig. 1 while measurements of H were in progress; the center line of the plate was along the y axis.

Measurements of plate and strips could be made with a Brown and Sharpe gauge which was

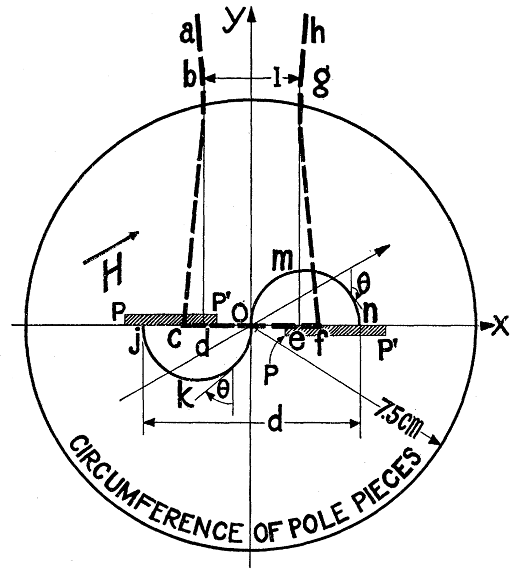

FIG. 1. PP' are photographic plates; O is the source of β-particles.

graduated in 10^{-4}'s of an inch and which could be reliably estimated to a 10^{-5} of an inch, with a Gaertner comparator and invar steel standard scale 10 cm long, and with a new Brown and Sharpe 31 mm standard disk. The invar scale was stated by the Bureau of Standards to be accurate to 1 part in 10^6. All measurements of the plate, strips, and photographic plates are expressed in terms of this standard scale when used as absolute lengths.

When the current in the strips was reversed, the change in force on them was measured by balancing it with a mass m on the balance. m was nearly 400 mg and was obtained in the main by combining the 200, 100 and 100 mg weights of a new and previously unused set of platinum, assayers' weights by Becker; each of these weights is quoted by the manufacturer as being accurate to within 0.005 mg. To get m really accurately, small balance deflections (occurring when the current in the strips was reversed) could be measured by the motion of a spot of light reflected from a small mirror on the balance beam to a scale about two meters from the balance. The location of the spot of light could be read to within 1/5 mm with a magnifying glass. When fully loaded in the experiment, the balance had a sensitivity of 0.409 mg/cm, which was entirely sufficient to get m accurately.

Current was supplied to the strips around the plate by two 160 ampere-hour lead storage batteries connected in parallel. The current was sent through two standard resistors of 4.0001 international ohms at 25°C and having negligible temperature corrections near 25°C. These resistors were connected in parallel by heavy copper bars and the current through them adjusted until a galvanometer in series with a standard cell (the two being connected across the resistors) showed no deflection. A measurement of H using one resistor balanced by two standard cells yielded the same value of H as this arrangement, so the resistance between the resistors and the copper bars was entirely negligible. The resistors were calibrated to 0.005 percent by the Bureau of Standards in February, 1934, and, except for Scott's work then, have not been used until the present experiment. The standard cell along with two others was calibrated to 0.01 percent by the Bureau of Standards at the same time and was found to have an e.m.f. of 1.01884 international volts at 25°C; at the time of this experiment these cells had not changed their relative e.m.f.'s by more than $\frac{1}{2}\times 10^{-5}$ volts, so it was assumed that their absolute values had not altered appreciably. The galvanometer used to indicate the existence of the correct current was of sensitivity 1.84×10^{-5} amp./cm, readable to 1/5 mm easily. The current I could be adjusted by means of a variable resistance network to within $\frac{1}{2}\times$mm scale deflection, or to within 10^{-6} amp.

The Ra B β-particles (from the active deposit from the radon of 23 mg of radium) were emitted from the surface of a platinum wire placed at 0 (Fig. 1); they traveled in opposite directions as allowed by slits in the camera and fell on photographic plates PP'. The distances between the outer edges of the three sets of clearly defined "images" on the plates could be measured with the Gaertner comparator and standard scale. The camera and its enclosing box are of brass tested previously for magnetic effects and found to have none; Scott fully describes them in his paper.[3] The vacuum in the box was got by a Cenco Hyvac pump running continuously during the three and one-half hour exposures.

All observations and measurements were made in two rooms which were maintained at temperatures between 23° and 25°C.

Theory

Preliminary measurements have shown that when the glass plate is in position for field measurements, the center of the silver strip around the glass plate lies, in the large, along a curve C of the form (exaggerated in the diagram) of $abcdefgh$ in Fig. 1; this curve is sufficiently nearly symmetric to be treated for our purposes as symmetric. In Fig. 1 the magnetic field H is perpendicular to the plane of the paper and directed into the paper; x and y axes perpendicular to H are chosen with their origin in line with the centers of the pole faces and half way between them, the x axis horizontally; ds with x component dx is an element of length along C. The width l of the current loop is measured at some convenient location (as bg) along the glass plate.

To get the field H_0 at the origin by the Cotton balance method, note that the force acting on the current I (e.m.u.) in the strip is, besides gravity, $I\int_c Hdx$; when I is reversed, the change in force on the strip is measured by the weight of the mass m which will just counterbalance it; then

$$mg = 2I\int_c Hdx.$$

Since H and C are necessarily continuous, we can write

$$\int_c = \int_a^c + \int_c^d + \int_d^e + \int_e^f + \int_f^h;$$

and since subsequent experiment shows that

$$\int_a^c, \quad \int_c^d, \quad \int_e^f, \quad \int_f^h < 10^{-3}\int_d^e,$$

approximate evaluation of these integrals is satisfactory. Let

$$H = H_0 + h(x) \text{ along } de,$$
$$H = H_0 F(y) \text{ along } ac \text{ and } fh,$$
$$H = H_0 \text{ along } cd \text{ and } ef;$$

then

$$mg = 2I\left\{ H_0\int_a^c F(y)dx + H_0\int_c^d dx + \int_d^e [H_0 + h(x)]dx + H_0\int_e^f dx + H_0\int_f^h F(y)dx \right\}.$$

Also let C be represented from a to c by $x=f(y)$, and let

$$\int_c^d dx = \tfrac{1}{2}\Delta l;$$

then in view of the symmetry of C with respect to the y axis,

$$mg = 2I\left[2H_0\int_a^c F(y)f'(y)dy + H_0\Delta l + H_0 l + \int_{-l/2}^{l/2} h(x)dx\right].$$

By solving for the coefficient of l,

$$H_0 = \frac{1}{l}\left[\frac{mg}{2I} - 2H_0\int_a^c F(y)f'(y)dy - H_0\Delta l - \int_{-l/2}^{l/2} h(x)dx\right];$$

here m, g, l and I are measurable with high accuracy; the last three terms in the right member are of the nature of corrections to the first and (as previously stated) are fairly small, so in them only the approximate value of H_0 need be used.

To this Cotton balance equation for H_0 must be added for this experiment the Hartree correction for non-uniformity of H along the β-particle paths. In Fig. 1 two semi-circular paths for the same β-particle energy are Okj and Omn; measuring θ along them as shown in the figure, let

$$H = H_0 + h_1(\theta) \text{ along } Okj,$$
$$H = H_0 + h_2(\theta) \text{ along } Omn.$$

Then according to Hartree,[4] the first order correction to be applied to H_0 to give the constant value of H which, if it were acting all along the β-particle path Okj, would produce the observed deviation Oj is

$$\tfrac{1}{2}\int_0^\pi h_1(\theta)\sin\theta d\theta.$$

Applying to the above equation for H_0 the mean of the two Hartree corrections for the two paths of the same energy of β-particle, we get

$$H_0 = \frac{1}{l}\left[\frac{mg}{2I} - 2H_0\int_a^c F(y)f'(y)dy - H_0\Delta l - \int_{-l/2}^{l/2} h(x)dx\right] + \frac{1}{4}\left(\int_0^\pi h_1(\theta)\sin\theta d\theta + \int_0^\pi h_2(\theta)\sin\theta d\theta\right), \quad (1)$$

which is the expression adopted for H_0 to be used in $H\rho$.

To get ρ, the distance d between the outer, clearly defined edges of corresponding images on the photographic plates is measured; from it is subtracted the diameter D of the platinum wire supporting the source at O. Then

$$\rho = (d-D)/4. \quad (2)$$

$H\rho$ for a β-particle energy is the product of the appropriate value of expressions (1) and (2).

The relative error to be expected in a single determination of H_0 follows from two principles concerning errors:[5] the relative error in a product of quantities having known relative errors is the sum of the relative errors of the multiplicands, and the absolute error of a sum of quantities having known absolute errors is the sum of the absolute errors of the summands. Let ϵ_i, $i=1$ through 10, be the absolute errors which might possibly be present in single determinations of d, D, l, m, g, I, $H_0\int Ff'dy$, $H_0\Delta l$, $\int h(x)dx$, and $\int h(\theta)\sin\theta d\theta$ respectively; these must be estimated from the precisions of the observations on the several quantities entering into expressions (1) and (2). Then since (by the definition of relative error) there corresponds to the relative error in a quantity an absolute error whose magnitude is equal to the product of the magnitude of the quantity into the relative error,

$$(mg/2I)(\epsilon_4/m + \epsilon_5/g + \epsilon_6/I) + 2\epsilon_7 + \epsilon_8 + \epsilon_9$$

is the absolute error to be expected in the bracketed part of expression (1); to it corresponds quite nearly the relative error

$$\epsilon_4/m + \epsilon_5/g + \epsilon_6/I + (2\epsilon_7 + \epsilon_8 + \epsilon_9)/(mg/2I).$$

Hence the absolute error in the first term of the

[4] Hartree, Proc. Camb. Phil. Soc. **21**, 746 (1923).

[5] Gibbs, *Adjustment of Errors in Practical Science* (Oxford Press, 1929), pp. 76–78.

right member of (1) is approximately

$$H_0\left(\frac{\epsilon_3}{l}+\frac{\epsilon_4}{m}+\frac{\epsilon_5}{g}+\frac{\epsilon_6}{I}+\frac{2\epsilon_7+\epsilon_8+\epsilon_9}{mg/2I}\right);$$

added to the absolute error

$$\tfrac{1}{2}(\tfrac{1}{2}\epsilon_{10}+\tfrac{1}{2}\epsilon_{10})=\epsilon_{10}/2,$$

in the second term and divided by H_0, we have as the relative error to be expected in a single determination of H_0:

$$e_1=\frac{\epsilon_3}{l}+\frac{\epsilon_4}{m}+\frac{\epsilon_5}{g}+\frac{\epsilon_6}{I}+\frac{2\epsilon_7+\epsilon_8+\epsilon_9}{mg/2I}+\frac{\epsilon_{10}}{2H_0}. \quad (3)$$

The relative error to be expected in a single determination of ρ is obviously

$$e_2=(\epsilon_1+\epsilon_2)/(d-D). \quad (4)$$

If ϵ_1, $i=1$ through 10, are estimated conservatively as the maximum absolute errors to be expected in their parent magnitudes, the corresponding maximum relative error to be expected in an accepted $H\rho$ is

$$e_3=e_1+e_2. \quad (5)$$

To get the energy E of a β-particle of known $H\rho$, combine with the relativistic expression

$$E=m_0c^2[1/(1-v^2/c^2)^{\frac{1}{2}}-1]$$

for E the relativistic force equation

$$Hev=mv^2/\rho=m_0v^2/\rho(1-v^2/c^2)^{\frac{1}{2}}$$

so as to eliminate v. Here v is the velocity of the β-particles, m_0 their rest mass, e their charge, and c the velocity of light. The resulting expression

$$E=m_0c^2[(A+1)^{\frac{1}{2}}-1],\ A=(H\rho e/m_0)^2(1/c)^2,$$

is the one commonly used. If e/m_0 is in e.m.u. and if e is in e.s.u., then $m=(m_0/e)(e/c)$; also if E is expressed (as V) in electron volts, its value in ergs must be multiplied by $10^{-8}\ c/e$; thus if $H\rho$ is in gauss cm, the energy in electron volts is

$$v=(m_0/e)c^2[(A+1)^{\frac{1}{2}}-1]\times 10^{-8}. \quad (6)$$

This can be put more conveniently for computation if we define ϕ by

$$\phi=\tan^{-1}A^{\frac{1}{2}}=\tan^{-1}(H\rho e/m_0)(1/c),$$

then

$$v=(m_0/e)c^2(\sec\phi-1)\times 10^{-8}. \quad (7)$$

If ϵ_{11} and ϵ_{12} are the absolute errors to be expected in e/m_0 and c respectively, the relative error to be expected in $(A+1)$ (see Eq. (6)) is

$$\frac{A}{A+1}\left(2e_3+\frac{2\epsilon_{11}}{e/m_0}+\frac{2\epsilon_{12}}{c}\right).$$

Since the extraction of the square root of a quantity with a known relative error yields roots with relative errors only half that of their square,[6] the absolute error in $(A+1)^{\frac{1}{2}}$ is

$$\frac{A}{(A+1)^{\frac{1}{2}}}\left(e_3+\frac{\epsilon_{11}}{e/m_0}+\frac{\epsilon_{12}}{c}\right),$$

which is also to be expected in $[(A+1)^{\frac{1}{2}}-1]$. Hence by addition of the relative errors in m_0/e, c^2, and $[(A+1)^{\frac{1}{2}}-1]$, the relative error to be expected in V is

$$e_4=\frac{\epsilon_{11}}{e/m_0}+\frac{2\epsilon_{12}}{c}+\frac{A}{(A+1)-(A+1)^{\frac{1}{2}}}\left(e_3+\frac{\epsilon_{11}}{e/m_0}+\frac{\epsilon_{12}}{c}\right). \quad (8)$$

The values of ϵ_{11} and ϵ_{12} are preferably the maximum errors judged to be possible in the accepted values of their parent magnitudes; if they are not, but are the probable errors quoted with the accepted values, e_4 is of the nature of an hybrid which is only a lower bound of the maximum relative error to be expected in V.

It should be remembered that expressions (3), (4), (5) and (8) do not take into account any systematic errors which might have been present and undetected in the determinations of the quantities appearing in expressions (1), (2) and (7).

Measurements

The experiment consisted in measuring the quantities appearing in expressions (1) and (2) so as to determine, respectively, H and the ρ's

[6] For if a is a quantity with a small known absolute error Δa, then

$$(a+\Delta a)^{\frac{1}{2}}=a^{\frac{1}{2}}+\Delta a/2a^{\frac{1}{2}}+\cdots\cong a^{\frac{1}{2}}+\Delta a/2a^{\frac{1}{2}}.$$

Thus the relative error in $a^{\frac{1}{2}}$ is

$$(1/a^{\frac{1}{2}})\times(\Delta a/2a^{\frac{1}{2}})=1/2(\Delta a/a),$$

or is one-half the relative error in a.

for the three lines. H_0 was determined from forty determinations of m and each ρ was determined seven times in order to reduce the maximum possible relative error to be expected in $H\rho$ as far as is reasonably possible. The accepted $H\rho$ for an energy is the product of the H_0 value (from the forty determinations of m) as corrected for the proper path by the Hartree correction, into the appropriate radius of curvature of the path. The correction quantities were in general determined first. The error quantities ϵ_i were estimated as each quantity was measured. All measurements were repeated at least once, usually more than once. Data are usually recorded with more figures than are strictly significant, in order to prevent the entrance of errors in computation.

The functions $h(x)$ and $h(\theta)$ were determined from changes in H revealed by the 1000 turn coil and the sensitive galvanometer. Table I is the data obtained for $\int h(x)dx$ at $H_0=1250$ gauss. Integration was effected numerically by assuming that $h(x)$ is linear between adjacent experimentally determined points; the result is that

$$\int_{-l/2}^{l/2} h(x)dx = -0.319 \text{ gauss cm.}$$

It is estimated that $\epsilon_9=0.01$. Table II is the data used in calculating $\int h(\theta) \sin \theta d\theta$ for $H_0=1250$ gauss and $\rho=1.54$ cm; $h(\theta)$ are given in gauss. Integration is exactly as before and the results for this radius and for the two other radii used are tabulated in Table III; the Hartree correction as tabulated is the mean of the two corrections for the two paths from 0. It is estimated from experience with these that $\epsilon_{10}=0.01$.

The value of l was measured at the location bg (Fig. 1), which is between $y=8.0$ and $y=9.0$. It was measured first with the comparator and standard scale and found to be 3.07918 cm. After the experiment was terminated, l was measured again, but with the 31 mm disk and the Brown and Sharpe gauge. The value of 3.07891 cm was obtained in this way. The finally accepted value of l is the mean of these two: $l=3.07905$ cm. These values of l are got by subtracting from the overall width of the glass plate and silver strips at bg, half the thicknesses of the silver strips at that location. To accompany l we assign $\epsilon_3=0.00015$.

TABLE I. *Data for* $\int h(x)dx$.

x (cm)	$h(x)$ (gauss)
−1.5	−0.018
−1.0	−0.106
−0.5	−0.076
0.0	−0.000
0.5	−0.113
1.0	−0.187
1.5	−0.263

TABLE II. *Data for* $\int h(\theta) \sin \theta d\theta$, $\rho=1.54$ *cm*.

	$\rho=1.54$ cm	
θ	$h_1(\theta)$	$h_2(\theta)$
0°	0.000	0.000
30°	−0.027	−0.263
60°	−0.115	−0.566
90°	−0.110	−0.667
120°	−0.234	−0.501
150°	−0.130	−0.314
180°	−0.189	−0.150

TABLE III. *Hartree corrections.*

ρ (cm)	$\frac{1}{2}\int h_1 \sin \theta d\theta$ (gauss)	$\frac{1}{2}\int h_2 \sin \theta d\theta$ (gauss)	Correction (gauss)
1.12	−0.098	−0.352	−0.225
1.34	−0.128	−0.418	−0.273
1.54	−0.122	−0.471	−0.296

To evaluate $f'dy$ and Δl, the total width of the glass plate and of the silver strips in place was measured at many locations along the plate with the 10^{-4} inch gauge; as above, from these widths were subtracted half the thicknesses at the same locations of the silver strips. In Table IV are representative data from the large amount actually obtained by these measurements. As before it is assumed that $f(y)$ is linear between points determined experimentally. Since l was chosen as the width of the current loop between $y=8.0$ and $y=9.0$, Δl is obtained by subtracting the width of the current loop at $y=8.5$ from the width at $y=0.0$; from Table IV we have $\Delta l=3.08251-3.07905=0.00346$ cm.

The function $F(y)$ was obtained with the less sensitive fluxmeter arrangement; its values are given in Table V. For purposes of integration, $F(y)$ was also assumed to be linear between observed points. It is found that $2\int Ff'dy=$

TABLE IV. *Width of current loop.*

y (cm)	Width of strips and plate (cm)	$\frac{1}{2}$ sum of strip thicknesses (cm)	$2f(y)$ = width of current loop (cm)
0.0	3.09195	0.00944	3.08251
0.5	3.08914	0.00945	3.07969
1.0	3.08901	0.00945	3.07956
2.0	3.08853	0.00947	3.07906
2.3	3.08835	0.00948	3.07887
2.4	3.08876	0.00949	3.07927
2.8	3.08905	0.00950	3.07955
2.9	3.08862	0.00951	3.07911
3.0	3.08849	0.00952	3.07897
4.3	3.08966	0.00952	3.08014
4.5	3.08953	0.00953	3.08000
5.5	3.08878	0.00952	3.07926
6.0	3.08877	0.00951	3.07926
6.7	3.08903	0.00949	3.07954
7.0	3.08911	0.00950	3.07961
8.5	3.08853	0.00948	3.07905
9.5	3.08837	0.00946	3.07891
11.5	3.08861	0.00950	3.07911
12.0	3.08873	0.00949	3.07924
12.4	3.08854	0.00948	3.07906
12.6	3.08876	0.00946	3.07932
13.2	3.08905	0.00944	3.07963
13.5	3.08861	0.00944	3.07917
15.0	3.08861	0.00945	3.07916
17.0	3.08869	0.00945	3.07924
17.5	3.08903	0.00943	3.07960
18.0	3.08854	0.00943	3.07911
20.0	3.08847	0.00942	3.07905
25.0	3.08872	0.00944	3.07928

-0.00354 using the entire set of data of which Table IV is a representative portion; the Table IV data give a slightly lower value than this.

The actual values of the corrections $H_0\Delta l$ and $2H_0\int Ff'dy$ are obtained directly as

$$H_0\Delta l = 1250\times 0.00346 = 4.33 \text{ gauss cm},$$
$$2H_0\int Ff'dy = 1250\times(-0.00354) = -4.43 \text{ gauss cm}.$$

It is estimated that $\epsilon_7=\epsilon_8=0.05$.

Since the resistors used in getting I are calibrated to only 0.005 percent and since the standard cells, to only 0.01 percent, the maximum error to be expected in I is at least 0.015 percent. The error inherent in adjusting for I with the resistance network is at most 10^{-7} e.m.u. By Ohm's law I is 0.509403 international ampere; the conversion[7] to e.m.u. gives $I=0.0509433$ e.m.u. Thus $\epsilon_6=0.0000076+0.0000025=0.00001$; the second term is added because of the uncertainty in the factor of conversion from international to electromagnetic units.

[7] *The Smithsonian Physical Tables*, eighth edition (1933), p. 81, gives one international ampere equal to (0.99995 ±0.00005) absolute ampere.

Because three weights are required to produce the 400 mg of m, ϵ_4 must contain 0.015 mg. Because of errors possible in locating the ends of the balance deflections when I is reversed, which come to $2\times 0.02\times 0.41=0.0164$ mg, ϵ_4 is larger than 0.015. Since m was measured nine times in each of the forty determinations of it, we take[8] $\epsilon_4=(0.015+0.0164/9^{\frac{1}{2}})/40^{\frac{1}{2}}=0.0032$ mg.

The value of g in the locality of the experiment is 979.28 cm/sec.^{2} and $\epsilon_5=0.005$ cm/sec.^{2}.

By observing the balance deflections when I was reversed and when the 400 mg were placed on the balance, the correction to be applied to the 400 mg to give m accurately could be found. These deflections were observed nine times for each determination of m. The forty mean observations of the balance deflection are recorded with their mean in Table VI. The evaluation of H_0 by taking for m the mean of a large number

TABLE V. *Function F(y).*

y (cm)	$F(y)$
0.0	1.00
5.6	1.00
7.5	0.91
8.5	0.50
9.5	0.26
10.5	0.17
11.5	0.13
12.5	0.11
15.0	0.07
17.5	0.04
22.5	0.02

TABLE VI. *Observations of balance deflections for m.*

Trial	Defl. (cm)	Trial	Defl. (cm)	Trial	Defl. (cm)	Trial	Defl. (cm)
1	1.53	11	1.52	21	1.52	31	1.52
2	1.52	12	1.54	22	1.50	32	1.52
3	1.52	13	1.53	23	1.52	33	1.52
4	1.53	14	1.54	24	1.51	34	1.52
5	1.52	15	1.54	25	1.51	35	1.51
6	1.53	16	1.50	26	1.51	36	1.51
7	1.51	17	1.51	27	1.52	37	1.54
8	1.52	18	1.51	28	1.53	38	1.51
9	1.53	19	1.52	29	1.52	39	1.52
10	1.54	20	1.52	30	1.52	40	1.50

Mean = 1.520

[8] Analogously to the customary procedure used in the operation of classical error computations, we assume that the relative error of the mean of n determinations of a quantity varies inversely as $n^{\frac{1}{2}}$. See, e.g., Bartlett, *The Method of Least Squares*, third edition (1915), p. 44, Eq. (46).

of observations which extended over several days is valid because Scott[3] showed in his experiment that over a period of weeks the value of H_0 remained sensibly constant. The value of m to be used is thus $0.4000000+1.52\times 0.000409 = 0.400622$ gm. By use of Eq. (1) and the previously evaluated correction quantities it is seen that, except for the Hartree corrections,

$$H_0=\frac{1}{3.07905}\left(\frac{0.400622\times 979.28}{2\times 0.0509433}+4.43-4.33+0.32\right)=1250.70 \text{ gauss.}$$

Values of d were read from the photographic plates with the Gaertner comparator corrected by the standard scale; they could be repeated to within 0.0001 cm usually. Since each was done seven separate times, $\epsilon_1=0.0001/7^{\frac{1}{2}}=0.000038$. The value of D was found with a direct-reading 10^{-4}-inch Brown and Sharpe micrometer caliper and is 0.02405 cm to within $\epsilon_2=0.0001$ cm. In Table VII are presented the seven values of each ρ got by Eq. (2) and their three means. Since H_0 is constant, these should be constant within the proper groups in the table.

Knowledge of the H_0 from Table VI, of the mean ρ's of Table VII, of the Hartree corrections of Table III, of the various correction terms, and of the estimates of the possible errors is sufficient to provide the desired values of $H\rho$ and of the possible errors present.

Results

Tables VIII, IX, X, and XI contain the results of the experiment. The accepted $H\rho$'s of Table VIII are from the value of H_0 from Table VI and corrected by Table III and from the ρ's of Table VII. In view of the good agreement (1 part in 6400) of the present value of $H\rho$ for the most intense β-particles and that of Scott, it seems safe to consider the $H\rho$'s listed in Table VIII as sufficiently adequately determined in absolute value (within the possible limits of relative error given below) to serve as rather reliable standards of β-particle $H\rho$'s.

In Table IX are the possible relative errors necessary to the computations based on Eqs. (3), (4) and (5) for the least $H\rho$; they are collected from the section on Measurements. Thus the values of $H\rho$ in Table VIII cannot be considered as any more accurate than 1 part in 3000, even though each measurement used in getting $H\rho$ is more accurate than 1 part in 3000. Evidently some 60 percent of the uncertainty is accounted for by the uncertainty in the calibrations of the standard cells and standard resistors and in the uncertainty in the conversion factor for international and electromagnetic units of current. Within this limit of 1 part in 3000 the present experiment yields the same value of $H\rho$ for the most intense Ra B β-particles as Scott's experiment, though it is seen that Scott's hopes for an accuracy of 1 part in 10,000 are rather too optimistic.

The energies corresponding to the $H\rho$'s we have established are obtained from Eq. (7) using[9]

Table VII.

Trial	ρ (cm)	Mean ρ (cm)
1	1.12442	
2	1.12446	
3	1.12442	
4	1.12440	1.12440
5	1.12442	
6	1.12440	
7	1.12426	
1	1.33642	
2	1.33642	
3	1.33650	
4	1.33645	1.33644
5	1.33657	
6	1.33635	
7	1.33642	
1	1.54471	
2	1.54476	
3	1.54468	
4	1.54462	1.54472
5	1.54478	
6	1.54470	
7	1.54471	

Table VIII. *$H\rho$'s of three intense Ra B β-ray lines.*

Line	$H\rho$ (gauss cm)
Least intense	1406.0
Of medium intensity	1671.1
Most intense	1931.5

[9] The value of e/m_0 is that given by Birge, Phys. Rev. **49**, 204 (1936). c is taken from *The Smithsonian Physical Tables*, eighth edition (1933), p. 74.

TABLE IX. *Maximum relative errors to be expected in least $H\rho$.*

Error	Magnitude	Magnitude (parts per 10,000)
ϵ_3/l	0.00015/3.079	0.49
ϵ_4/m	0.00002/.4007	0.08
ϵ_5/g	0.005/979.3	0.05
ϵ_6/I	.00001/.05094	1.97
$2I(2\epsilon_7+\epsilon_8+\epsilon_9)/mg$	0.16/3850	0.41
$\epsilon_{10}/2H_0$	0.01/2500	0.04
$\epsilon_1/(d-D)$	.000038/4.496	0.08
$\epsilon_2/(d-D)$	.0001/4.496	0.22
Sum $=e_3$		3.34

$$e/m_0=1.7562\times10^7 \text{ e.m.u./g},$$
$$c=2.99796\times10^{10} \text{ cm/sec.}$$

with $\epsilon_{11}=0.00027\times10^7$ and $\epsilon_{12}=0.00004\times10^{10}$. They are in Table X.

In Table XI are the numbers necessary for the calculation of the lower bound e_4 of the relative error to be expected in the least V according to Eq. (8). They follow from the preceding two paragraphs. Eq. (8) becomes by substitution of these numbers

$$e_4=\frac{1.54}{10{,}000}+\frac{0.24}{10{,}000}+\frac{0.678}{0.383}\left(\frac{3.34}{10{,}000}+\frac{1.54}{10{,}000}+\frac{0.24}{10{,}000}\right)=\frac{1.1}{1000}.$$

Thus the energies V cannot be considered as more accurate than 1 part in 1000. In fact, since ϵ_{11} and ϵ_{12} are only classical probable errors, e_4 may be even larger than 1 part in 1000, the increase depending only upon how much larger than the probable errors the corresponding absolute errors possible in e/m_0 and c are.

TABLE X. *Energies of three intense Ra B β-rays.*

$H\rho$ (gauss cm)	V (electron volts)
1406.0	1.512×10^5
1671.1	2.044×10^5
1931.5	2.610×10^5

TABLE XI. *Quantities for calculation of e_4 for least V.*

Quantity	Magnitude	Magnitude (parts per 10,000)
$\epsilon_{11}/(e/m_0)$	0.00027/1.7562	1.54
ϵ_{12}/c	0.00004/2.998	0.12
e_3	3.34/10,000	3.34
A	$\left(\frac{1.406\times1.756}{2.998}\right)^2=0.679$	

It is also noteworthy that even though $H\rho$ is accurate to within 1 part in 3000, the major part of the 1 part in 1000 accuracy allowed for V is accounted by possible errors in $H\rho$.

Finally, it should be noted that the improvements of this experiment over that of Scott lie in the facts that: a new balance of higher sensitivity, a new set of weights for m, straight silver strips, the standard scale, and a new gauge graduated in 10^{-4}'s of an inch were available; twice as large a current, determined by a far simpler potentiometer, was used in the Cotton balance; it includes measurements for the three most intense Ra B lines rather than for only one; and it provides a conservative, explicit estimate of the precision of which it is capable, 1 part in 3000 in $H\rho$, 1 in 1000 in V.

The writer also wishes in conclusion to record his appreciation both for the suggesting of this experiment and for the valuable aid given him during the experiment, by Dr. H. A. Wilson.