A consideration of the other corrections indicates that the only ones which might be appreciable in the normal state are those given by Dunham[9] for the anharmonicity terms. They involve a power series in $B_e{}^2/\omega_e{}^2$, which is of the order of 10^{-6} for the CuD molecule, making even these corrections negligible.

The values of ρ calculated from the vibrational and rotational constants are given in Table V and compared with the value of $(u/u_i)^{\frac{1}{2}}$ from mass-spectrograph data. The agreement is excellent with the exception of $(B_e{}^i/B_e)^{\frac{1}{2}}$ in the $^1\Sigma^*$ state. Here the value is 0.99976 for $Cu^{65}D$ to $Cu^{63}D$, which differs by 0.00024 from the calculated value. This difference may be due in part to difficulty in reduction of the data as is indicated above since only the low values of J can be used in this state. The fact that $(B_e{}^i/B_e)^{\frac{1}{2}}$ for CuD to CuH in the $^1\Sigma^*$ state is also much larger than the calculated value indicates that the discrepancy is real, however, and that B_e should in this case be corrected as mentioned above.

Rather limited data are available for the determination of the $\Delta G_{v+\frac{1}{2}}$ values for $Cu^{65}D$, especially in the normal state. This makes the uncertainty in the value of $\omega_e{}^i/\omega_e$ much greater than the uncertainty in $(B_e{}^i/B_e)^{\frac{1}{2}}$ for this state. An estimate of the probable error in $\omega_e{}^i/\omega_e$ based on the individual probable errors in the values of $\Delta G_{v+\frac{1}{2}}$ is 0.00015. In the $^1\Sigma$ state $\omega_e{}^i/\omega_e$ for $Cu^{65}D$ to $Cu^{63}D$ has the value 0.99947, and in the $^1\Sigma^*$ state 0.99954. The corresponding values for $Cu^{63}D$ to $Cu^{63}H$ are 0.7135 and 0.7129, respectively.

The large number of individual values used in the determination of the B_v constants makes $(B_e{}^i/B_e)^{\frac{1}{2}}$ for the normal state the most reliable value of ρ determined from the present work. This value for the molecules $Cu^{65}D$ and $Cu^{63}D$ is 0.999503. The calculated values of the probable errors of the values of B_e as given by Eqs. (4) and (5) are 0.000071 and 0.000018, respectively. A consideration of the probable error of the individual values of B_v would indicate that these values are too small. It is believed, however, that $(B_e{}^i/B_e)^{\frac{1}{2}}$ for $Cu^{65}D$ to $Cu^{63}D$ can have a probable error no greater than 0.00006. The value of this ratio for $Cu^{63}D$ to $Cu^{63}H$ for the $^1\Sigma$ state is 0.7131.

In conclusion the author wishes to express his sincere thanks to the Physics Department of The Johns Hopkins University for the use of the spectrograph on which the plates were taken.

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[9] J. L. Dunham, Phys. Rev. **41**, 721 (1932).

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The Temperature Variation of the Elastic Moduli of NaCl, KCl and MgO

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The principal adiabatic and isothermal elastic moduli and elastic constants of NaCl, KCl and MgO have been measured over the temperature ranges 80°K to 270°K (Balamuth and Rose) and 270°K to 480°K for NaCl; 80°K to 280°K for KCl; and 80°K to 560°K for MgO. The Debye characteristic temperatures of these substances at 80°K, calculated from the adiabatic elastic constants, are 320°K, 246°K and 946°K, respectively. The variation of the isothermal c_{11} and c_{44} with temperature is described very accurately by the formula: $(c_{ii})_u=(c_{ii})_0 \exp(-A_{ii}F_{ii}(u))$, where $u=(T/\theta)$, T is the absolute temperature and θ the characteristic temperature, $(c_{ii})_0$ denotes the value at absolute zero, A_{ii} is a constant of the material, and $F_{ii}(u)$ is a function of u which assumes the value unity when $u=1$, *and which is the same for all three substances.* Experimental values of $F_{11}(u)$ and $F_{44}(u)$ are given for values of u between 0.1 and 1.5. The isothermal c_{12} does not vary with temperature, over the specified ranges, by more than the error of measurement.

The temperature variations of the three principal elastic constants of sodium chloride between 80°K and 270°K have been reported in this journal by Balamuth[1] and Rose.[2] These investigators perfected their experimental method for use only at temperatures below that of melting ice. The object of the present research is, in the first instance, to extend the scope of the method and the observations on sodium chloride to higher temperatures. In addition, observations

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[1] Balamuth, Phys. Rev. **45**, 715 (1934).

[2] Rose, Phys. Rev. **49**, 50 (1936).

have been made on single crystals of potassium chloride and, over the extended temperature range, of magnesium oxide, both of which have the same lattice structure as sodium chloride. A scrutiny of the data reveals a remarkable correlation between the thermo-elastic properties of the three substances, of such a sort, in fact, as to suggest that it is characteristic at least of the lattice type. Apart from its theoretical interest, this correlation permits the calculation of any one of the elastic constants at a given temperature when its value at two other temperatures is known, and so yields plausible values for the constants at absolute zero.

EXPERIMENTAL METHOD

The experimental method, which is described in detail in the papers of Balamuth and Rose heretofore cited, utilizes the properties of a separately excited composite piezoelectric oscillator constructed by cementing a suitably cut, right circular cylinder of crystalline quartz to one end of a cylinder of specimen material whose cross section is the same as that of the quartz. The diameter of the cylinders is about 4 mm and their lengths vary between 1 cm and 5 cm. The fundamental frequency of free longitudinal or torsional vibration of this system, hereafter called the "resonance frequency," is measured by observing the variation of the electrical impedance of the piezoelectric oscillator with the frequency of the applied voltage, and the Young's or the rigidity modulus of the specimen material for the direction of the cylinder axis is calculated from this and other readily measurable quantities.

The present research is concerned with only two directions in a cubic lattice, namely, (0,0,1) and (0,1,1). Accordingly, the Young's modulus, G, and the rigidity modulus, μ, are related to the principal moduli of the crystal by the formulae,[3]

$$\begin{aligned} 1/G&=s_{11}, \\ 1/G'&=s_{11}'=\tfrac{1}{2}(s_{11}+s_{12}+\tfrac{1}{2}s_{44}), \\ 1/\mu&=s_{44}, \\ 1/\mu'&=s_{11}-s_{12}+\tfrac{1}{2}s_{44}, \end{aligned} \tag{1}$$

in which the unprimed and the primed quantities refer, respectively, to the (0,0,1) and the (0,1,1) directions. It follows from these equations that

$$s_{12}=1/G'-1/2\mu', \tag{2}$$

or

$$=1/G+1/2\mu-1/\mu'. \tag{3}$$

The elastic moduli and elastic constants are related by the formulae,

$$\begin{aligned} c_{11}&=(s_{11}+s_{12})/(s_{11}-s_{12})(s_{11}+2s_{12}), \\ c_{12}&=-s_{12}/(s_{11}-s_{12})(s_{11}+2s_{12}), \\ c_{44}&=1/s_{44}\,. \end{aligned} \tag{4}$$

Lastly, the compressibility, κ, is given by the formulae,

$$\begin{aligned} \kappa&=3(s_{11}+2s_{12}), \\ &=3/(c_{11}+2c_{12}). \end{aligned} \tag{5}$$

The method yields immediately the adiabatic moduli and constants. These are related to the isothermal quantities by the formulae,[4]

$$\begin{aligned} (s_{11})_{\text{ad.}}-(s_{11})_{\text{is.}}&=(s_{12})_{\text{ad.}}-(s_{12})_{\text{is.}}=-T\alpha^2/\rho c_p, \\ (c_{11})_{\text{ad.}}-(c_{11})_{\text{is.}}&=(c_{12})_{\text{ad.}}-(c_{12})_{\text{is.}}, \\ &=T\alpha^2/(\rho c_p)(s_{11}+2s_{12})^2, \end{aligned} \tag{6}$$

where T is the absolute temperature, α is the coefficient of linear expansion, c_p is the specific heat at constant pressure, and ρ is the density. The adiabatic and isothermal s_{44} and c_{44} are the same.

At temperatures below 273°K the present experimental method is entirely the same as that of Rose, except that the cryostatic bath liquid is petroleum ether cooled by liquid nitrogen.

The initial problem which arises when an attempt is made to extend the method to higher temperatures is the discovery of a satisfactory adhesive with which to cement the quartz to the specimen cylinder. That now in use is a thin paste of sodium silicate solution and talcum powder. The cylinders are cemented together under a pressure of 4.5 kg/cm², and the join is baked under this pressure for 15 minutes in air at a temperature of 150°C. Great care must be exercised to ensure that a minimum amount of cement is left adhering to the sides of the oscillator. The effect of such excess material is to load the oscillator, so that the entire resonance frequency *vs.* temperature curve is depressed parallel to itself by a few tenths of a percent. This shift can be, and is, measured and corrected for by adopting as the correct resonance frequency at 273.1°K that which is observed when

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[3] Voigt, *Lehrbuch der Kristallphysik*, p. 739.

[4] Voigt, reference 3, p. 789.

the adhesive is a very thin film of para rubber dissolved in vaseline, and the oscillator is immersed in melting ice.

Numerous tests were made of the satisfactory performance of the talcum-silicate cement. Perhaps the most significant of these is the following: Two cylinders of quartz were chosen whose resonance frequencies for the *torsional* mode of vibration were, respectively, 53 kc and 69 kc. Separate frequency *vs.* temperature observations were made on these up to a temperature of 560°K. The two were then cemented together with their optic axes at right angles (to increase the differential thermal expansion at the interface), and the resonance frequency of the composite oscillator, driven by one of the cylinders, was measured over the same temperature range. The formulae of the method then yielded the resonance frequency of the other cylinder, which was compared with the correct value previously ascertained. The two values agreed to 0.13 percent, without systematic variation. Now Balamuth has pointed out that the effect of the cement is negligible *provided* the lengths of the quartz and specimen cylinders are so adjusted that the fundamental frequencies of free vibration of, respectively, the cylinders separately and the composite oscillator are the same to better than ten percent. Furthermore, Rose observed that the effect of the cement on the resonance frequency is greater for the torsional mode of vibration than for the longitudinal. Hence the foregoing test may be regarded as exceptionally severe.

A further stringent internal check on the measurements made on whatever substance is afforded by calculating s_{12} from measured values of G' and μ' with Eq. (2), and again from measured values of G, μ and μ' with Eq. (3). This test is applied in all cases.

Between 273°K and 373°K the cryostatic bath liquid is a mixture of 75 percent glycerin and 25 percent water; above 373°K it is Socony Super Hecla Mineral Oil. The baths are electrically heated, and the temperature is stabilized to 0.1°K by manual control of the current in an auxiliary heating coil of low heat capacity.

When the thermal expansion coefficient of a fragile specimen is greater than that of quartz, it is not necessary to use at higher temperatures the triple oscillator devised by Rose to prevent splitting of the specimen cylinder caused by differential thermal expansion at the interface. As the oscillator is heated above the temperature at which it was prepared, this differential expansion compresses the specimen instead of rending it.

RESULTS

Sodium chloride

The specimen cylinders of sodium chloride were cut from the same large crystal as were those used by Balamuth and Rose. The elastic moduli and elastic constants of this material over the temperature range 270°K to 480°K are given in Table I. These data fit smoothly onto the observations between 80°K and 270°K which appear in Rose's paper.[2] They represent the mean of measurements of G, μ, and μ', each on four different specimens. The average deviation in the calculated values of s_{11}, s_{44} and s_{44}' is 0.2 percent. The values of s_{12} were calculated with Eq. (3), and are reliable to about two percent. The elastic constants were calculated from the moduli with Eqs. (4), and the isothermal quantities with Eqs. (6).

Between 273°K and 414°K the formula used in computing the density was[5]

$$\rho = 2.1680(1 - 10.6 \times 10^{-5} t - 0.3 \times 10^{-6} t^2).$$

Above 414°K the coefficient of cubical expansion is, simply, 14×10^{-5}. Values of c_p were taken from the *International Critical Tables* throughout.

Potassium chloride

The specimen cylinders of this material were cut from a crystal obtained through the courtesy of Dr. Paul F. Kerr of the Department of Mineralogy at Columbia University. According to his records, the source was Kolusz, Silesia. Minute bubbles appeared in this crystal, but otherwise it was clear.

The elastic constants and elastic moduli of potassium chloride over the temperature range 80°K to 280°K are given in Table II. Because of the scarcity of the available material, it was possible to measure G, μ, and μ' each on only two specimens. The values of these quantities

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[5] Saini, Helv. Phys. Acta **7**, 494 (1934).

TABLE I. *The elastic moduli and constants of NaCl.*

T°K	ADIABATIC MODULI ×10^{13} (cm^2/dynes)			ADIABATIC CONSTANTS ×10^{-11} (dyne/cm^2)			ISO-THERMAL c_{11}×10^{-11}
	S_{11}	$-S_{12}$	S_{44}	C_{11}	C_{12}	C_{44}	
270	22.08	4.49	78.26	5.06	1.30	1.278	4.95
280	22.29	4.58	78.47	5.03	1.30	1.274	4.90
290	22.51	4.67	78.67	4.99	1.31	1.271	4.86
300	22.73	4.76	78.87	4.96	1.31	1.268	4.82
310	22.96	4.85	79.08	4.92	1.32	1.265	4.78
320	23.19	4.94	79.29	4.88	1.32	1.261	4.74
330	23.42	5.03	79.50	4.84	1.32	1.258	4.69
340	23.66	5.13	79.71	4.81	1.33	1.255	4.65
350	23.90	5.23	79.92	4.77	1.33	1.251	4.61
360	24.15	5.33	80.14	4.73	1.34	1.248	4.57
370	24.40	5.43	80.36	4.69	1.34	1.244	4.53
380	24.66	5.54	80.58	4.66	1.35	1.241	4.49
390	24.93	5.65	80.80	4.62	1.35	1.238	4.44
400	25.20	5.76	81.02	4.58	1.36	1.234	4.40
410	25.48	5.87	81.26	4.55	1.36	1.231	4.36
420	25.78	5.99	81.50	4.51	1.36	1.227	4.32
430	26.08	6.11	81.76	4.47	1.37	1.223	4.28
440	26.38	6.23	82.02	4.43	1.37	1.219	4.23
450	26.69	6.36	82.29	4.39	1.38	1.215	4.19
460	27.01	6.49	82.57	4.36	1.38	1.211	4.15
470	27.33	6.62	82.85	4.32	1.39	1.207	4.11
480	27.66	6.75	83.13	4.28	1.39	1.203	4.07

The isothermal c_{12} is 1.17×10^{11} over the entire temperature range.

TABLE II. *The elastic moduli and constants of KCl.*

T°K	ADIABATIC MODULI ×10^{13} (cm^2/dynes)			ADIABATIC CONSTANTS ×10^{-11} (dyne/cm^2)		ISO-THERMAL c_{11}×10^{-11}
	S_{11}	$-S_{12}$	S_{44}	C_{11}	C_{44}	
80	21.36	2.3	150.7	4.81	0.664	4.81
90	21.50	2.4	150.9	4.79	.663	4.78
130	22.18	2.6	151.8	4.65	.659	4.64
140	22.37	2.7	152.1	4.62	.658	4.61
150	22.57	2.7	152.4	4.59	.656	4.57
160	22.78	2.7	152.7	4.55	.655	4.53
170	22.99	2.8	153.1	4.52	.653	4.49
180	23.20	2.8	153.5	4.48	.651	4.45
190	23.42	2.8	153.9	4.44	.650	4.41
200	23.64	2.9	154.3	4.40	.648	4.37
210	23.86	2.9	154.7	4.36	.647	4.33
220	24.09	3.0	155.1	4.33	.645	4.29
230	24.31	3.0	155.5	4.29	.643	4.25
240	24.54	3.1	156.0	4.24	.641	4.21
250	24.77	3.2	156.4	4.20	.639	4.17
260	25.00	3.2	156.9	4.16	.637	4.13
270	25.23	3.3	157.3	4.12	.636	4.09
280	25.46	3.3	157.8	4.08	.634	4.05

The adiabatic and isothermal c_{12} are 0.6×10^{11} over the entire temperature range.

differed between specimens by the amounts 0.5 percent, 0.3 percent, and 0.5 percent respectively. Accordingly the value of s_{12} is reliably to only about five percent. The formula used in computing the density was[6]

$$\rho = 1.9920(1 - 10.6t - 0.4 \times 10^{-7} t^2).$$

Steinebach[7] found the ratios of the value of s_{11} and s_{44} at room temperature to those at 88°K to be 1.195±0.02 and 1.05±0.02, respectively. Those calculated from Table II are 1.205 and 1.044, respectively. Values of the isothermal moduli at 30°C reported by different observers are given in Table III.

Slater[8] found the isothermal compressibility at 30°C to be 56.3×10^{-13}, as compared with the present extrapolated value, 56.7×10^{-13}. It should be noted that the crystals examined by Bridgman and Slater were grown artificially, while those used by Voigt and the writer were natural.

Magnesium oxide

The specimen cylinders of magnesium oxide were cut from large, clear, well formed crystals supplied by the Norton Company, of Worcester, Mass. Because of its hardness this material cannot be turned on a lathe, and these cylinders, like the quartz, were formed by hand grinding in a series of split cylindrical laps. Two part oscillators were used over the entire temperature range.

The elastic constants and elastic moduli of magnesium oxide over the temperature range 80°K to 560°K are given in Table IV. They represent the mean of measurements of G and μ on four specimens, and μ' on three. The average deviations were 0.15 percent, 0.15 percent, and 0.1 percent. Accordingly the value of s_{12} is reliable to about 2 percent.

The density of magnesium oxide at 25°C was measured with a specific gravity bottle. The mean of observations on three sets of specimens was

$$\rho_{25} = 3.5761 \pm 0.0005.$$

The density at other temperatures was computed from this value, with the coefficient of thermal expansion reported by Durand[9] and Austin.[10]

Madlung and Fuchs[11] found the isothermal

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[6] Henglein, Zeits. f. physik. Chemie **115**, 91 (1925).

[7] Steinebach, Zeits. f. Physik **33**, 674 (1925).

[8] Slater, Phys. Rev. **23**, 488 (1924).

[9] Durand, Physics **7**, 297 (1936).

[10] Austin, J. Am. Ceramic Soc. **14**, 795 (1931). The data on MgO are reprinted in reference 9.

[11] Madlung and Fuchs, Ann. d. Physik **65**, 289 (1921).

TABLE III. *Values of isothermal moduli at 30°C reported by different observers.*

Observer	Elastic moduli $\times 10^{13}$ s_{11}	s_{12}	s_{44}
Durand	26.26	−3.7	158.9
Voigt[1]	27.37	−1.4	156.0
Bridgman[2]	29.4	−5.3	127

[1] Voigt, reference 3, p. 741.

[2] Bridgman, Proc. Am. Acad. 64, 19 (1929).

compressibility of their best specimens of magnesium oxide to be 6.4×10^{-13}. The present value is 6.54×10^{-13}.

It is of interest to note that the behavior of this substance is widely at variance with Cauchy's relation, $c_{12} = c_{44}$, between the isothermal elastic constants.

CALCULATION OF THE DEBYE CHARACTERISTIC TEMPERATURE

The Debye characteristic temperatures of the several substances were calculated from the adiabatic elastic constants at 80°K by an obvious modification of the method of Hopf and Lechner.[12] Thus, for magnesium oxide, $\theta_{80} = 946$°K; for sodium chloride, $\theta_{80} = 320$°K; and for potassium chloride, $\theta_{80} = 246$°K. These values agree with those obtained from specific heat measurements at low temperatures within the limits of error of the latter.

CORRELATION OF THE THERMOELASTIC PROPERTIES

The temperature variation of the isothermal c_{11} and c_{44} for NaCl, KCl and MgO can be expressed by the formula

$$(c_{ii})_u = (c_{ii})_0 \exp\left(-A_{ii}F_{ii}(u)\right), \tag{7}$$

where $u = T/\theta$, T is the absolute temperature and θ the characteristic temperature, $(c_{ii})_0$ denotes the value at absolute zero, A_{ii} is a constant of the material, and $F_{ii}(u)$ is a function of (u) which assumes the value unity when $u = 1$, *and which is the same for all three substances.*

The forms of the functions $F_{11}(u)$ and $F_{44}(u)$ are indicated in Figs. 1 and 2, in which the observed values of $F_{ii}(u) = (1/A_{ii}) \ln\left((c_{ii})_0/(c_{ii})_u\right)$ for NaCl, KCl and MgO are plotted as ordinates and corresponding values of u as abscissae. Numerical values of $F_{11}(u)$ and $F_{44}(u)$ are given in Table V. It will be seen that the two are nearly the same, but the difference between them is probably real, for, apart from other considerations, there seems to be no way of ascribing it to systematic error of observation.

The isothermal c_{12} for all three substances is constant, within the accuracy of the measurements, over the entire temperature range investigated, and it can be argued that this

TABLE IV. *The elastic moduli and constants of MgO.*

T°K	ADIABATIC MODULI $\times 10^{13}$ (cm²/dynes) S_{11}	$-S_{12}$	S_{44}	ADIABATIC CONSTANTS $\times 10^{-11}$ (dyne/cm²) C_{11}	C_{12}	C_{44}	ISOTHERMAL $c_{11} \times 10^{-11}$
80	3.839	0.855	6.380	29.87	8.56	15.673	29.86
90	3.841	.857	6.381	29.87	8.58	15.671	29.86
150	3.872	.872	6.393	29.72	8.64	15.643	29.67
160	3.880	.875	6.396	29.67	8.65	15.635	29.62
170	3.888	.878	6.399	29.63	8.66	15.627	29.56
180	3.897	.882	6.403	29.58	8.67	15.618	29.51
190	3.906	.886	6.407	29.53	8.68	15.609	29.45
200	3.916	.890	6.412	29.48	8.68	15.599	29.39
210	3.926	.894	6.417	29.42	8.69	15.589	29.33
220	3.936	.899	6.421	29.37	8.70	15.578	29.27
230	3.946	.903	6.426	29.32	8.71	15.566	29.21
240	3.957	.908	6.431	29.27	8.72	15.554	29.14
250	3.968	.913	6.436	29.21	8.73	15.542	29.08
260	3.979	.917	6.442	29.16	8.74	15.529	29.02
270	3.991	.922	6.447	29.10	8.74	15.516	28.95
280	4.002	.927	6.452	29.05	8.75	15.503	28.89
290	4.013	.932	6.457	28.99	8.76	15.490	28.82
300	4.025	.937	6.463	28.93	8.77	15.477	28.75
310	4.037	.942	6.468	28.88	8.78	15.464	28.69
320	4.048	.947	6.474	28.82	8.79	15.451	28.62
330	4.060	.952	6.479	28.76	8.80	15.438	28.55
340	4.072	.957	6.485	28.70	8.80	15.424	28.48
350	4.084	.962	6.490	28.64	8.81	15.410	28.41
360	4.096	.967	6.496	28.58	8.82	15.396	28.34
370	4.109	.972	6.502	28.52	8.83	15.382	28.27
380	4.121	.977	6.508	28.46	8.84	15.368	28.20
390	4.134	.983	6.514	28.40	8.85	15.354	28.13
400	4.147	.988	6.520	28.33	8.85	15.339	28.06
410	4.159	.993	6.526	28.27	8.86	15.324	27.99
420	4.173	.999	6.532	28.21	8.87	15.309	27.91
430	4.187	1.004	6.538	28.14	8.88	15.294	27.84
440	4.200	1.010	6.545	28.08	8.88	15.279	27.76
450	4.214	1.015	6.551	28.01	8.89	15.264	27.69
460	4.229	1.021	6.558	27.95	8.90	15.249	27.61
470	4.243	1.027	6.564	27.88	8.91	15.234	27.53
480	4.258	1.033	6.571	27.81	8.91	15.219	27.46
490	4.273	1.038	6.578	27.74	8.92	15.204	27.38
500	4.289	1.044	6.584	27.67	8.93	15.188	27.30
510	4.304	1.050	6.591	27.60	8.93	15.172	27.22
520	4.320	1.057	6.598	27.54	8.94	15.156	27.14
530	4.335	1.063	6.605	27.47	8.94	15.140	27.06
540	4.351	1.070	6.612	27.40	8.95	15.124	26.98
550	4.367	1.077	6.619	27.32	8.96	15.108	26.90
560	4.383	1.085	6.626	27.25	8.96	15.092	26.82

The isothermal c_{12} is $8.57 \pm 0.04 \times 10^{11}$ over the entire temperature range.

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[12] A description of this method is given by Born, Enc. d. Math. Wiss., Vol. 3, p. 649. The modification consists in identifying $(z+\zeta)^{-3/2}$ with a fifth degree polynomial over the actual range of values assumed by z, instead of over the range $z = 0$ to $z = 1$.

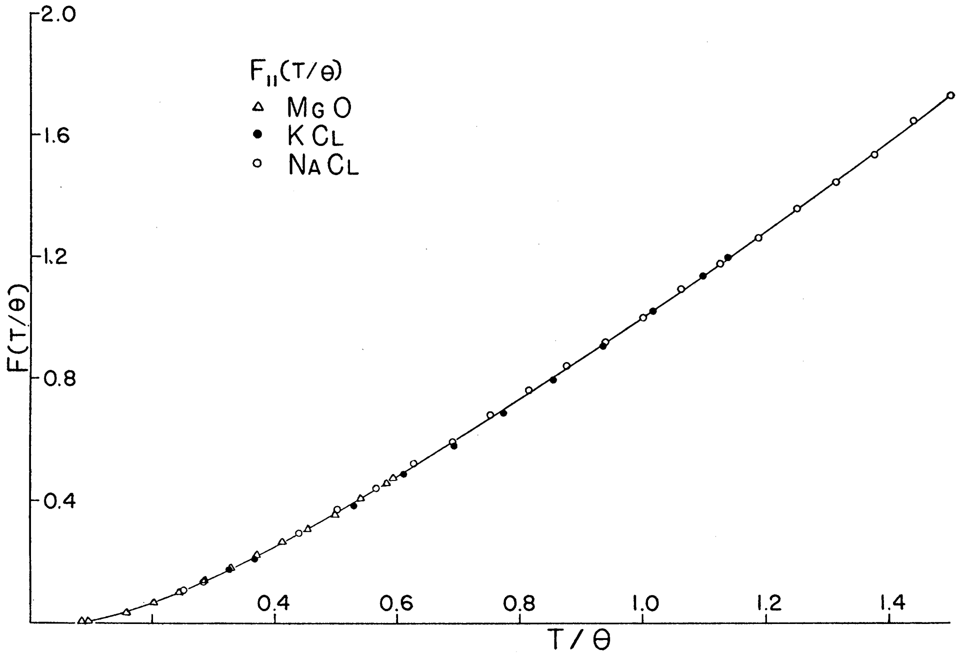

FIG. 1. The characteristic function for c_{11}.

constant value persists clear up to the melting point of the crystal. For at this temperature $c_{11}=c_{12}$, and if the present data are extrapolated, on the assumptions that c_{11} varies linearly with the temperature and c_{12} not at all, to the temperature at which they become equal, one obtains for the melting point of NaCl, 1170°K; of KCl, 1145°K; and of MgO, 2850°K. The observed values are, respectively, 1077°K, 1045°K, and 2800°K to 3100°K.

In order to represent the data as in Figs. 1 and 2 it is necessary to know the values of $(c_{11})_0$ and $(c_{44})_0$ for one of the substances only. Magnesium oxide is chosen, since, as will be evident from a scrutiny of Fig. 3, in this case extremely accurate values can be obtained by extrapolation of the observations above 80°K. The elastic constants of the other substances at absolute zero are calculated from these numbers and the values of the elastic constants which correspond to any two other values of u for which data on magnesium oxide are available. Thus, let $R_1(u', u'')$ denote the quantity $\ln((c_{ii})_{u'}/(c_{ii})_{u''})$ for the substance (1), and $R_2(u', u'')$ the same quantity for the substance (2). Then it follows immediately from Eq. (7) that

$$R_1(0, u'')/R_2(0, u'')=R_1(u', u'')/R_2(u', u'').$$

A_{ii} is calculated with the formula

$$A_{ii}=\ln((c_{ii})_0/(c_{ii})_{u=1})$$

provided $(c_{ii})_{u=1}$ is known, and $F_{ii}(u)$ is then obtained from Eq. (7). Since $(c_{ii})_{u=1}$ for MgO is not known, the A_{ii}'s for this substance are calculated by forcing the single observations at

TABLE V. *Values of the characteristic functions.*

u	$F_{11}(u)$	$F_{44}(u)$	u	$F_{11}(u)$	$F_{44}(u)$
0.1	0.007	0.007	0.9	0.865	0.845
.2	.062	.052	1.0	1.000	1.000
.3	.148	.123	1.1	1.136	1.146
.4	.247	.210	1.2	1.280	1.296
.5	.362	.311	1.3	1.427	1.447
.6	.480	.424	1.4	1.577	1.597
.7	.605	.553	1.5	1.730	1.750
.8	.735	.695			

TABLE VI. *Values of $(c_{ii})_0$ and A_{ii} for the three crystals.*

	$(c_{11})_0\times10^{-11}$	$(c_{44})_0\times10^{-11}$	A_{11}	A_{44}
MgO	29.89	15.679	0.230	0.0920
NaCl	5.85	1.339	.210	.0601
KCl	4.95	.669	.168	.0440

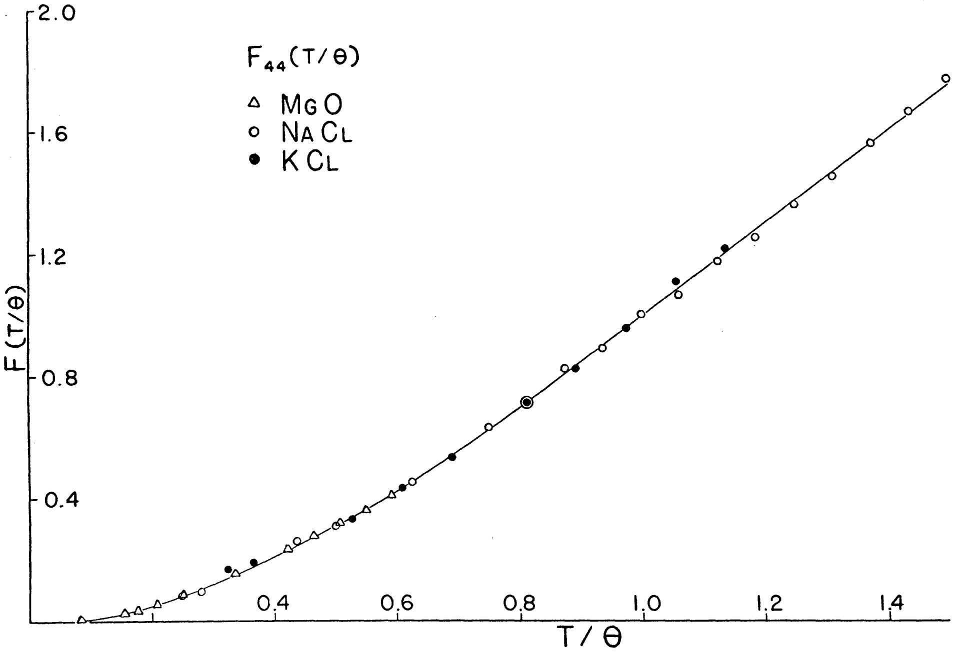

FIG. 2. The characteristic function for c_{44}.

$u=0.592(T=560°\mathrm{K})$ to fit the combined $F_{ii}(u)$ curves of NaCl and KCl. It then appears that the $F_{ii}(u)$ curves of MgO are identical with those of the other two substances. Values of $(c_{ii})_0$ and A_{ii} for the three crystals are given in Table VI.

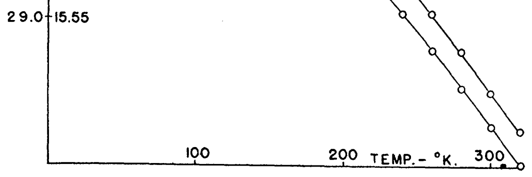

FIG. 3. Showing the method for obtaining the elastic constants of MgO at absolute zero.

It should be noted that a very small variation in the measured value of c_{ii} results in a large departure of the calculated value of $F_{ii}(u)$ from the curves of Figs. 1 and 2. The largest departure there shown indicates a variation of at most 0.2 percent in c_{ii}, and arises from the uncertainty in the terminal digit of the tabulated value. Conversely, the temperature variation of the elastic constants can be calculated from Eq. (7) with greater precision than it can be measured.

In conclusion, the writer desires to acknowledge his indebtedness to Dr. Paul F. Kerr, for the gift of the potassium chloride crystal; to Mr. Raymond R. Ridgway of the Norton Company, who was instrumental in securing the magnesium oxide crystals; and to Dr. S. L. Quimby, at whose suggestion the present research was undertaken, and who followed its progress with helpful counsel and encouragement.