Concerning the Shape of the Compton Lines
(From a Letter to A. H. Compton)

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The structure of the spectrum of the modified x-radiation scattered by bound electrons is discussed. It is shown that the well-known continuous Compton band must have a definite limit on the short wave side (i.e., toward the unmodified line). Between this limit and the unmodified Rayleigh line, theory predicts the existence of a spectrum of discrete lines, which are appropriately classed as Raman lines. They are composed of those scattered photons which have excited, but not ionized the scattering atom. The conditions under which it might be possible to observe these Raman lines are discussed. There is no abrupt change in the specific intensity (per unit wave-length interval) as the limit of the discrete spectrum is crossed. It is suggested that the lines reported by B. B. Ray in 1930 may be interpreted as residue of Compton bands cut off by the foresaid limit.

While the Compton radiation from free electrons is monochromatically sharp, that from bound electrons consists of a continuous band. The shape of this band in the case of hydrogen can be calculated explicitly on the basis of certain simplifying assumptions.[1] If $\Delta\lambda$ is the distance in wave-length from the center of the band, Δl the value of $\Delta\lambda$ at half-maximum, and $x=\Delta\lambda/\Delta l$, then the specific intensity is

$$J=J_{\max}/[1+(2^{\frac{1}{3}}-1)x^2]^3. \quad (1)$$

For elements other than hydrogen, one must use methods of numerical calculation.[2]

The point to which I would direct attention is that the Compton band should have a definite limit, g, on the short wave side, i.e., toward the unmodified Rayleigh line. The existence of this limit follows directly from the energy equation

$$h(\nu-\nu')=W+W_0 \quad (2)$$

if one sets $W=0$. (W=kinetic energy of the recoil electron, W_0 its binding energy, ν=frequency of the unmodified Rayleigh line, ν'=frequency of an individual point in the modified Compton band.) The distance of this limit from the Rayleigh line, $\Delta\lambda_g$, is thus found to be

$$\Delta\lambda_g=\lambda^2 W_0/hc. \quad (3)$$

We compare this with the distance $\Delta\lambda_c$ of the center of the Compton band from the Rayleigh line:

$$\Delta\lambda_c=2(h/mc)\sin^2(\theta/2) \quad (4)$$

and obtain from (3) and (4) with $W_0=e^2/2a$, $a=\hbar^2/me^2$ for hydrogen:

$$\Delta\lambda_c/\Delta\lambda_g=((4\pi a/\lambda)\sin(\theta/2))^2. \quad (5)$$

If we disregard extremely soft radiation, then $\lambda<4\pi a$ ($4\pi a\sim 6$A for hydrogen). If one chooses $\theta\sim 180°$, then

$$\Delta\lambda_g<\Delta\lambda_c,$$

i.e., our limit g cuts *a small piece from the short wave side* of the Compton band (Fig. 1). But if one makes

$$\sin(\theta/2)=\lambda/4\pi a, \quad (6)$$

e.g., $\theta=13\frac{1}{2}°$ for $\lambda=1.5$A (Cu $K\alpha$) and $a=\frac{1}{2}$A, then the limit cuts the Compton band *precisely in its center* (Fig. 2). If we proceed further in this direction, for $\theta\sim 0$, the Compton band will be cut on the other side of its center and the *remaining part may*, in extreme cases, *be very small* (Fig. 3).

What becomes of the intensity of the Compton band that is thus suppressed? We know that the combined Rayleigh and modified scattering is equal in intensity to the classical Thomson scattering. We know further, that in calculating the total incoherent scattering we must sum over all energy values of the *complete* system of characteristic functions. The *discrete* energy levels belong to this system as well as do the *continuous* ones. While the transition to a continuous level signified the production of a recoil electron, a transition to a discrete level represents the case in which the electron remains *bound in an excited state*. Since, in the case of hydrogen the nth excited state has the energy $W_n=-W_0/n^2$, the energy Eq. (2) must be replaced by

[1] Fritz Schnaidt, Dissertation, Munich, Ann. d. Physik **21**, 89 (1934).
[2] G. Burkhardt, Dissertation, Munich, to appear in Ann. d. Physik (1935).

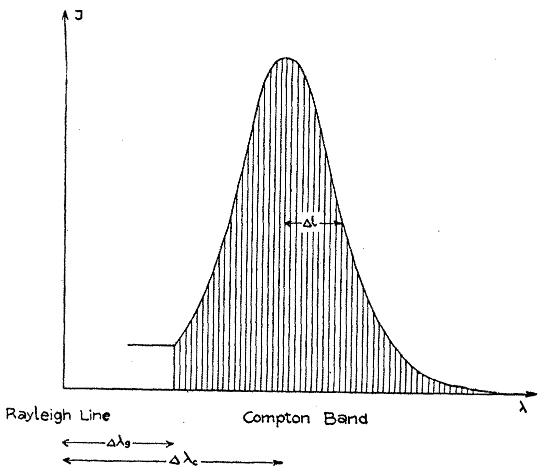

FIG. 1. Showing limit of Compton line on short wave-length side for $\theta = 180°$.

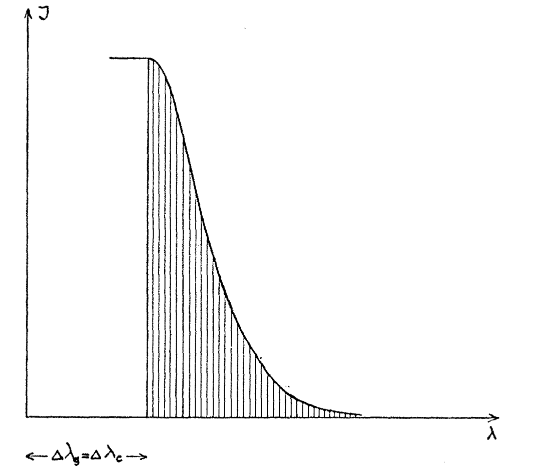

FIG. 2. Showing limit of Compton line on short wave-length side for $\theta = 13.5°$.

$$h\nu_n' = h\nu - W_0(1 - 1/n^2). \quad (7)$$

This ν_n' is greater than the ν' at our limit g, which corresponds to $W = 0$. The corresponding wave-lengths of the radiation are thus less than the wave-length λ_g of this limit, and approach it for $n \to \infty$. The continuous Compton band is thus extended beyond g as a series of discrete lines. In Figs. 1, 2, and 3, the unshaded extensions of the Compton band toward the left indicate that the intensity of this series of lines also joins continuously on to the intensity of the band, insofar as one distributes the finite intensity of each line over the interval between it and its neighbor. This is quite analogous to the well-known continuity of intensity in passing from the Balmer series to the Balmer continuum.

The discrete lines which have just been described will be called the *Raman spectrum* of hydrogen. We can therefore say: *The continuous Compton band is extended on the short wave side by a discrete Raman spectrum.*[3]

The question arises whether this Raman radiation can be detected experimentally, and what conditions will be favorable for this purpose. From Fig. 2 one will suspect: If the Compton band is cut through the center, then the Raman spectrum must—because of the continuity of intensity—be particularly strong. The corresponding angle of scattering, we have seen in a special case to be $\theta \pm 13.5°$. But it is clear that with such small angles of scattering, the resolution of the Rayleigh line and Compton band, and *a fortiori* the resolution of the Rayleigh and Raman lines, will be difficult. One must recall that by Eq. (4) the center of the Compton band moves toward the Rayleigh line as θ diminishes. In Fig. 4 the position and *line* intensity of the Rayleigh and Raman lines are drawn approximately to scale (i.e., heights of the Raman lines are to the

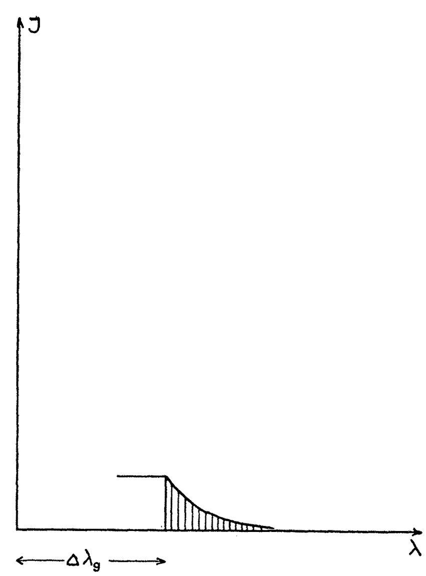

FIG. 3. Showing limit of Compton line on short wave-length side for $\theta = 0°$.

[3] Compare, Compton and Allison, *X-Rays in Theory and Experiment* (Van Nostrand, 1935), p. 239.

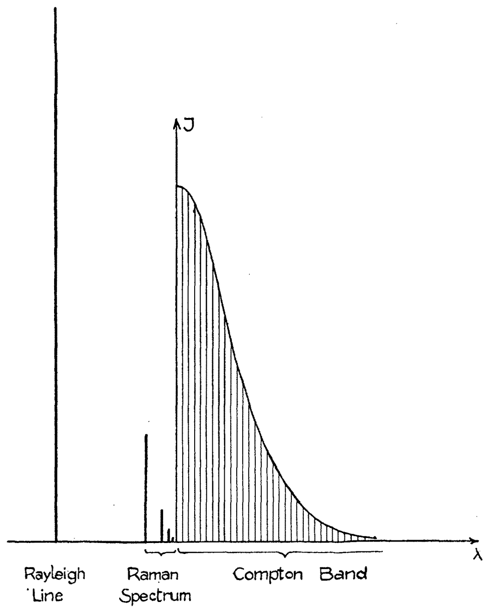

FIG. 4. Showing relative intensities of Rayleigh and Raman lines.

height of the Rayleigh line in the ratio of their intensities; the *specific* intensity of the Compton band is plotted on the scale of the previous figures, and is not directly comparable to the lines). The first Raman line (transition $n=1\rightarrow 2$) is more intense than all others; in the most favorable case, it has $\frac{1}{5}$ the intensity of the Rayleigh line.[1] This first Raman line is shown by Eq. (7), to be separated from the Rayleigh line by $\frac{3}{4}$ of the distance between the latter and the limit g. The succeeding Raman lines diminish rapidly in intensity (proportionally with $1/n^3$). With sufficient resolving power, the first line might perhaps be resolved.

This conclusion concerning the most favorable angle θ, which has been reached from a consideration of Fig. 2, is confirmed by the more exact calculations of Schnaidt.[1] Our present considerations and those of Schnaidt are, strictly speaking, valid only for hydrogen. They may be extended qualitatively to other atoms if the altered structure of the Raman spectrum is appropriately taken into consideration.

In conclusion, I would call attention to observations of B. B. Ray,[4] which have not, however, been confirmed by others.[5] They concern the passage of Röntgen rays through very thin layers, e.g., of carbon, which resulted in the formation of a "very weak, broad, diffuse" line on the long wave-length side of the primary line, which "appeared to have a more or less definite edge on its short wave side." This description reminds one of our Fig. 3, which was drawn to represent very small angles of scattering. The Compton band is reduced to a very small residue by the limit g. In the case of carbon this residue would correspond to the K electrons, and the limit g to the K limit (the L electrons, being much less firmly bound, would contribute only to the central part of the Compton band, which, in turn is cut off by our limit g). This coincides with Ray's observation that his line was separated from the primary one by the wave number of carbon $K\alpha$, which differs very little from that of the carbon K limit. The corresponding separation was also observed when nitrogen and oxygen were used as scattering substance. We would therefore like to interpret these lines as the residues of the K electron contribution to the Compton band (in distinction to Mr. Ray, who speaks of a "partial absorption of the Röntgen rays"). No mention of the Raman lines is to be found in Ray's paper. These should lie between the Compton band and the Rayleigh line and it is probable that the conditions were not favorable for their observation.

We do not consider it to be impossible that future repetitions of these experiments may lead to a positive result, if they are performed with reference to the theoretical views given here. In general terms, the object of such investigations should be to exhibit the anomalous forms of the Compton band (Figs. 2 and 3) and its extension as a Raman spectrum.

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[4] B. B. Ray, Zeits. f. Physik **66**, 261 (1930) and subsequent notes of R. C. Majumdar, S. Bargava and J. B. Muckerjee, Nature **127**, 92, 273 (1931).

[5] Compare, e.g., J. M. Cork, Phys. Rev. **37**, 1555 (1931), O'Leary, ibid. p. 873.