Relativity and Nuclear Forces

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(Received June 19, 1936)

To obtain an estimate of the relativity correction in nuclear problems, the Klein-Gordon theory is applied to the common forms of interaction between the neutron and the proton. Because there is an appreciable region of space in which the zero-point energy is enormous, the relativity effect is larger than would be supposed by mere inspection of binding energies. For the deuteron, it amounts to several times the binding energy if the forces have a range around 10^{-13} cm; for ranges about 3×10^{-13} cm it decreases to a few percent of the binding energy.—The calculations were based on two types of assumed interactions: the rectangular potential hole and the error function potential.

ACCORDING to the calculations of Wigner,[1] Massey and Mohr,[2] Feenberg,[3] and others the binding energy of light nuclei can be accounted for with reasonable accuracy by supposing the existence of interactions of various types between the elementary particles. It is common to these calculations that they represent the total energies as differences between large potential and slightly smaller kinetic energies, thus involving zero-point energies which are many times as great as the total energy of the nucleus. This circumstance is responsible for the fact that the binding energies are quite insensitive to the form chosen for the interaction; it allows the interaction to be specified very simply by two parameters, of which one is the width and the other the depth of the potential hole. But it also raises a question as to the basic meaning of the assumed interactions, whose precise form is rendered practically unobservable by the large amount of zero-point energy which they generate. Yet, while one might wonder about the ultimate satisfaction which the use of potential functions in connection with nuclear particles is likely to give, attention should be called to another point which is not quite so trivial and a little more definite.

It has to do with the extent to which one may safely neglect relativity corrections in the calculations mentioned. To make the argument precise let us refer to the simplest, the ordinary potential type of interaction, between a proton and a neutron, and represent it first as a potential well with vertical walls, of width a and depth D. The depth of this hole which will produce the correct binding energy of the deuteron (2.1 MEV) is easily calculated with the use of Schrödinger's equation for any given width a. It is also easy to compute, in simple classical fashion, the relative speed, v, of proton and neutron as a function of a. One then obtains the set of values listed in the first column of Table I. It is thus clear that the problem here under discussion invites inquiry.

There exists no rigorous relativistic theory for the problem of several bodies. Even if spins are entirely ignored, the two-body problem as it presents itself here, is incapable of solution. For the purpose of estimating the order of magnitude of the error in a classical calculation it may be adequate, however, to replace the two-body problem by a one-body problem using the *relativistically incorrect* procedure of introducing the reduced mass and relative coordinates. The results thus obtained are not to be trusted numerically, but they lie within a range of uncertainty peculiar to the nonrelativistic calculations. The one-body problem may then be solved by use of a relativistic modification of the Schrödinger equation. Since we do not know how to deal with the spins anyway, the suggested form is that of Klein-Gordon:

$$\nabla^2\psi+(1/\hbar^2c^2)[(E-V)^2-E_0^2]\psi=0. \qquad (1)$$

This, then, takes account of no other relativistic effects than the change of mass with velocity. If a well of width a and depth D is again chosen for V, the solution is of the same form as that of the ordinary Schrödinger equation, the energy

[1] E. Wigner, Phys. Rev. **43**, 252 (1933).
[2] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc **A152**, 693 (1935).
[3] E. Feenberg, Phys. Rev. **47**, 850 (1935).

TABLE I. *Binding energies of the deuteron computed under various conditions.*

$a\times10^{13}$ (cm)	$\frac{v}{c}$	D_{class} (MEV)	$D_{\text{rel.}}$ (MEV)	$D_{\text{class}}-D_{\text{rel.}}$ (MEV)
1	0.71	121.2	108.9	12.3
1.5	.49	58.7	55.8	2.9
2.0	.38	35.7	34.8	0.9
2.5	.31	24.93	24.37	0.56
3.0	.27	18.69	18.40	0.29

being determined by

$$\tan(\kappa a) = -\kappa/k.$$

But κ and k have slightly different meanings:

$$k=\left(\frac{2M}{\hbar^2}W-\frac{W^2}{\hbar^2c^2}\right)^{\frac{1}{2}},$$

$$\kappa=\left[\frac{2M}{\hbar^2}(D-W)+\frac{(D-W)^2}{\hbar^2c^2}\right]^{\frac{1}{2}},$$

where W is the binding energy.

Entries in the second and third columns of Table I show how, for a given value of a and the experimental binding energy of the deuteron, D is modified by the procedure here outlined. It is to be observed that the two values of D differ by more than 5 times the binding energy of the deuteron for $a=10^{-13}$ cm, but only by 13 percent of the binding energy for $a=3\times10^{-13}$ cm.

To compare the differences between the second and third columns of Table I with the binding energy of the deuteron (2.1 MEV) is not an altogether fair test of the theories. One should rather compute the effect of the decrease in D upon W. This was done for a variety of values of a, and it was found that the absolute error in W is about one-half the difference between the two D's.

The potential functions used in the literature (cf. in particular references 1, 2, 3) are not always of the simple type we have chosen. We wish to point out that there is no simple way of relating the results obtained for a rectangular hole to other potential functions. The average kinetic energies for the same binding energy may be widely different for two different potential troughs having approximately equal half-widths.

Feenberg[3] has done considerable work with potential functions of the type—$Ae^{-\alpha r^2}$. He has, in particular, succeeded in fitting the constants A and α to the observed binding energies for the deuteron and the alpha-particle, and suggests the values $A=165\ Mc^2$, $1/\alpha^{\frac{1}{2}}=1.34\times10^{-13}$ cm. It was thought of interest to compute the magnitude of relativity corrections for this case.

A simple method for determining the effect on the binding energy is the following. We write Eq. (1) in the form

$$(H+W+F)\varphi=0. \tag{2}$$

Here $W=Mc^2-E$, the binding energy, and $(H+W)\psi=0$ is the Schrödinger equation. The "perturbing" function F is then seen to be

$$F=-(1/2Mc^2)(W+V)^2. \tag{3}$$

If the integral over this function, weighted by ψ^2, is small, we may apply ordinary perturbation theory to compute the effect of F on W, and we may substitute the empirical value for W in (3). Thus

$$\Delta W=-(1/2Mc^2)\int(W+V)^2\psi^2 d\tau. \tag{4}$$

This expression was evaluated for Feenberg's potential function with the values of A and α cited above, ψ having been found by numerical integration of the Schrödinger equation. The result obtained was $\Delta W=-1.17Mc^2\approx0.6$ MEV. It amounts to an error of 28 percent of the binding energy. While this seems large, it must of course be remembered that a much smaller relative change in the constants of the potential function would annul it.

Relativity effects are evidently not serious as long as the range of the nuclear forces is as large as or larger than 3×10^{-13} cm. It may be that relativity corrections are of greater importance in heavy than in light nuclei.

Unfortunately, an attempt to make a correct relativistic calculation of the binding energies is blocked, chiefly by the following facts:

1. The use of *simultaneous* potentials is inconsistent with the relativistic point of view. For interactions between electrical charges this difficulty can be circumvented by the use of retarded potentials. Can this be done for nuclear constituents? *Formally* it is possible. The Hamiltonian for a group of particles then contains, besides the classical terms, an infinite sequence of terms in negative powers of c, the velocity of

light, that in c^{-1} being absent. But such procedure is, strictly speaking, not permissible. In classical electrodynamics, the retarded potentials which one encounters are solutions of Maxwell's equations; they appear as necessary generalizations of Coulomb's law. In nuclear physics, where the potential functions used hitherto are entirely arbitrary, the use of retarded functions has no meaning whatever.

2. Even if, following the classical analogy, retarded potentials are introduced, no exact calculation can be made. For there will then appear damping terms corresponding to the classical radiation reaction, permitting no stationary solution. The first of these terms is of the order c^{-3}. To avoid them, the sequence must therefore be broken off after the term in c^{-2}. But in that case, the approximation would hardly be satisfactory and the calculation of doubtful utility.

AUGUST 15, 1936 PHYSICAL REVIEW VOLUME 50

The Relation Between Electron Field Emission and Contact Electromotive Force for Liquid Mercury

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(Received June 13, 1936)

The relation between contact e.m.f. and the impulsive potential necessary to initiate a vacuum spark has been studied for a liquid mercury cathode. The magnitude and time of application of the impulsive potential were determined by a cathode-ray oscillograph so that possible distortion of the mercury surface produced by the electric field could be evaluated. For impure mercury no definite relation could be found. However, for carefully distilled mercury the relation between the work function and breakdown field was in qualitative but not in quantitative agreement with theory.

THE impulsive potential necessary to produce a vacuum spark has been studied by Beams[1] and Quarles.[2] Beams has investigated the field emission from a liquid mercury cathode using impulsive potentials of approximately 10^{-6} sec. duration and, by means of a rotating mirror, has shown that the luminosity appears at the anode before the cathode which suggested that the breakdown was initiated by field emission from the cathode. Quarles has measured the breakdown fields between a mercury cathode and a molybdenum anode along with the accompanying variation in the work function of the mercury. His results, although qualitative, were not in quantitative agreement with the theoretical predictions of Fowler and Nordheim.[3]

The present investigation was undertaken to extend the work of Quarles over a wider range and to find, if possible, the effect of impurities on the relation between field emission and the work function. Also the applied impulses were investigated with a high speed cathode-ray oscillograph in order that the effect of possible distortion of the mercury surface on the field measurements, as suggested by Tonks,[4] could be evaluated.

Outline of Procedure

The method consisted in measuring both the contact potential difference between a hot platinum filament and the mercury surface also the potential necessary to cause a vacuum spark between the mercury and a molybdenum sphere. The contact potential is a measure of the difference between the work functions of the two surfaces.[5, 6] The work function of platinum is assumed constant. Moreover, Cassel and Glückauf[7] have found that mercury vapor does not affect the work function of platinum. There-

[1] Beams, Phys. Rev. **44**, 803 (1933).
[2] Quarles, Phys. Rev. **48**, 260 (1935).
[3] Fowler and Nordheim, Proc. Roy. Soc. **A119**, 173 (1928).
[4] Tonks, Phys. Rev. **48**, 562 (1935).
[5] Mönch, Zeits. f. Physik **65**, 233 (1930).
[6] Eckart, Zeits. f. Physik **47**, 38 (1929).
[7] Cassel and Glückauf, Zeits. f. physik. Chemie 18 Abt. B 4–5, 347 (1932).