Velocity Distribution of Atomic Electrons by the Method of Electron Impact

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When gas atoms are bombarded with electrons traveling with a speed which is large compared to the orbital speeds of the atomic electrons, a bombarding electron collides either with the nucleus or an atomic electron. In the latter case the distribution-in-energy of the electrons scattered at a given angle gives directly the distribution-in-component-velocity of the atomic electrons. The two distribution functions are of the same form.

SEVERAL years ago the author[1] extended the simple photon theory of the Compton effect to the case of the impact between an x-ray photon and a moving atomic electron and showed that the Compton modified line at a certain angle of scattering should have a width due to the motion of the atomic electrons. More recently the author[2] has shown that the distribution-in-intensity in the Compton modified band measures directly, after multiplication by a proportionality constant, the distribution-in-component velocity of the atomic electrons giving rise to the band. In these papers, the author has used the following assumptions: (a) The problem may be treated as that of the impact of a photon with an electron moving in free space but having the same velocity as the actual atomic electron; and (b) if, after applying the principles of conservation of energy and momentum in the substitute problem of (a), the kinetic energy of the recoiling atomic electron is insufficient for it to escape from the atom, the process of the substitute problem becomes inoperative and the photon must then be treated as impinging on the atom as a whole with the result that the scattered photon shows no change (or rather, a negligible change) in wave-length.

The assumption (a) is valid in the case of photon scattering because the photon having no charge is not affected by the field of the nucleus either before or after impact with the atomic electron. We now consider the impact of an outside electron with an atomic electron. An assumption similar to (a) in the case of electron scattering is only valid if the velocity of the impinging electron is so great that the diameters of the "collision areas" of the nucleus and atomic electron are both small compared with the average distance between the nucleus and atomic electron. In the preceding paper of this issue of the *Physical Review*, Hughes and West describe the scattering of fast electrons by helium. Under their experimental conditions the impinging electron is, except in very rare cases, scattered by either the nucleus or the atomic electron. We shall therefore use assumptions similar to (a) and (b) in the problem of the scattering of fast electrons by the relatively slow-moving atomic electrons.

Let v_0 be the velocity in cm/sec. of the impinging electron just before impact and v that of the scattered electron[3] just after impact. Let the direction of v be that of the x-axis and let the v_0v plane (the plane of scattering) be the xz plane. Let ϕ be the angle of scattering, u_x, u_y, u_z the velocity components of the atomic electron just before impact and u_x', u_y', u_z' the components just after impact. The direction cosines of v_0 are $\cos\phi$, 0, $-\sin\phi$. Conservation of kinetic energy gives

$$v_0^2+u_x^2+u_y^2+u_z^2=v^2+u_x'^2+u_y'^2+u_z'^2, \quad (1)$$

while conservation of momentum requires

$$v_0\cos\phi+u_x=v+u_x', \quad (2)$$

$$u_y=u_y', \quad (3)$$

$$u_z-v_0\sin\phi=u_z'. \quad (4)$$

Eliminating u_x', u_y', u_z', we obtain

$$v^2-v(v_0\cos\phi+u_x)+v_0(u_x\cos\phi-u_z\sin\phi)=0. \quad (5)$$

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[1] G. E. M. Jauncey, Phil. Mag. **49**, 427 (1925); Phys. Rev. **25**, 314 (1925); Phys. Rev. **25**, 723 (1925).

[2] G. E. M. Jauncey, Phys. Rev. **46**, 667 (1934).

[3] Following the usual practice, the scattered electron is distinguished from the recoil electron by the greater velocity of the former.

This is a quadratic in v/v_0 and its solution is given by formula. In Hughes and West's experiments with helium u_x/v_0 and u_z/v_0 are small with respect to unity while $\cos\phi$ and $\sin\phi$ are of the order unity. Hence, to the second power of u_x/v_0 and u_z/v_0, we have, using the plus sign of the ambiguity,

$$v/v_0=\cos\phi+\frac{u_z\tan\phi}{v_0}+\frac{(u_xu_z\sin\phi\sec^2\phi-u_z^2\sin^2\phi\sec^3\phi)}{v_0^2}, \quad (6)$$

while to the first power of the quantities we have for high speed impinging electrons

$$v/v_0=\cos\phi+\frac{u_z\tan\phi}{v_0}. \quad (7)$$

Now v_0 and v are the velocities just before and just after impact, respectively. They are not the actual velocities of the impinging and the scattered electron outside the atom. However, if both velocities are high, we may make the approximation that v_0 and v are the speeds outside the atom. Experimentally, velocities are measured in "volts," V_0 and V. Hence, squaring (7), we obtain to the first power of u_z/v_0

$$V/V_0=\cos^2\phi+u_z(2m/V_0e)^{\frac{1}{2}}\sin\phi. \quad (8)$$

For a given angle of scattering ϕ and a given accelerating voltage V_0 for the impinging electrons, we have from (8) a *linear* relation between V and u_z. From this it follows that, if the number of atoms per cm^3 having electrons with z-component velocities u_z in the range du_z is $f(u_z)du_z$ and if a definite fraction of these electrons gives rise to $F(V)dV$ scattered electrons of energy V volts in the range dV volts,

$$f(u_z)du_z=\text{const } F(V)dV. \quad (9)$$

In virtue of the *linear* relation (8),

$$f(u_z)=\text{const } F(V_0\cos^2\phi+u_z(2mV_0/e)^{\frac{1}{2}}\sin\phi). \quad (10)$$

If $F(V)$ can be determined experimentally, $f(u_z)$ can immediately be obtained from (10), the curves of $f(u_z)$ *vs.* u_z and $F(V)$ *vs.* V having the same shape. On the other hand, if $f(u_z)$ is known from theory,

$$F(V)=\text{ const } f((e/2mV_0\sin^2\phi)^{\frac{1}{2}}(V-V_0\cos^2\phi)). \quad (11)$$

In the original Bohr theory of the helium atom the two electrons were at a constant radial distance from the nucleus and therefore had velocities constant in magnitude but random in direction. In such a case[4]

$$\begin{aligned} f(u_z)&=\text{const } (\neq 0), \text{ for } |u_z|\leqq C,\\ &=0 \qquad\qquad\quad, \text{ for } |u_z|>C, \end{aligned} \quad (12)$$

where C is the magnitude of the orbital velocity in cm/sec. Consequently, for this case, (11) and (8) give

$$\begin{aligned} F(V)&=\text{const } (\neq 0), \text{ for } |(V-V_0\cos^2\phi)| \leqq C(2mV_0/e)^{\frac{1}{2}}\sin\phi\\ &=0, \text{ for } |(V-V_0\cos^2\phi)| > C(2mV_0/e)^{\frac{1}{2}}\sin\phi. \end{aligned} \quad (13)$$

Moreover, in this case, we may speak of a spread in the velocity (measured in volts) of the electrons scattered in the direction ϕ. From (13) it is seen that this spread is

$$V_1-V_2=4\sin\phi(UV_0)^{\frac{1}{2}}, \quad (14)$$

where V_1 and V_2 are the retarding voltages (as in Hughes and West's experiments) respectively necessary to prevent all scattered electrons from entering the Faraday cylinder and to allow all such electrons to enter the cylinder when the cylinder is placed at a scattering angle ϕ. The quantity U in (14) is the voltage equivalent of the kinetic energy of the orbital electron.

Finally, it should be noted that the conclusions of this paper are only valid under the various restrictions mentioned in the paper.

[4] In the more general case of a radially symmetrical atom, the probability distribution function $n(C)$ for the speed C (magnitude only) of an atomic electron is related to the probability distribution function $f(u)$ for the component velocity u in a particular direction by

$$f(u)=\frac{1}{2}\int_u^\infty\frac{n(C)dC}{C}.$$

The inverse relation is

$$n(C)=-2Cf'(C).$$

These relations are used in the kinetic theory of gases.