Scattering of Fast Electrons by Helium

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When a charged particle collides with another charged particle, the force between them varying inversely as the square of the distance, wave mechanics and classical mechanics give the same scattering formula when the particles are unlike, and different formulas when they are identical. In the particular case of scattering of electrons by electrons, wave mechanics predicts a smaller scattering than does classical mechanics, in a ratio which has a minimum of 1 to 2 at 45°. The scattering of 2000 and 4000 volt electrons by helium atoms has been investigated. For such energy values, the scattering at considerable angles (>20°) is due to the nuclei and to atomic electrons acting independently of each other. Thus it is possible to measure the ratio of the scattering by the atomic electrons to that by the nucleus and compare it with the values given by the two theories. The elastically scattered electrons are to be identified with those scattered by the nucleus while the inelastically scattered electrons are to be identified with those scattered by atomic electrons. The results are in quantitative agreement with the wave mechanical theory of collisions between electrons. A subsidiary result of the investigation is that the distribution of energies among the inelastically scattered electrons may be used to show that the velocity of the atomic electrons has a value close to that given by the Bohr theory.

THE problem of the scattering of one particle by another particle, attracting or repelling each other according to the inverse square law, has been investigated on the classical particle theory and on the wave mechanical theory. When the particles are unlike both theories lead to the same scattering formulas, but when the particles are identical the two theories lead to different scattering formulas. In this paper it is shown that, when electrons are scattered by electrons, the experimental results are decisively in favor of the wave mechanical description of the scattering process.

The formula for the scattering of electrons by bare nuclei is

$$\alpha=(Z^2e^4/4m^2v^4)\ \mathrm{cosec}^4\,(\phi/2), \tag{1}$$

where α is the probability that an electron of mass m and velocity v will be scattered into unit solid angle, at an angle ϕ with its original direction, by a nucleus of charge Ze. (It is assumed that the electron may be moving before collision with equal probability along any path, within and parallel to the axis of a column of unit cross section, which contains the scattering nucleus.) This formula was derived by Rutherford on the classical particle theory.[1] Wave mechanics leads to precisely the same formula.[2]

For the scattering of electrons by electrons initially at rest, Darwin found that the classical particle theory gave

$$\alpha_1=(e^4/m^2v^4)\ 4\cos\phi\ (\mathrm{cosec}^4\,\phi+\sec^4\phi). \tag{2}$$

The recoil electrons which after collision are indistinguishable experimentally from the impinging electrons are included in this formula. When, however, the problem is considered from the standpoint of wave mechanics, Mott[3] showed that it was necessary to take into account the spin of the electron. This led to the formula

$$\alpha_2=(e^4/m^2v^4)\ 4\cos\phi\ (\mathrm{cosec}^4\,\phi+\sec^4\phi-\Phi\ \mathrm{cosec}^2\,\phi\ \sec^2\phi), \tag{3}$$

where

$$\Phi=\cos\,((2\pi e^2/hv)\log\tan^2\phi).$$

Under our experimental conditions (angles over 20° and electron energies in excess of 2000 volts), Φ is within 2 percent of unity, and we shall therefore write $\Phi=1$. The ratio between the values of α_1 and α_2 reaches a maximum of 2 to 1 at 45°. The relationship at other angles may be inferred from Fig. 7.

Since it is easier experimentally to make relative measurements than to make absolute measurements, we arranged to measure the *ratio* of the scattering by electrons to that by nuclei, and then to compare the ratios found experimentally with those given by the two theories. We cannot secure free nuclei and free electrons for scattering measurements. The next best

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[1] E. Rutherford, Phil. Mag. **21**, 669 (1911).

[2] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, Chap. III (Oxford University Press, 1933).

[3] N. F. Mott, Proc. Roy. Soc. **A126**, 259 (1930).

thing to do is to use hydrogen or helium in which the electrons are bound so loosely to the nuclei that the binding energy can be neglected when the electrons in the beam have energies exceeding 1000 volts. In order that we may consider the gas as essentially a mixture of nuclei and electrons acting as independent scattering centers (as though they were not grouped together in atoms), the collision parameters for an appreciable deflection must be such that any one scattering event must be due *either* to a nucleus, *or* to an atomic electron, but never to both. In Fig. 1, we have drawn to scale the paths along which a 2000 volt electron must move in order to be deflected through 30° by a helium nucleus and by an electron at rest. The distance between the two centers is the most probable distance between the nucleus and an atomic electron in the helium atom. It is evident that, when a 2000 volt electron is scattered through 30° or more by a helium atom, it must have been scattered either by the nucleus, or by an atomic electron, except in those very infrequent cases in which the particles constituting the atom happen to be orientated so that the incoming electron, after being deflected by one particle, is then deflected again by a second particle before it leaves the atom.

Since there are two atomic electrons in helium to each nucleus, the ratio of the scattering by the atomic electrons in an atom to that by the nucleus will be $\alpha_1'/\alpha=2\alpha_1/\alpha$ on the classical particle theory and $\alpha_2'/\alpha=2\alpha_2/\alpha$ on the wave mechanical theory. α_1'/α and α_2'/α, so defined, are the quantities to be compared with the experimentally determined ratio α'/α.

We can distinguish readily between scattering by nuclei and scattering by atomic electrons by the fact that electrons lose no energy when they are scattered by nuclei but do lose energy when they are scattered by atomic electrons. In the latter case the energy of the electrons after collision is $V=V_0\cos^2\phi$, where V_0 is the energy of the impinging electrons, and ϕ is the angle which the path of an electron makes after collision with the path of the impinging electron before collision. Consequently in the graph showing the scattered electron current as a function of the retarding potential, there should be two steps, one corresponding to those electrons

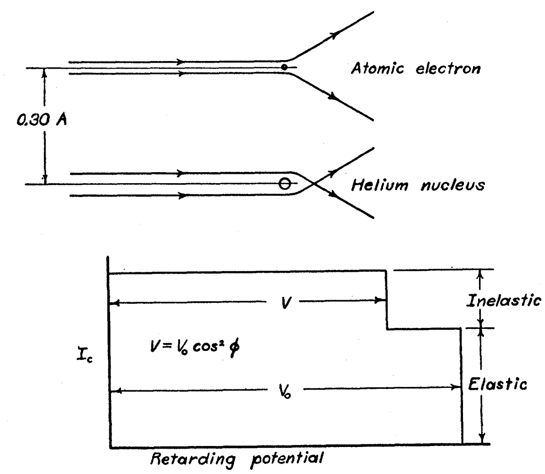

FIG. 1. *Above:* Scattering of a 2000 volt electron by an atomic electron and helium nucleus. *Below:* Theoretical scattered electron current *v.* retarding voltage curve.

which have been scattered without loss of energy and the other corresponding to those which have been scattered with loss of energy (Fig. 1). If the atomic electrons are in random motion, instead of being at rest, as we have assumed up to the present, the discontinuous step will be replaced by a sloping line indicating a range of energies instead of a single energy $V=V_0\cos^2\phi$. The ratio of the number of electrons scattered inelastically to the number scattered elastically can be read off the experimental curves and compared with the theoretical values.

Klemperer[4] investigated the scattering of 30 to 45 kilovolt electrons by a thin collodion film. The energies of the inelastically scattered electrons were far from being grouped around the energy given by $V=V_0\cos^2\phi$, as theory would suggest. It is probable therefore that some degree of multiple scattering was unavoidable in his experiments. It may be concluded that his results are in better accord with the wave mechanical theory than with the classical theory, the agreement being qualitative rather than quantitative. Mohr and Nicol[5] studied the angular distribution of the electrons scattered inelastically by hydrogen and helium and found ill defined humps at the angles corresponding to scattering by isolated electrons. Since their electron energies were not over 300 volts, one

[4] O. Klemperer, Ann. d. Physik **15**, 361 (1932).
[5] C. B. O. Mohr and F. H. Nicol, Proc. Roy. Soc. **A144**, 596 (1934).

cannot expect to find the scattering by the nuclei clearly separated from the scattering by the atomic electrons.

Apparatus and Experimental Procedure

The essential features of the apparatus are shown in Fig. 2. The scattering chamber consists of a brass cylinder, which was made vacuum tight by beeswax and rosin mixture. Through one end of this cylinder (perpendicular to the plane of the figure) passes the shaft of the turntable which carries the electron gun. The shaft forms part of the inside member of a tapered greased joint, the outer member being a piece of clear Bakelite. The brass cylinder of the analyzer is attached at one side of the scattering chamber. Two tubes open into the chamber, one from the pumps and one from a capillary leak connected to the gas storage reservoir. The analyzer has no opening into the main chamber except the narrow defining slits and is evacuated by means of a separate connection to the pumps.

The electron gun is mounted on a brass plate perpendicular to the turntable and extending nearly to the walls on all sides, thus rendering the scattering region field-free. On this plate is mounted a cylinder containing three slits of thin sheet platinum of dimensions 0.4×2.0 mm and 10 mm apart, and behind these is a square shell containing the filament. In the one mm thick front face of this shell is a slit of dimensions 1.0×5.0 mm with longer edges beveled 45° on the outside of the face. Between the filament and shell is applied a potential difference V_1 of 2 to 25 volts and between the shell and the rear one of the narrow defining slits a potential difference V_0 of 2000 or 4000 volts. The gun is aligned with the axis of the analyzer by removing the square shell and filament and observing through the analyzer a light placed behind the gun slits. However, the actual center of the electron beam is later found electrically by means of the analyzer, and the scattering angle is measured from that point. The angular spread of the electron beam by the latter method corresponds to about 2° rotation of the turntable. The current in the beam was kept between one and five microamperes.

The homogeneity of the electron beam with respect to energy can be seen from the solid curve in Fig. 3. However, when scattering measurements are being made under a fairly high pressure, the energy distribution of the incident electrons is that shown by the dashed curve. This spread of energies arises in the accelerating field within the gun and is unavoidable when the gun is operated in gas at an appreciable pressure. The same sort of inhomogeneity is observed in the elastically scattered electrons of Fig. 4 to 6, for these cannot be expected to be more homogeneous than the primary electrons.

The analyzer is essentially a pair of slits to define the solid angle from which electrons are received, a retarding field to measure electron energies, and a Faraday cylinder to collect the scattered electrons. The defining slits (S_1 and S_2, Fig. 2) are 0.3×2.0 mm, the slit S_3 2×9 mm, and the slit S_4 3×7 mm. The distance from S_1 to S_2 is 28 mm, from S_2 to S_3 7 mm, from S_3 to S_4 12 mm, and from S_4 to the Faraday cylinder 3 mm. The inside diameter of the Faraday cylinder is 10 mm and its depth 35 mm. S_3 is put at a potential of +45 volts with respect to S_2 to turn back any positive ions that might pass through the defining slits. Between S_3 and S_4 is applied the retarding field. The lead from the Faraday cylinder passes through an amber plug to the grid of the FP 54. The defining slits are of thin platinum sheet, other slits of thin brass.

The high voltage supply is a full-wave rectifier with filter system capable to reducing the ripple to less than 0.3 percent of the total voltage. The retarding voltage is taken off across a 50T vacuum tube with filament connected through a high resistance to the negative side of the hv

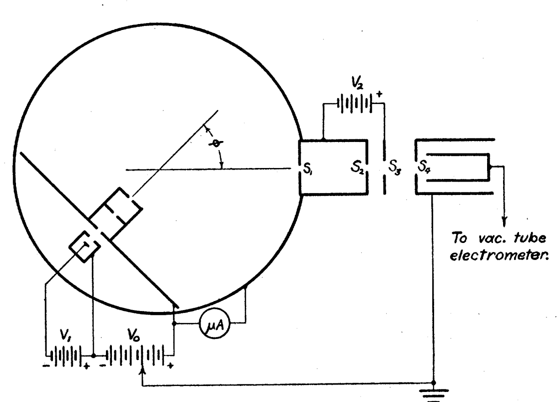

Fig. 2. Apparatus.

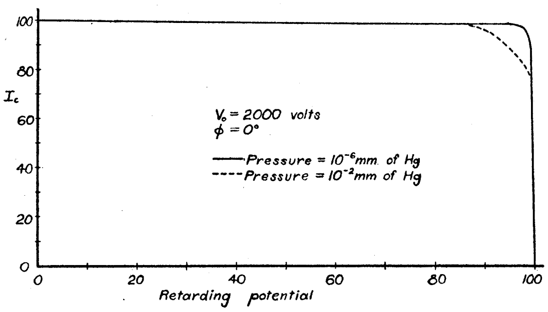

FIG. 3. Homogeneity of the electron beam.

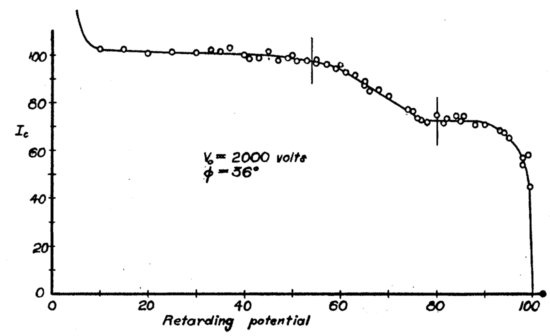

FIG. 4. Experimental curve. 2000 volts, 36°.

supply and plate to the positive side. By varying the grid bias of the 50T tube, one can obtain any retarding voltage from 2 percent to at least 99.5 percent of the accelerating voltage. Higher retarding voltages were obtained by inserting batteries in series with the HV supply. Accelerating and retarding voltages are each measured by a 0–1.0 milliammeter in series with a resistor across the line, the resistor having one megohm for each thousand volts of accelerating voltage. For the retarding voltage the resistor is that used in series with the 50T tube. The unit is capable of giving up to 6000 volts at 2 milliamperes.

Helmholtz coils are used to neutralize the earth's magnetic field. For such fast electrons as we used the adjustment of the current through the coils is not critical, since even without the coils the deflection of the beam is not large.

The gas used was purified by passing it through two charcoal traps immersed in liquid air when filling the reservoir. The helium was the commercial product, which is 95 percent pure. The reservoir is a large sylphon bellows so that the gas can be kept at atmospheric pressure as it is used. The gas flows through a capillary leak into the scattering chamber, the pressure in the chamber being kept at a constant value determined by the influx through this leak and the exhaust through a constricted line to the pumps. The pressure in the chamber can be decreased if desired by expanding the sylphon to decrease the pressure in the reservoir. A charcoal trap cooled with solid CO_2 was inserted in the intake line between leak and chamber. With the flow of gas shut off, the pressure in the chamber was less than 10^{-6} mm of mercury as measured on a McLeod gauge, whereas, with the gas fed in and the pumps on, the pressure averaged 0.025 mm of mercury. Hence for this constant flow method the impurity due to gas evolved in the scattering chamber will certainly be less than one part in a thousand. During scattering measurements the pressure of the gas can be observed constantly by means of a Pirani gauge.

The current to the Faraday cylinder of the analyzer is measured by means of an FP 54 electrometer tube with the DuBridge and Brown balanced circuit at a sensitivity of 120,000 mm/volt. No change was made in the original circuit except to insert a fine adjustment for the grid bias in order to make the open-key and closed-key zeros coincide. The tube and grounding key are enclosed in an evacuated brass case, and all leads, batteries and accessory apparatus are well shielded. The 18 volts for the operation of the circuit are supplied by large glass-cell storage batteries. The rate-of-charge method (with no grid resistor) was used, since it is more sensitive than the constant deflection method and less liable to subjective error. The maximum currents measured in the runs shown in Fig. 4 to 6 were of the order of 10^{-15} amp.

With no gas in the apparatus the observed background current for any of the scattering angles used was apparently due entirely to secondary electrons emitted where the electron beam struck metal surfaces. Except for the case of energies less than about 20 percent of the maximum energy of the beam, this background was never more than one-half percent of the current scattered from the gas and was therefore negligible.

Experimental Results

Typical retarding potential curves are shown in Fig. 4. An ordinate of 100 represents the current corresponding to an arbitrary standard reference retarding voltage, say 20 percent or 30 percent of the accelerating voltage. An abscissa of 100 represents the accelerating voltage V_0. Points on these curves were mostly taken in pairs, one at a retarding voltage V_x, say, and the next at $V_x+V_0(1-\cos^2\phi)$. From each of these pairs a value of the ratio of inelastic to elastic scattering can be calculated. (Only those pairs of which one value is to be found on the upper flat portion of the curve, and the other on the lower flat portion, are used in computing the ratio.) The deviations of these values from their mean give a measure of the error due to fluctuations in the FP 54 circuit and the beam current, which far outweighs any other error. The inelastic and elastic groups are clearly separated and enough of the flat portions is present in each case to fix their magnitudes. Note that the inelastic distribution has an energy spread which is entirely different in character from the spread in the elastic distribution due to the inhomogeneity of the original beam (see Figs. 3, and 4, 5, or 6).

Table I shows the values of the ratio α'/α of inelastic scattering to elastic scattering in helium for various scattering angles. The values in this table are plotted in Fig. 7, in which are also the theoretical curves based on the classical theory and on the Mott theory.

Discussion

It is evident from Table I or Fig. 7 that, when the impinging electrons have 2000 to 4000 volt energies, the experimental values of α'/α, the ratio of the inelastic electron scattering to the elastic electron scattering, agree well with the values calculated on the Mott theory (α_2'/α), and differ widely from the values calculated on the classical particle theory (α_1'/α). The conclusion to be drawn then is that the results are decisively in favor of the wave mechanical description of the collision of two electrons.

We now turn to a subsidiary result of the investigation. When the scattered electron current is plotted as a function of the retarding voltage, as in Figs. 4 to 6, it is found that the inelastically scattered electrons have a spread in energy which (a) is larger the greater the angle of scattering, and (b) is smaller the greater the energy of the impinging electron. From the equations for the conservation of energy and momentum of two colliding electrons, one of which has a definite velocity and the other (in our case the atomic electron) a velocity which may have any direction at the moment of impact, the range of velocities among the electrons scattered at any angle can be calculated. Jauncey[6] gave a theory of the width of the modified line in the Compton effect on the assumption that the atomic electrons, with which the photons collide, are in random motion with the speed given by the Bohr theory. The same theory gives the range in energies of scattered electrons on substituting electrons for photons. Jauncey's formula for the range in energies is

$$V_1-V_2=4\sin\phi(UV_0)^{\frac{1}{2}},$$

where V_0 is the energy of the impinging electron,

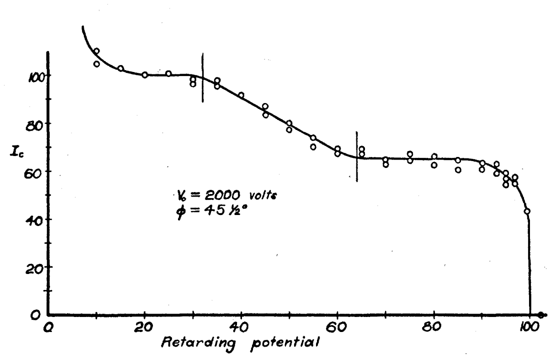

FIG. 5. Experimental curve. 2000 volts, 45.5°.

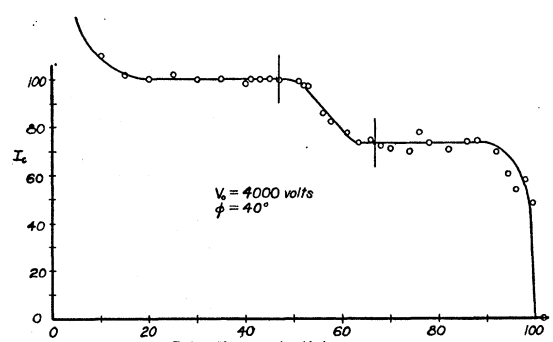

FIG. 6. Experimental curve. 4000 volts, 40°.

[6] G. E. M. Jauncey, Phil. Mag. **49**, 427 (1925).

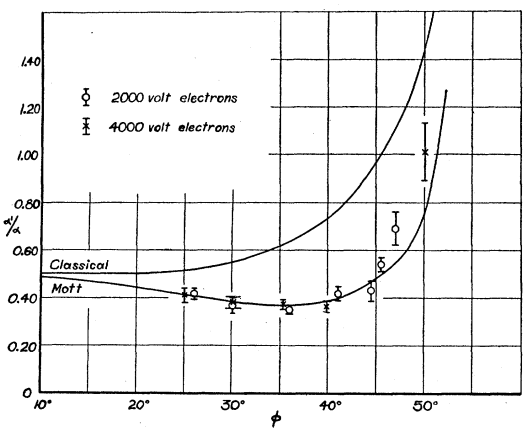

FIG. 7. Comparison of experimental ratios with theoretical ratios (continuous lines).

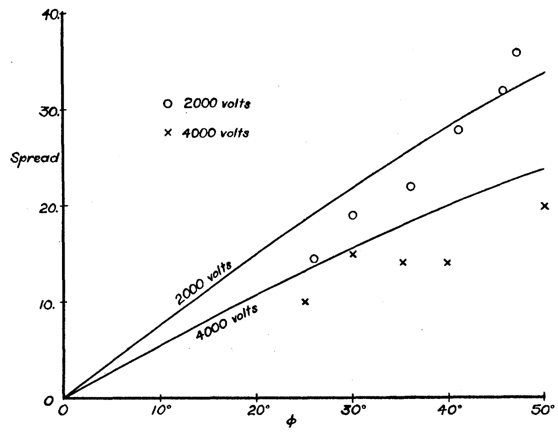

FIG. 8. Spread of energies of the inelastically scattered electrons. *Continuous lines:* theoretical ratios. *Circles and crosses:* experimental points for 2000 and 4000 volt electrons, and respectively.

U the energy of the atomic electrons, V_1 and V_2 the maximum and minimum energies among the electrons scattered inelastically at ϕ.[7] For U we take the ionization potential, and for V_0 the value of the energy of the impinging electrons in volts. These theoretical ranges in the energies of the inelastically scattered electrons are shown in Figs. 4 to 6 by two vertical lines. The results for all our curves are summarized in Fig. 8. It is clear that the theory describes at least the order of magnitude of the range of energies.

It is possible in principle to calculate the distribution of energy among the atomic electrons from the shape of the curve giving the distribution of energies among the scattered electrons.[7] For example, if the atomic electrons were moving in random directions with a constant velocity, then the part of the curves in Figs. 4 to 6 corresponding to the inelastically scattered electrons should be straight lines. The fact that they are practically straight lines may be taken to indicate that the velocities of the atomic electrons in helium do not deviate much from the average velocity. It must be stated, however, that, while our experimental arrangement is particularly effective for measuring the *ratio* of the inelastically scattered electrons to the elastically scattered electrons, it is not so suitable for the accurate measurement of the *distribution* of energies among the inelastically scattered electrons. Consequently, in order to get accurate experimental data from which to get information about the distribution of velocities among atomic electrons, a different type of apparatus should be used. It would be interesting to apply this method to determine accurately the velocities of atomic electrons and to find out whether or not, in atoms containing electrons of more than one kind, there is evidence of well separated velocities.

[7] G. E. M. Jauncey, following paper.

TABLE I. *Ratio of inelastic scattering to elastic scattering.*

Electron energy (volts)		Experimental α'/α	Theoretical α_1'/α	Theoretical α_2'/α
2000	26°	0.419±0.020	0.527	0.433
	30°	0.370±0.034	0.553	0.386
	36°	0.353±0.020	0.632	0.371
	41°	0.419±0.027	0.771	0.404
	44.5°	0.424±0.040	0.939	0.470
	45.5°	0.542±0.027	0.999	0.500
	47	0.690±0.047	1.120	0.558
4000	25°	0.408±0.034	0.523	0.414
	30°	0.379±0.024	0.553	0.386
	35.25°	0.375±0.022	0.608	0.372
	39.75°	0.361±0.026	0.726	0.386
	50°	1.010±0.081	1.437	0.761