increase in the applied field at which successive breakdowns occur can be attributed to a conditioning of the anode. A definite part of this progressive increase in field can be attributed to conditioning of glass surfaces when they are exposed to the discharge.

11. A few tubes eventually would not give breakdown at about 26 kv which was the highest voltage available. Tests with one, in which there was opportunity for much gas contamination did not result in the subsequent occurrence of breakdown.

12. The highest electric field that could be applied to a cathode without breakdown occurring was about 4.7×10^6 v/cm.

13. The evidence favors the conclusion that the electric field applied to the cathode surface rather than the applied voltage is the more important factor in producing breakdown. The evidence likewise favors the conclusion that the anode has no effect on the breakdown and that when the shielding against glass surfaces is complete, the breakdown is determined primarily by conditions at the cathode.

14. The suggestion is made that breakdown involves a rupturing of the cathode surface under the action of local heating and mechanical strain associated with the electric field.

The author acknowledges with pleasure the continued interest of Dr. H. E. Mendenhall during the progress of the experiments and is indebted to Dr. J. A. Becker for many suggestions regarding the interpretation of the data and the form of the manuscript.

AUGUST 1, 1936 PHYSICAL REVIEW VOLUME 50

The Approximate Solution of Nuclear Three and Four Particle Eigenvalue Problems

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(Received May 23, 1936)

The problem of determining intranuclear forces from the mass defects of the hydrogen and helium isotopes is investigated under the assumption that the interaction potentials are proportional to a function $f(\alpha r^2)$ having the general form of a potential well and possessing the power series expansion

$$f(\alpha r^2)=1-\alpha r^2+c_1(\alpha r^2)^2/2!-c_2(\alpha r^2)^3/3!+\cdots$$

about the origin. With this assumption the Rayleigh-Schroedinger perturbation theory is applicable to the two, three and four particle eigenvalue problems. The perturbation calculation yields small corrections to the eigenvalues given by the "equivalent" two particle method. The corrections are checked very satisfactorily in a special case by means of a complicated variational calculation. Numerical results are given for two extreme forms of the neutron-proton model: Model I—Interaction between unlike particles only, Model II—Equal interactions between all pairs of particles. These results put close upper and lower bounds on the strength of the interaction between like particles in the model, intermediate between (I) and (II), which corresponds most closely to the experimental facts.

I. INTRODUCTION

THE mass defects of the hydrogen and helium isotopes appear to require a nuclear model with strong attractive forces between neutrons and protons and somewhat weaker attractive forces between like particles.[1, 2, 3] However in the study of the eigenvalue problems it is convenient to consider two extreme forms of the neutron-proton model with

(I) Interactions between neutrons and protons only;

(II) Identical interactions between all pairs of particles.

The model which corresponds most closely to the experimental facts is intermediate between (I) and (II). If the eigenvalue problems associated with (I) and (II) are solved, the corresponding solutions for intermediate models can be obtained by a simple process of interpolation. The Hamiltonian operators for models (I) and (II)

[1] Feenberg and Knipp, Phys. Rev. **48**, 906 (1935).
[2] R. D. Present, Phys. Rev. **49**, 640 (1936).
[3] Massey and Mohr, Proc. Roy. Soc. **A152**, 693 (1935).

are taken to have the form

$$H_{\mathrm{I}}(\mathrm{H}^3)=-\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3)-A_{\mathrm{I}}f(\alpha r_{12}^2)-A_{\mathrm{I}}f(\alpha r_{13}^2), \quad (1)$$

$$H_{\mathrm{I}}(\mathrm{He}^4)=-\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3+\Delta_4)-A_{\mathrm{I}}f(\alpha r_{13}^2)-A_{\mathrm{I}}f(\alpha r_{14}^2)-A_{\mathrm{I}}f(\alpha r_{23}^2)-A_{\mathrm{I}}f(\alpha r_{24}^2), \quad (2)$$

$$H_{\mathrm{II}}(\mathrm{H}^3)=-\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{12}^2)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{13}^2)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{23}^2), \quad (3)$$

$$H_{\mathrm{II}}(\mathrm{He}^4)=-\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3+\Delta_4)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{12}^2)-\cdots-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{34}^2), \quad (4)$$

in which $f(\alpha r^2)$ is a function possessing the power series expansion

$$f(\alpha r^2)=1-\alpha r^2+c_1(\alpha r^2)^2/2!-c_2(\alpha r^2)^3/3!+\cdots \quad (5)$$

about the origin and vanishing rapidly for large values of r. The factor of two-thirds in Eqs. (3) and (4) is introduced in order to make A_{I} and A_{II} directly comparable. We wish to determine the normal state eigenvalues $E_{\mathrm{I}}(\mathrm{H}^3)$, $E_{\mathrm{I}}(\mathrm{He}^4)$, $E_{\mathrm{II}}(\mathrm{H}^3)$, $E_{\mathrm{II}}(\mathrm{He}^4)$ as functions of the parameters in the interaction potential.

II. The Perturbation Method

A detailed discussion of the symmetrical three particle problem is given to illustrate the general method. The starting point is the Schroedinger equation

$$\{-\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{12}^2)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{13}^2)-\tfrac{2}{3}A_{\mathrm{II}}f(\alpha r_{23}^2)\}\psi=E_{\mathrm{II}}(\mathrm{H}^3)\psi. \quad (6)$$

For small values of α the potential function $f(\alpha r^2)$ may be replaced by the first two terms in the power series expansion (5). With this approximation Eq. (6) reduces to

$$\{-\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3)-2A_{\mathrm{II}}+\tfrac{2}{3}\alpha A_{\mathrm{II}}(r_{12}^2+r_{13}^2+r_{23}^2)\}\psi^0=E_{\mathrm{II}}{}^0\psi^0. \quad (7)$$

In terms of the internal coordinates

$$\xi_1=(16\alpha A_{\mathrm{II}}/9)^{\frac{1}{4}}(x_1-\tfrac{1}{2}(x_2+x_3)),\quad \xi_2=(\alpha A_{\mathrm{II}})^{\frac{1}{4}}(x_2-x_3), \quad (8)$$

Eq. (7) takes the simple form

$$\{\Delta_1+\Delta_2+\epsilon^0-s_1^2-s_2^2\}\psi^0=0, \quad (9)$$

which we recognize as the wave equation for a system of six independent linear harmonic oscillators.[4] The same coordinate transformation applied directly to Eq. (6) yields the equation

$$\{\Delta_1+\Delta_2+\epsilon_{\mathrm{II}}(\mathrm{H}^3)-s_1^2-s_2^2+(3/8)c_1(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}\times(s_1^4+4(s_1\cdot s_2)^2/3+2s_1^2s_2^2/3+s_2^4)-(1/96)c_2(\alpha/A_{\mathrm{II}})(9s_1^6+\cdots+11s_2^6)+\cdots\}\psi=0, \quad (10)$$

in which

$$\epsilon_{\mathrm{II}}(\mathrm{H}^3)=[E_{\mathrm{II}}(\mathrm{H}^3)+2A_{\mathrm{II}}]/(\alpha A_{\mathrm{II}})^{\frac{1}{2}}. \quad (11)$$

The wave function ψ can be expanded in terms of the solutions of the oscillator Eq. (9) and the usual perturbation theory applied to obtain $\epsilon_{\mathrm{II}}(\mathrm{H}^3)$ as a power series in $(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}$, c_1, c_2:

$$\epsilon_{\mathrm{II}}(\mathrm{H}^3)=\epsilon^0+(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}\epsilon^{(1)}+(\alpha/A_{\mathrm{II}})\epsilon^{(2)}+\cdots. \quad (12)$$

The results of the perturbation calculation are

$$\epsilon^0=6,\quad \epsilon^{(1)}=-(15/4)c_1,\quad \epsilon^{(2)}=(5/128)\{112c_2-129c_1^2\}. \quad (13)$$

The same procedure when applied to the other three problems yields results similar to (13). The energy eigenvalues are given by the expressions

$$E_{\mathrm{I}}(\mathrm{H}^3)=-2A_{\mathrm{I}}\{1-3(0.933\alpha/A_{\mathrm{I}})^{\frac{1}{2}}+(15/8)c_1(0.933\alpha/A_{\mathrm{I}})+(2.5664c_1^2-(35/16)c_2)(0.933\alpha/A_{\mathrm{I}})^{\frac{3}{2}}+\cdots\}, \quad (14)$$

$$E_{\mathrm{I}}(\mathrm{He}^4)=-4A_{\mathrm{I}}\{1-3(0.7286\alpha/A_{\mathrm{I}})^{\frac{1}{2}}+(15/8)c_1(0.7286\alpha/A_{\mathrm{I}})+(2.5089c_1^2-(35/16)c_2)(0.7286\alpha/A_{\mathrm{I}})^{\frac{3}{2}}+\cdots\}, \quad (15)$$

$$E_{\mathrm{II}}(\mathrm{H}^3)=-2A_{\mathrm{II}}\{1-3(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}+(15/8)c_1(\alpha/A_{\mathrm{II}})+(2.5195c_1^2-(35/16)c_2)(\alpha/A_{\mathrm{II}})^{\frac{3}{2}}+\cdots\}, \quad (16)$$

$$E_{\mathrm{II}}(\mathrm{He}^4)=-4A_{\mathrm{II}}\{1-3(0.75\alpha/A_{\mathrm{II}})^{\frac{1}{2}}+(15/8)c_1(0.75\alpha/A_{\mathrm{II}})+(2.4902c_1^2-(35/16)c_2)(0.75\alpha/A_{\mathrm{II}})^{\frac{3}{2}}+\cdots\}. \quad (17)$$

In the special case

$$f(\alpha r^2)=e^{-\alpha r^2},\quad (c_1=c_2=1), \quad (18)$$

the expansions in powers of $(\alpha/A)^{\frac{1}{2}}$ appear to converge quite rapidly. However the range in which Eqs. (14), (15), (16), (17) may be expected to be moderately accurate does not extend into the physically interesting region ($\alpha^{-\frac{1}{2}}<2.8\times10^{-13}$ cm).

[4] See W. V. Houston, Phys. Rev. **47**, 942 (1935) for a discussion of the general "harmonic oscillator" model.

III. The "Equivalent" Two-Particle Equations

The procedure described in the preceding section can be applied to obtain the normal state eigenvalue of the two-particle Hamiltonian operator

$$\mathrm{H} = -\Delta - Bf(\beta r^2). \tag{19}$$

The resulting expression for the eigenvalue is

$$\mathcal{E} = -B\{1-3(\beta/B)^{\frac{1}{2}}+(15/8)c_1(\beta/B) + (2.5781c_1^2-(35/16)c_2)(\beta/B)^{\frac{3}{2}}+\cdots\}. \tag{20}$$

We get "equivalent" two particle problems by giving B and β the values

Model I

$$\left.\begin{matrix} B=2A_{\mathrm{I}} \\ \beta=1.8660\alpha \end{matrix}\right\}\mathrm{H}^3, \tag{21}$$

$$\left.\begin{matrix} B=4A_{\mathrm{I}} \\ \beta=2.9142\alpha \end{matrix}\right\}\mathrm{He}^4. \tag{22}$$

Model II

$$\left.\begin{matrix} B=2A_{\mathrm{II}} \\ \beta=2\alpha \end{matrix}\right\}\mathrm{H}^3, \tag{23}$$

$$\left.\begin{matrix} B=4A_{\mathrm{II}} \\ \beta=3\alpha \end{matrix}\right\}\mathrm{He}^4. \tag{24}$$

These results were originally found by a variational method[5] which does not require that any restrictions be placed on the potential function $f(\alpha r^2)$; in particular the power series expansions about the origin may contain odd powers of r. It is clear from the original variational derivation that the eigenvalues $\mathcal{E}_{\mathrm{I}}(\mathrm{H}^3)$, $\mathcal{E}_{\mathrm{I}}(\mathrm{He}^4)$, $\mathcal{E}_{\mathrm{II}}(\mathrm{H}^3)$, $\mathcal{E}_{\mathrm{II}}(\mathrm{He}^4)$ of the "equivalent" problems do not differ very much from the eigenvalues of the corresponding three and four particle problems. The differences can now be estimated since the two sets of eigenvalues are related by the equations

$$E_{\mathrm{I}}(\mathrm{H}^3) = \mathcal{E}_{\mathrm{I}}(\mathrm{H}^3) + 0.021\alpha(\alpha/A_{\mathrm{I}})^{\frac{1}{2}}c_1^2+\cdots, \tag{25}$$

$$E_{\mathrm{I}}(\mathrm{He}^4) = \mathcal{E}_{\mathrm{I}}(\mathrm{He}^4) + 0.172\alpha(\alpha/A_{\mathrm{I}})^{\frac{1}{2}}c_1^2+\cdots, \tag{26}$$

$$E_{\mathrm{II}}(\mathrm{H}^3) = \mathcal{E}_{\mathrm{II}}(\mathrm{H}^3) + 0.117\alpha(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}c_1^2+\cdots, \tag{27}$$

$$E_{\mathrm{II}}(\mathrm{He}^4) = \mathcal{E}_{\mathrm{II}}(\mathrm{He}^4) + 0.228\alpha(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}c_1^2+\cdots. \tag{28}$$

The Eqs. (25), (26), (27), (28) reduce the three and four particle eigenvalue problems to the much simpler problem of computing the lowest eigenvalue of the one dimensional equation[6]

$$\{d^2/dr^2+\mathcal{E}+Bf(\beta r^2)\}\varphi=0. \tag{29}$$

Calculations to test the accuracy of the Eqs. (25), (26), (27), (28) are described in the appendix. The results, as far as they go, indicate that these equations are quite accurate in the physically interesting range of α values.

IV. Numerical Results

A table from which any one of the quantities $\mathcal{E}$, B, β can be computed when the other two are known is available in the case of the Gaussian potential (Eq. 18).[7] The conditions

$$E_{\mathrm{I}}(\mathrm{He}^4) = E_{\mathrm{II}}(\mathrm{He}^4),$$

$$-E_{\mathrm{I}}(\mathrm{He}^4) + (\text{coulomb correction})^8 = 54mc^2 \tag{30}$$

are sufficient to determine A_{I} and A_{II} as functions of α. These quantities in turn determine $E_{\mathrm{I}}(\mathrm{H}^3)$ and $E_{\mathrm{II}}(\mathrm{H}^3)$, which are given in Table I.

Table II gives the depth of the neutron-proton well, $A_{\nu\pi}$, as a function of α and the binding energy of the deuteron.

Attractive forces between like particles are necessary to account for the difference between $A_{\nu\pi}$ and A_{I}, A_{II}. Let $A_{\nu\nu}$ and $A_{\pi\pi}$ represent the depths of the neutron-neutron and proton-proton wells. The inequality[9]

$$2(A_{\mathrm{I}}-A_{\nu\pi}) \leqq A_{\nu\nu} \leqq 2(A_{\mathrm{II}}-A_{\nu\pi}) \tag{31}$$

is an immediate consequence of the two assumptions

Table I.* A_{I}, A_{II}, $E_{\mathrm{I}}(\mathrm{H}^3)$, $E_{\mathrm{II}}(\mathrm{H}^3)$.

$\alpha^{-\frac{1}{2}}$	α	A_{I}	A_{II}	$E_{\mathrm{I}}(\mathrm{H}^3)$	$E_{\mathrm{II}}(\mathrm{H}^3)$
2.8×10^{-13} cm	10	64	65	19.8	17.9
2.0×10^{-13} cm	20	97	98	17.1	14.3
1.6×10^{-13} cm	30	126	129	14.9	11.6

* Units—Energy—$mc^2=510{,}000$ e.v.
Length—$\hbar/c(Mm)^{\frac{1}{2}}=8.97\times10^{-13}$ cm.

[5] Feenberg, Phys. Rev. **47**, 850 (1935), Eq. (26); reference 1, Eq. (13).

[6] An "equivalent" two particle equation was used by Wigner in his pioneer investigation of the three and four particle eigenvalue problems, Phys. Rev. **43**, 252 (1933).

[7] Reference 1, Table I; also reference 5, section II.

[8] Reference 1, Table II.

[9] Feenberg, Phys. Rev. **49**, 273 (1936).

TABLE II. $A_{\nu\pi}(\alpha, E(H^2))$.

α \ $E(H^2)$	$-3.5mc^2$	$-4.0mc^2$	$-4.5mc^2$
10	48	50	51
20	82	84	86
30	114	117	120

TABLE IV. $g(\alpha, E(H^2))$.

α \ $E(H^2)$	$-3.5mc^2$	$-4.0mc^2$	$-4.5mc^2$
10	0.22	0.23	0.24
20	0.17	0.18	0.19
30	0.14	0.15	0.16

TABLE III. *Upper and lower limits on* $A_{\nu\nu}$ *(Eq. 31).*

α \ $E(H^2)$	$-3.5mc^2$		$-4.0mc^2$		$-4.5mc^2$	
	U	L	U	L	U	L
10	33	31	30	28	26	24
20	32	29	28	24	24	20
30	30	24	25	18	20	13

TABLE V. *Upper and lower limits on* $A_{\nu\nu}$ *(Eq. 33).*

α \ $E(H^2)$	$-3.5mc^2$		$-4.0mc^2$		$-4.5mc^2$	
	U	L	U	L	U	L
10	44	42	41	39	38	36
20	46	43	43	40	40	37
30	46	40	42	36	39	32

(a) $A_{\nu\nu} \sim A_{\pi\pi}$,

(b) Neutron-proton interaction independent of spin orientation. The upper and lower limits on $A_{\nu\nu}$, computed from Eq. (31) are shown in Table III.

Assumption (b) is probably incorrect. It appears necessary to assume a dependence on spin orientation in the neutron-proton interaction as suggested by Wigner in order to account for the very large scattering cross section of slow neutrons in hydrogen.[10, 11] We replace (b) by the assumption:

(c) Depth of the neutron-proton well
$= A_{\nu\pi}$ (triplet interaction),
$= (1-2g)A_{\nu\pi}$ (singlet interaction).

The constant g is determined by the condition[12]

$$E(H^2, \text{singlet}) \sim 0. \qquad (32)$$

Values of g computed from Eq. (32) are listed in Table IV.

The inequality (31) must be replaced by

$$2(A_{I}-A_{\nu\pi})+gA_{\nu\pi} \leqq A_{\nu\nu} \leqq 2(A_{II}-A_{\nu\pi})+gA_{\nu\pi} \qquad (33)$$

and Table III by a similar table based on Eq. (33).

For $\alpha=10$, the like particle depth $A_{\nu\nu}$ is almost identical with the effective neutron-proton depth $(1-g/2)A_{\nu\pi}$. Consequently at this point

$$A_{\nu\nu}=2(A_{II}-A_{\nu\pi})+gA_{\nu\pi}. \qquad (34)$$

In the range $10 \leqq \alpha \leqq 20$ the upper limit on $A_{\nu\nu}$ coincides with the accurate value. Also in this range the eigenvalue $E_{II}(H^3)$ does not differ very much from the eigenvalue $E(H^3)$ of the physical model under assumption (c). No correction is needed at $\alpha=10$. At $\alpha=20$ the departure from complete symmetry lowers the eigenvalue by the amount $0.3mc^2$. This correction includes the effect of changing from ordinary to Majorana forces between unlike particles.[13] For intermediate points the linear interpolation formula

$$E(H^3)=-17.9+0.33(\alpha-10), \quad 10 \leqq \alpha \leqq 20, \qquad (35)$$

is sufficiently accurate.

Let $-E(He^4)$ represent the correct (unknown) binding energy of the alpha-particle. The quantity

$$\delta E(He^4)=E(He^4)+54 \qquad (36)$$

measures the deviation from the value $54mc^2$ used in Eq. (30). There are corresponding deviations $\delta A_{\nu\nu}$ and $\delta E(H^3)$ from the tabulated values of $A_{\nu\nu}$ and $E(H^3)$. The relations

$$\delta A_{\nu\nu} \sim -1.1\,\delta E(He^4), \quad \delta E(H^3) \sim 0.4\,\delta E(He^4), \quad 10 \leqq \alpha \leqq 20, \qquad (37)$$

permit the calculation of corrections to the tabulated values when $\delta E(He^4)$ is known.

The nuclear masses and energies given recently by Aston,[14] Oliphant,[15] K. T. Bainbridge,[16] Bethe and Livingston[17] determine the energies

[10] Reference 1, section V.
[11] L. A. Young, Phys. Rev. **48**, 913 (1935).
[12] The numerical integration of Eq. (28) with $\mathcal{E}=0$ yields the result $(1-2g)A_{\nu\pi} \sim 2.7\alpha$.

[13] Eqs. (8) and (9), reference 1, are used to compute the corrections.
[14] F. W. Aston, Nature **137**, 357 (1936).
[15] M. L. Oliphant, Nature **137**, 396 (1936).
[16] Cornell Symposium (July 1936).
[17] Bethe and Livingston, unpublished (we are indebted to Dr. Bethe for communicating to us the results tabulated in (38)).

$$
\begin{aligned}
E(\mathrm{H}^2)&=-4.34\pm0.12mc^2,\\
E(\mathrm{H}^3)&=2E(\mathrm{H}^2)-7.82\pm0.03mc^2,\\
E(\mathrm{He}^3)&=2E(\mathrm{H}^2)-6.26\pm0.20mc^2,\\
E(\mathrm{He}^4)&=2E(\mathrm{H}^2)-4.65\pm0.3mc^2.
\end{aligned}
\tag{38}
$$

Using Eqs. (35) and (37) the experimental binding energy of H^3 determines α to have the value $16(\alpha^{-\frac{1}{2}}\sim 2.25\times10^{-13}$ cm).[18] From Table V and Eq. (37), $A_{\nu\nu}\sim 41mc^2$ at $\alpha=16$.[19] These values fit moderately well with new results on the anomalous scattering of fast protons in hydrogen.[20, 21]

Appendix. Variational Calculations

The Gaussian wave function

$$\psi^0=N\exp\left[-(\nu/2)(r_{12}^2+r_{13}^2+r_{23}^2)\right] \tag{39}$$

may be taken as an approximation to the normal state eigenfunction of the Hamiltonian operator $\mathrm{H}_{\mathrm{II}}(\mathrm{H}^3)$. A better approximation which should give fairly accurate values for the energy is

$$\psi=(1+c\mathrm{H}_{\mathrm{II}})\psi^0. \tag{40}$$

In the variation method of calculating the energy the following matrix elements are required

$$
\begin{aligned}
&(0|1|0)=1,\quad (0|\mathrm{H}_{\mathrm{II}}|0)=3\alpha\sigma-2A_{\mathrm{II}}(\sigma/(\sigma+1))^{\frac{3}{2}},\\
&(0|\mathrm{H}_{\mathrm{II}}{}^2|0)=12\alpha^2\sigma^2-6\alpha A_{\mathrm{II}}\{-3\sigma(\sigma/(\sigma+1))^{\frac{3}{2}}+\sigma(\sigma/(\sigma+1))^{5/2}\}\\
&\qquad+(4/3)A_{\mathrm{II}}{}^2\{(\sigma/(\sigma+2))^{\frac{3}{2}}+16\sigma^3/(2\sigma+1)^{\frac{3}{2}}(2\sigma+3)^{\frac{3}{2}}\},\\
&(0|\mathrm{H}_{\mathrm{II}}{}^3|0)=60\alpha^3\sigma^3-(3/2)A_{\mathrm{II}}\alpha^2(\sigma/(\sigma+1))^{7/2}(48\sigma^2+136\sigma+103)\\
&\qquad+4A_{\mathrm{II}}{}^2\alpha\{(\sigma/(\sigma+2))^{\frac{3}{2}}(3\sigma^2+9\sigma+2)/(\sigma+2)+64\sigma^3(3\sigma^3+9\sigma^2+5\sigma)/(2\sigma+1)^{5/2}(2\sigma+3)^{5/2}\}\\
&\qquad-(8/9)A_{\mathrm{II}}{}^3\{48\sigma^3/(4\sigma^2+12\sigma+6)^{\frac{3}{2}}+16\sigma^3/(2\sigma+3)^3+(\sigma/(\sigma+3))^{\frac{3}{2}}\}
\end{aligned}
\tag{41}
$$

where ν has been replaced by $\frac{2}{3}\alpha\sigma$.

The corresponding treatment of the "equivalent" two particle Hamiltonian given by Eq. (19) is based on the approximate wave function

$$\psi^0=N\exp\left[-(\mu/2)r_{12}^2.\right. \tag{42}$$

The matrix element $(0|1|0)$ is the same as in Eqs. (41). If we replace B by $2A_{\mathrm{II}}$ and β by 2α, $(0|\mathrm{H}|0)$ is also the same as in (41). The other matrix elements are

$$
\begin{aligned}
&(0|\mathrm{H}^2|0)=15\alpha^2\sigma^2-12A_{\mathrm{II}}\alpha(\sigma/(\sigma+1))^{5/2}(\sigma+2)+4A_{\mathrm{II}}{}^2(\sigma/(\sigma+2))^{\frac{3}{2}},\\
&(0|\mathrm{H}^3|0)=105\alpha^3\sigma^3-2A_{\mathrm{II}}\alpha^2(\sigma/(\sigma+1))^{7/2}(45\sigma^2+156\sigma+156)\\
&\qquad+12A_{\mathrm{II}}{}^2\alpha(\sigma/(\sigma+2))^{\frac{3}{2}}(3\sigma^2+12\sigma+4)/(\sigma+2)-8A_{\mathrm{II}}{}^3(\sigma/(\sigma+3))^{\frac{3}{2}},
\end{aligned}
\tag{43}
$$

where μ has been replaced by $2\alpha\sigma$. In using (41) and (43) it is convenient to give A_{II} values which make the exact eigenvalue of the "equivalent" problem independent of α. This will occur if we set $A_{\mathrm{II}}=A_{\nu\pi}(E(\mathrm{H}^2)=-4.0)$. Then the exact eigenvalue of Eq. (29) is $-8mc^2$.

Table VI. *Comparison of the values of E and $\mathcal{E}$ calculated from (43) with the term $0.117\alpha(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}$ in Eq. (27).*

α	$(0\|\mathrm{H}\|0)$	$\mathcal{E}$	E	$E-\mathcal{E}$	$0.117\alpha(\alpha/A_{\nu\pi})^{\frac{1}{2}}$
10	-5.46	-7.34	-6.78	0.56	0.53
20	-2.77	-6.53	-5.40	1.13	1.14
30	-0.07	-5.60	-3.91	1.69	1.78

We now expect that the difference in the energies E calculated from (41) and $\mathcal{E}$ calculated from (43) should agree with the term $0.117\alpha(\alpha/A_{\mathrm{II}})^{\frac{1}{2}}$ in Eq. (27) if this equation is accurate. Table VI shows that Eq. (27) is quite accurate in the physically important range of α values.

[18] The coulomb interaction between the protons in He^3 accounts for about 80 percent of the experimental difference between $E(\mathrm{H}^3)$ and $E(\mathrm{He}^3)$. See reference 1, Table III.

[19] The condition $A_{\nu\nu}<(1-2g)A_{\nu\pi}$ which is satisfied for $\alpha>15$ precludes the existence of a di-neutron or di-proton. At $\alpha=16$, $A_{\nu\nu}$ is about 5 percent smaller than the neutron-proton singlet interaction with $E(\mathrm{H}^2$, singlet$)=0$.

[20] Tuve, Heydenburg, Hafstad, Phys. Rev. **49**, 402 (1936).

[21] Breit, Condon and Present (to appear in the Physical Review).