The Force at an Anchored Cathode Spot

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The steadying effect of a metallic anchor for the cathode spot of a mercury-pool arc has been used to make measurements on the force at the cathode. Two methods have been used. One was based on the depression of the meniscus edge carrying the cathode line, due to the pressure on it. This measured only the horizontal component which was exerted on the liquid and gave 19 dynes/amp. The other used a torsion pendulum to measure the total horizontal force component and gave 33 to 43.5 dynes/amp. over the current range zero to 10 amp. This was supplemented by a determination of the direction of the force based on the deflection of the cathode line by a magnetic field, giving 35 to 66 dynes/amp. for the resultant in the same current range. The results show interaction of current streams from different emitting areas to increase the force, and are not inconsistent with other determinations.

THE measurement of the force against a liquid mercury-arc cathode is inherently difficult. The force is probably not normal to the general liquid level but is more likely to be perpendicular to small portions of that surface which, due to the violent agitation, make considerable angles with the horizontal. And when, as seems to happen occasionally,[1] a spot plunges quite beneath the surface, the matter looks still more hopeless.

Recourse to confining the spot to a small area of mercury surface as Kobel has done[2] may well increase the pressure through the constriction of the arc path above the spot, although the arc tube could be designed so as not to do this. There is an additional objection to Kobel's method which will appear later.

An anchored[1] cathode spot gives a more stable and better defined condition thereby lending itself the better to measurements. Incidentally, the equality of arc drop between free and anchored spot arcs probably means that the results for the anchored spot are applicable to the free spot.

MEASUREMENTS USING THE DEPRESSION OF THE MENISCUS EDGE

Two methods were available. Direct observation of the cathode line formed at the junction of mercury with a vertical Mo strip immersed in it showed that as the arc current was increased, the meniscus edge and cathode line were depressed. By measuring the lowering of the line as the current was varied and applying the theory of the meniscus shape, it was possible to calculate the force per cm at the cathode line.

Consider in Fig. 1 the Mo sheet A against which the mercury meniscus M rises to tangency at point Q at height y_0 above the free liquid surface. Let the angle between the meniscus and the vertical at any point be θ, the height be y, the pressure against the liquid be P. Then

$$\sin\theta = 1 - y^2/\alpha - \gamma^{-1}\int_0^y P dy, \tag{1}$$

where $\alpha = 2\gamma/\rho g$, γ = surface tension, ρ = density.

For simplicity we suppose first that P is constant between θ_0, at the meniscus edge, and θ_1, corresponding to some point below the edge, and that it is zero beyond; and second, that the trace of the meniscus seen in the figure is circular between θ_0 and θ_1, the radius of curvature being r. With these assumptions

$$\int_0^{y_0} P dy = -Pr\int_{\theta_1}^{\theta_0} \cos\theta d\theta$$
$$= Pr(\sin\theta_1 - \sin\theta_0). \tag{2}$$

If we denote the width of the pressure strip between θ_0 and θ_1 by w, then

$$w = r(\theta_1 - \theta_0), \tag{3}$$

whence, combining,

$$\int P dy = Pw(\sin\theta_1 - \sin\theta_0)/(\theta_1 - \theta_0). \tag{4}$$

Applying Eq. (1) to the meniscus at y_0 and y_1, respectively, we then have,

[1] L. Tonks, Physics **6**, 294, 298 (1935).
[2] E. Kobel, Phys. Rev. **36**, 1636 (1930).

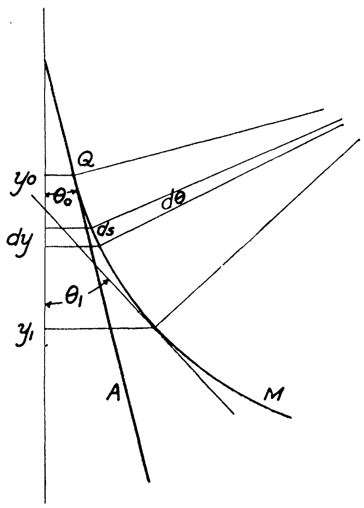

FIG. 1. Analysis of meniscus shape.

$$y_0^2/\alpha = 1 - \sin\theta_0 - (Pw/\gamma)\frac{\sin\theta_1 - \sin\theta_0}{\theta_1 - \theta_0}, \quad (5)$$

$$y_1^2/\alpha = 1 - \sin\theta_1. \quad (6)$$

Suppose, further, that w is so small that y_0 does not differ appreciably from y_1. Subtraction leads to

$$Pw = \gamma(\theta_1 - \theta_0), \quad (7)$$

giving the force per unit length at the cathode spot in terms of θ_0, the angle of the anchor, and θ_1 as determined from Eq. (6), where y_1 is the observed height of the cathode line.

It is to be noted first that any force against the anchor itself will not be counted; second, that the force is assumed to be highly localized; and third, that it is certainly pulsating corresponding to the passage back and forth of the small emitting areas which compose the cathode line. The last consideration, however, would be without influence on the validity of the results if there were no consequent motion of the liquid. Actually such motion is small and mainly confined to the narrow zone occupied by the cathode line.

The second limitation can be lifted by not adopting such definite suppositions. We note that quite generally, aside from any constant radius of curvature of the liquid or localization of P,

$$Pdy = P\cos\theta ds.$$

Eq. (1) then becomes

$$y_0^2/\alpha = 1 - \sin\theta_0 - \gamma^{-1}\int P\cos\theta ds$$

in place of Eq. (6). Thus y_0^2 varies linearly with the integral so that

$$\int P\cos\theta ds = -\gamma\Delta y_0^2/\alpha,$$

where Δy_0^2 is the change in y_0^2 from its value for zero current.

The experiment was tried in the simple tube, shown in Fig. 2, which had been constructed for another purpose. The mercury contained a trace of Al to aid in wetting the Mo strip S, which projected through the Hg surface when the tube was tilted back slightly (8°). The cathode line occupied the whole 3-cm-long front side of straight portion of S for the current range used.

The angle of tilt, θ_0, was measured by reflecting light from the Hg-wet Mo surface. The cathode line and mercury level heights were measured with a cathetometer. The meniscus height for zero current was found by observing the height at which surface curvature ceased, using the reflection of a movable light source.

From this height, which was 0.24 cm above the pool level, and from θ_0, the surface tension of the

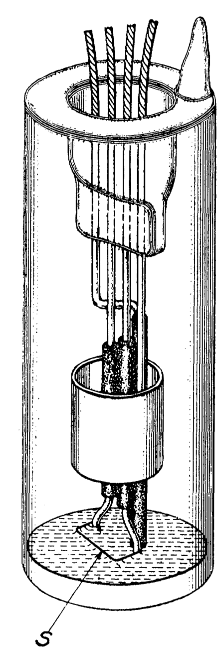

FIG. 2. Tube for measurement of depression of meniscus by cathode line.

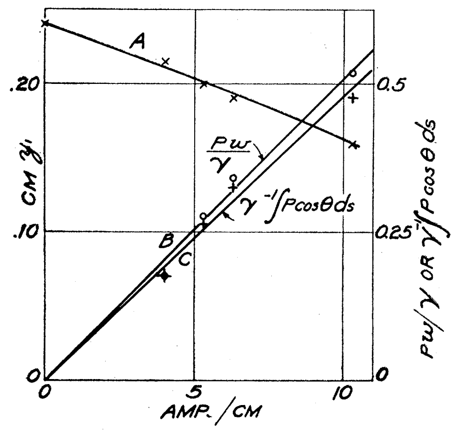

FIG. 3. Results of meniscus depression measurements.

mercury was found through α in Eq. (6). It came out to be 450 dynes/cm, which is acceptable within the limits of accuracy of this experiment and of uncertainty in the true value.

The depression of the cathode line with increasing current density is shown as Curve A in Fig. 3. Currents up to 31 amp. were used. The corresponding values of Pw/γ are plotted in Curve B as calculated from Eqs. (6) and (7). They show a linear variation of Pw/γ of 0.051 amp.$^{-1}$ cm, and differ only slightly from the more rigorous $\int P \cos \theta ds/\gamma$ represented by Curve C. This is no justification for the correctness of the assumptions underlying the calculation of Pw/γ, for a large pressure, exerted where the liquid had become almost horizontal, would not cause a large discrepancy. Using $\gamma = 450$ dynes cm^{-1} as determined above, the force on the cathode by this method is found to be

$$Pw/i = 23 \text{ dynes/amp.},$$

where i is the arc current.

Measurements of the Force Against an Anchor

The second method used was to measure the force against the anchor itself. This force includes not only that exerted directly on the exposed anchor surface, but also the $\int P \cos \theta ds$ of the first method. This is demonstrable either on the general ground of momentum balance or by an analysis of the hydrostatic pressures on the two sides of the anchor.

An experimental tube with the anchor hung from a simple gravity pendulum was built but deemed unsatisfactory because of the change in relation between anchor and Hg surface with deflection.

Finally a torsion pendulum was built in which a cathode spot, anchored at either of two symmetrically placed vanes, V_F or V_B of Fig. 4, caused a deflection of the mirror M against the torsion of the tungsten wires W_1 and W_2. Here the vanes always maintained the same position relative to the liquid surface. Two vanes were used because, while the moment arm of either was uncertain, the sum of the two, which became effective when both vanes were used in succession and the sum of their torques taken, was measurable and fixed. The glass ring R served to steady the rotating system with respect to the mercury surface. The mica disk extending across the whole tube kept the arc glow away from the mirror where its light would interfere with observations of the deflected light beam. The elastic constant of the system was determined by the oscillation method. The torsion wires were of tungsten 0.0178 cm in diameter and each 3 cm long, and the constant was 9.90×10^3 dynes cm/radian. V_F was 1.00 cm and V_B 1.08 cm in the clear. They were cleaned electrolytically in a H_2SO_4 bath, then hydrogen fired at 1200°C. They were mounted on the suspension without handling and then each was coated on one side, the back, with aquadag to prevent the cathode line from forming there; and in addition stripes 1 to 2 mm wide were painted along the vertical edges on the fronts of the vanes to confine the cathode line to the plane portion of the anchors.

The assembled tube was baked out on the pump for one hour at 500°C. Because of their unfavorable position, the vanes could only be heated to dull redness with the high-frequency coil, but the anode was brought almost to whiteness repeatedly. Despite the poor heating of the anchors, the spot anchored without undue difficulty. A permanent magnet helped by keeping the free spot in the vicinity of the vane on which it was to settle. The tube was run with 15 amp. from each vane in turn until a sticking vacuum was obtained; then the tube was sealed off.

A preliminary tube of similar design contained two anodes, one, A_1, in the azimuth of V_B, the other, A_2, 90° from it. In that tube it was determined that the cathode force was independent of the position of the anode. Changing the temperature of a water bath surrounding the cathode pool between 15°C and 33°C had no appreciable effect on the observed force.

On this same tube the cathode line was subjected to a magnetic field of some 25 gauss from an electromagnet. The magnet was supported in various positions with respect to the meniscus so that the field lay parallel to the edge, vertical, or horizontal and perpendicular to the edge. The field affected the position of the cathode line along the meniscus, thus changing the moment arm and consequently the deflection. After making the obvious correction, a small change in deflection remained with the parallel orientation, which could be explained by electrodynamic forces within the mercury. Other smaller differences are thought not to be significant.

In using the final tube it was noticed that when the arc current was suddenly stopped in order to confirm the zero following a deflection reading, the light beam consistently overshot the zero by several mm and only returned to the original zero in the course of a minute. The discrepancy increased with arc current. It was due to the convection currents set up in the mercury by the vapor blast from the cathode line. Their effect was to decrease the deflection, and by just the amount of the zero deflection. Accordingly, deflections were measured from the deflected zero to which the beam returned temporarily.

In this way deflections of the torsion system were measured for a series of arc currents, with the spot anchored on each vane for each current value. It was not possible to control which vane the spot would anchor on when it was started with the spark coil, but in several starts it would go to the desired vane in one of them. With each reading, a record was made of the "center of gravity" of the cathode line with respect to the vane itself in order that a more accurate value of the moment arm might be used in the calculations. This was desirable because it often happened that the whole available vane width was not occupied, particularly at the lower currents. Measurements were made from 1.5 amp., the lowest current which was sufficiently stable, up

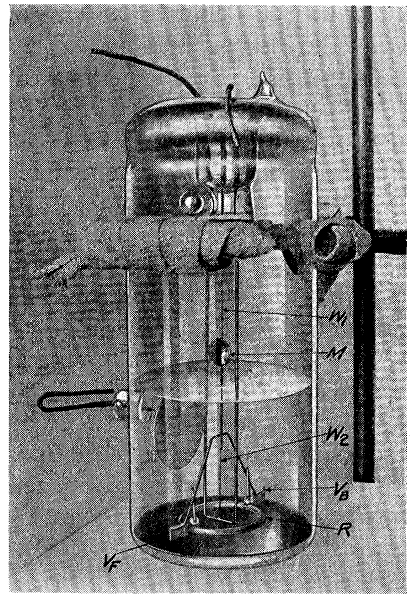

FIG. 4. Torsion tube.

to 10 amp., the largest current that the tube was designed for.

To use the compensation afforded by the double vane, the simple theory of the deflecting system, for an assumed axis displaced from the actual axis, was worked out. Since readings on the two vanes were not taken with equal currents, those at nearly equal currents were paired and the deflections corrected on the assumption that for the small differences involved, deflection was proportional to current. The force to be measured, f, is given by

$$f = i(L_1/i_1 + L_2/i_2)/l$$

and the error, δ, in axis position by

$$\delta = L_1/f - l_1 = l_2 - L_2/f$$

where L_1 and L_2 are the observed torques (given directly by deflections) on the two vanes, i_1 and i_2 the corresponding arc currents, $i(\sim i, \sim i_2)$ is the current to which f applies, l_1 and l_2 are the assumed radial distances of the two cathode lines, and l is the distance between the cathode lines on the two vanes. The analysis of the data on these lines gives values of δ ranging from $+0.08$ to -0.08 cm, compared to a value of l averaging 5.5 cm. This shows that the analysis is not justified by the accuracy of the measurements, but it sets a limit of 3 percent to the accuracy of the results.

The results of the measurements are shown in Fig. 5. It is evident that the force (horizontal component, of course) rises slightly faster than linearly with current. The ratio of force to current increases from 33 dynes/amp. at zero to 42.5 at 10 amp. It must, of course, be current density, rather than total current which is significant. But the observations of cathode line length which had been made during the course of the readings were too uncertain, and the distribution of spots along the line were too uneven to make direct correlation between force and current density possible. Suffice it to say that above 8 amp. the full available meniscus edge was occupied, a matter of 0.7 cm on V_F and 0.9 cm on V_B.

Measurement of the Direction of the Cathode Force

It has already been noted that the force here measured is only a horizontal component. It was observed, however, in applying the magnetic field to the cathode line in the earlier tests, that when the field direction was turned from the horizontal through the current flow direction to parallelism with the anchor, the displacement of the cathode spot reversed. Thus, at some position of the field intermediate between these two extremes, the cathode line was undisplaced. It is to be expected that the line of action of the mechanical force on the cathode coincides with the mean direction of current flow near the cathode. By deflecting this current, the magnetic field creates a force component parallel to the meniscus edge which moves the cathode line to one side or the other. The cathode line will be unaffected only when the field lies in a neutral direction coincident with that of the current, and this neutral direction is the direction of the cathode force itself. Knowledge of the neutral angle makes the calculation of this force from its horizontal component possible.

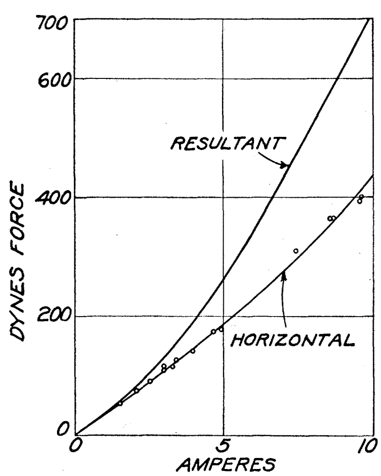

FIG. 5. Force on anchor.

In order to measure this direction, the simple arc tube shown in Fig. 6 was constructed. The Mo vane, 2 cm long by 1 cm wide, was tipped at 45° to the tube axis to permit a wide range of angles between anchor and pool by tilting the tube and using both sides of the anchor. The anchor was centered in the spherical bulb so that with the bulb half full of mercury the mercury surface always cut the anchor at the same level.

The Mo anchor was cleaned electrolytically in a H_2SO_4 bath, next it was hydrogen fired, then coated along the oblique edges with aquadag in order to confine the cathode line to the plane surface. The assembled tube, of hard glass, was baked out at 500°C for one hour; the Mo anode was repeatedly heated to whiteness by induction; the anode was thoroughly degassed; the mercury was distilled in; and the tube was run with a free spot until anchoring had been achieved on both anchor faces. The tube was run with 10 amp. until a sticking vacuum was obtained, then it was sealed off the pumping system.

The field for the actual tests was that from an old instrument magnet mounted on a horizontal shaft whose center line bisected the center line between N and S poles perpendicularly. The other end of the shaft carried an angular scale from which the orientation of the magnetic field was read. The experimental tube was mounted off the poles of the magnet and adjusted so that the meniscus edge lay in the extension of the center line of the shaft. The pool surface and a straight wire passing from pole to pole through two holes in the centers of the pole faces were useful in making the alignment and in determining the angular scale reading for a horizontal field. By sighting across vane and

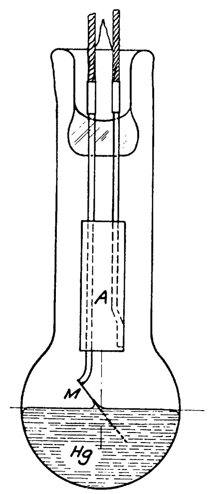

FIG. 6. Tube for observation of magnetic deflection of cathode spot.

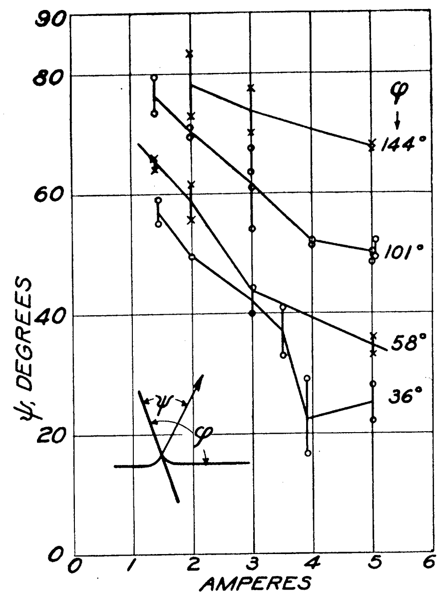

FIG. 7. Neutral magnetic field angles at about 36 oersteds.

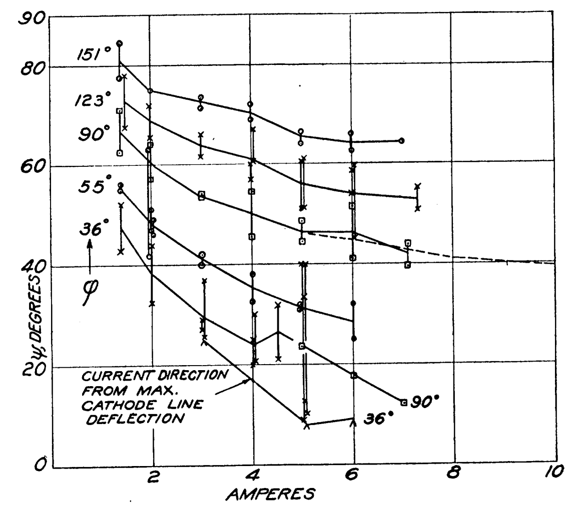

FIG. 8. Neutral magnetic field angles at about 17 oersteds.

wire, the angle φ between vane and liquid, as shown in Fig. 7, was then found.

Two distances between anchor center and pole ends, 27 and 43 mm, were used, resulting in field strengths of 36 and 17 oersteds, respectively. At a given anchor angle and current value, the two neutral positions of the field were determined alternately several times. The average angle ψ between neutral direction and that of anchor was calculated for each position, and the pairs of values were plotted, joined by a vertical line, in Figs. 7 and 8. Tests at one or two anchor angles indicated that the effect of water cooling was to decrease the difference between the two values of a pair. In view of the rapid evolution of Hg vapor from the cathode spot, it is unlikely that a difference in pressure of a few microns in the body of the tube, such as is occasioned by an increase in temperature, would affect the phenomena at the cathode. Accordingly, a difference in direction of current flow on this account is not to be expected. The sense of the magnetic field will, however, affect the flow of current through the body of the tube, which, in turn, can influence the neighborhood of the cathode spot. Such an effect appears very definitely in Fig. 8 between 5 and 7 amp. on the 90° curve. Besides the pairs of readings which are continuous with the lower current readings and lie near 45°, there is a set at 12° to 23° corresponding to a peculiar glow distribution which sometimes occurred with the field in one sense. Another cause for different angle values for the pair lies in the dissymmetry of the magnetic field which undoubtedly existed under the experimental conditions.

A few field angle readings were made for the extreme deflection of the cathode line using an anchor angle of 36°. The points designated in this way in Fig. 8 were then calculated on the assumption that the current direction lay at right angles to the magnetic field. But the deflected spot was far from horizontal so that the whole phenomenon would have to be referred to a different set of axes. It is therefore reasonable that the points found this way do not agree with those determined by a null deflection method.

The decrease in angle between neutral direction and anchor plane as the current increases reflects the depression of the meniscus edge and shows the effect of the cathode surface immediately adjacent to the cathode line in directing the current flow. If this were the only factor, affecting the flow in the region where the cathode pressure develops we should, however, find that the various curves for different anchor angles coincided; since the shape of the meniscus at the anchor is but slightly affected by the anchor angle. For instance, the assumption used in Eq. (2) leads to an almost coincident set of curves independent of anchor angle, except for a small region at the larger current values.

The decrease in ψ with anchor angle undoubtedly arises from effects at some distance from the meniscus edge, which tend to make the current pass midway between the two surfaces bounding the region. This requires an interaction between the current elements from the different elementary emitting areas so that the whole cathode phenomenon cannot be considered as the simple sum of individual elementary processes.

Calculation of the Resultant Cathode Force

The neutral magnetic field angles found for the 90° anchor angle can now be applied to the force data. Since it had not been possible to reach 10 amp. in the magnetic deflection tests because of spot freeing with the field applied, the 90° curve of Fig. 8 was extrapolated as shown by the dashed line. Dividing the horizontal force component as given by Fig. 5 by the sin ψ from Fig. 8 the resultant shown in Fig. 5 was found.

The force is seen to increase more than proportionally to the current. From a value of 35 dynes/amp. at small currents it rises to 66 at 10 amps. Apparently each emitting area of the cathode line exerts not only a pressure arising from it alone but also an additional force arising from the interaction of the currents from the neighboring areas, already evidenced above.

This action, taking place at a greater distance from the meniscus edge than that due to the individual spot, is consistent with the behavior of the neutral angle with increasing current.

If we ascribe the force F of 35 dynes/amp. arising from a single emitting area to the pressure of plasma electrons[3] we have

$$F=2.74(1-f)T_e^{\frac{1}{2}},$$

where $(1-f)$ is the fraction of the arc current carried by ions at the cathode and T_e is the electron temperature. Corresponding to an electron temperature of 20,000° this calls for a positive ion fraction of 9 percent, for 40,000° it calls for 6 percent. These values are within limits set by other considerations.

A preliminary experiment on the rate of vaporization q from an anchored cathode spot has given the very rough value of 2×10^{-4} g coulomb^{-1}. This is 12 times Kobel's value, but there is good reason for believing it to be the more reliable. Now the reaction from the cathode force must be the momentum carried toward the anode by particles leaving the cathode plasma. Thus their average velocity, neglecting angular distribution, must be F/q. If we suppose these particles to acquire this velocity as ions in the cathode plasma, the voltage through which they would have to be accelerated is

$$V=300(m/2e)(F/q)^2=3.2 \text{ v}$$

for $F=35$, $q=2\times10^{-4}$. It does not seem at all unreasonable that there should be a potential maximum of this order of magnitude in the plasma at the cathode and that the degree of ionization within the plasma should be high. Kobel's value of q would lead to a voltage difference of over 400 v.

The force value of 66 dynes/amp. at the largest current used, with the indication that still higher values occur for more concentrated emission, is at variance with Kobel's[2] value of 40 for a surface supposedly completely covered by a cathode spot. Anchoring experiments indicate, however, that possibly half or two-thirds of Kobel's arc current was coming from a cathode line at the edge of his small cathode pool. The large horizontal component of the corresponding force would not appear as a hydrostatic pressure and would, therefore, go undetected, thus ex-

[3] L. Tonks, Phys. Rev. **46**, 278 (1934).

plaining his lower experimental result for the concentrated cathode condition.

Working with a poorer vacuum and using solid cathodes, Easton, Lucas and Creedy[4] have found considerably lower force values than these, though in the same order of magnitude. Two copper cathodes gave 7 and 5.5 dynes per amp. respectively and zinc gave 14. Their curves bend upward slightly.

Summary

The horizontal component of the force at a cathode line between mercury and a vertical anchor has been measured, first roughly by the depression of the meniscus edge, then more accurately by means of a torsion system.

The orientations of a magnetic field which deflected the cathode line neither one way nor the other were determined as current and anchor angle were varied.

Assuming that these neutral directions gave the directions of the resultant cathode force, this resultant was calculated from the horizontal components and was found to vary from 35 dynes amp.$^{-1}$ at low currents from the 1-cm wide vane to 66 dynes amp.$^{-1}$ at 10 amp.

Both the neutral angle experiment and this result show that the effects of concentrating emission at the cathode line are not simply additive.

[4] Easton, Lucas and Creedy, Elec. Eng. **53**, 1454 (1934).

AUGUST 1, 1936 PHYSICAL REVIEW VOLUME 50

The Separation of Gaseous Isotopes by Diffusion

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The design and use of a 34-member apparatus of the Hertzian type for the separation of gaseous isotopes by diffusion are described. Results of tests of the speed of various types and numbers of separation members are given. Tests on mixtures of carbon dioxide and argon at a few millimeters pressure give the same results as tests on the neon isotopes. The equilibrium time, from ten to fifteen hours, and the separation factor, 80, of a 24-member apparatus, working on mixtures of such gases, are found to be independent of the initial mixing ratios. Results obtained with the 34-member apparatus on methane, to concentrate C^{13}, and on nitrogen, to concentrate N^{15}, are shown to be in reasonable agreement with appropriate theory. Methane containing 16 percent of $C^{13}H_4{}^1$, and nitrogen containing 6 percent of $N^{14}N^{15}$, instead of the normal 1 percent and 0.6 percent, respectively, have been produced.

In this laboratory[1] an apparatus has been constructed similar to that described by Hertz,[2] which utilizes the difference in the thermal velocities of molecules of different weights to separate the isotopes of a gas. In principle, the apparatus works as follows. The mixture of gaseous isotopes it is desired to separate is circulated through a series of porous-walled tubes by diffusion pumps, as shown in Fig. 1. The tubes, which are alternately short and long, are surrounded by individual Pyrex jackets, connected to the low-pressure side of mercury diffusion pumps, as shown in the figure. The gas which diffuses through the wall of a long tube is pumped back into the system at a point just to the right of the adjacent short tube. Now this gas will be a little richer in the lighter isotopes than that which passed on toward the left. Similarly, the portion of this gas which diffuses through the wall of the adjacent short tube on the right will be still lighter, and so on. Thus, on the average, there is a net transfer of the lighter isotopes toward the right, or "light" end of the apparatus, while the

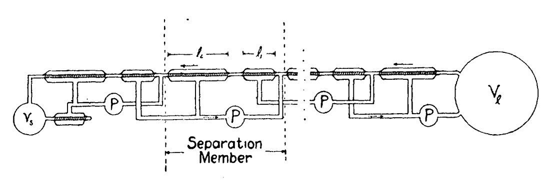

Fig. 1. Arrangement of porous walled tubes.

[1] Norman Bridge Laboratory of Physics, Pasadena.
[2] G. Hertz, Zeits. f. Physik **79**, 108 (1932).

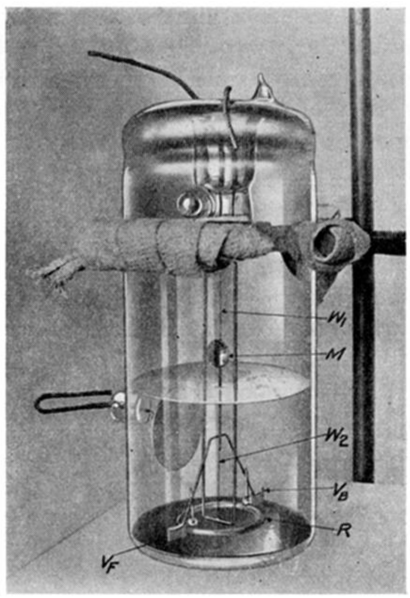

FIG. 4. Torsion tube.