The Variation of the Internal Friction and Elastic Constants with Magnetization in Iron. Part II

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The experimental method described in Part I of this paper is applied to measure the variation of the rigidity modulus and torsional decrement with magnetization in unannealed and annealed Armco iron. Formulae are derived which evaluate the contributions to the moduli and decrements arising from eddy currents induced in the vibrating specimen by the change in magnetization produced by the varying stress. These formulae contain the magnetostriction and Wiedemann effect coefficients, which, accordingly, have been measured. All effects of the eddy currents are thus eliminated from the final data, which are discussed in the light of recent ferromagnetic theory.

Rigidity Modulus and Torsional Decrement

In Part I of this paper[1] W. T. Cooke has described a precise experimental method for measuring the elastic moduli and coefficients of internal friction of solid substances, and has presented data which show the observed variation of Young's modulus E, and the longitudinal decrement Δ^E, with magnetization in specimens of unannealed and annealed Armco iron. The observed variation of the rigidity modulus G and the torsional decrement Δ^G with magnetization in the same specimens is shown in Figs. 1 to 3.

Both the longitudinal and torsional decrements are subject to correction, for, as was first pointed out by Kersten,[2] the energy dissipated in vibrating magnetized substances receives a contribution arising from eddy currents induced by the change in magnetization which accompanies the vibrational stress in the medium. It is evident that the magnitude of this contribution must depend on the shape of the specimen. In order, therefore, to arrive at a quantity characteristic of the material alone the observed values of Δ^E and Δ^G must be corrected by amounts which, it will be shown, depend in part on the longitudinal magnetostriction and on the Wiedemann effect, respectively. Accordingly these properties of the specimen material must be measured.

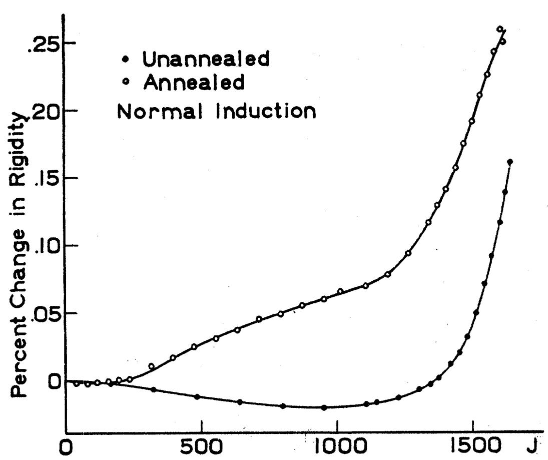

Fig. 1. Variation of the rigidity modulus with magnetization in unannealed and annealed Armco iron.

Magnetostriction and the Wiedemann Effect

Magnetostriction

The magnetostriction measurements were made on the ellipsoid which yielded the magnetic data reported in Part I.[3] The "form effect" dis-

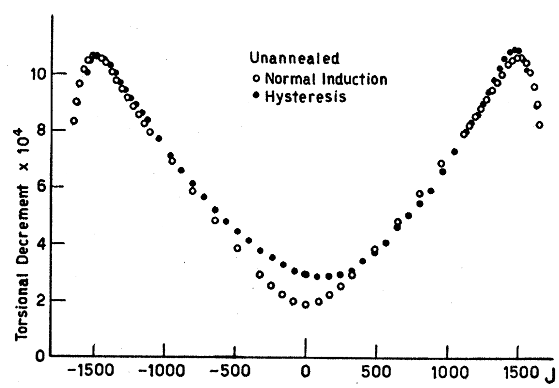

Fig. 2. Variation of the torsional decrement with magnetization in unannealed Armco iron.

[1] Cooke, Phys. Rev., preceding paper.

[2] Kersten, Zeits. f. tech. Physik **11**, 463 (1934).

[3] The experimental method is described in a forthcoming paper by D. Kirkham.

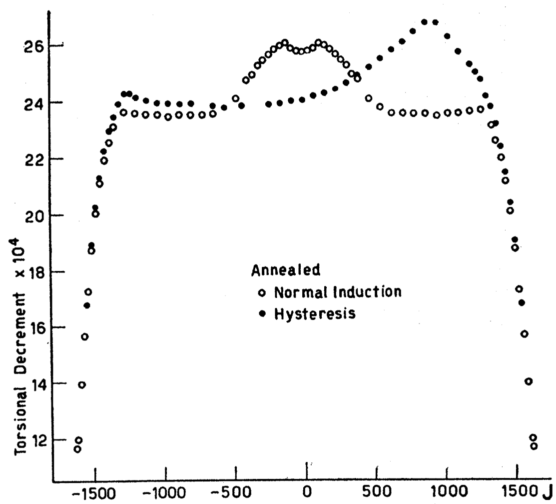

FIG. 3. Variation of the torsional decrement with magnetization in annealed Armco iron.

cussed by Becker[4] is negligible for such an ellipsoid. The results are shown in Fig. 4. The precision of the measurement of $\delta l/l$ is $\pm 0.10\times 10^{-6}$.

The unannealed specimen is evidently not isotropic, probably in consequence of a nonrandom distribution of internal stresses, or of microcrystalline axes. The anisotropy is manifest in an abnormally low value of Poisson's ratio, computed on the assumption of isotropy; in the value of the magnetization at high fields, which is larger before than after annealing; and in the fact that the Villari reversal does not appear in the unannealed material. The last named phenomenon has been discussed by Bitter.[5]

Wiedemann effect

It has been shown[6] that the Wiedemann effect is merely a complicated case of magnetostriction: the twist produced when a current is passed through a magnetized rod is a result of the rotation of the principal axes of strain in each volume element which occurs when a circular field, H_θ, is superposed on a longitudinal field, H_z. This twist may be regarded as due to a magnetostrictive shearing stress $-\eta' H_\theta$, where η' is a function of H_z alone provided $H_\theta^2 \ll H_z^2$. For a solid cylinder of radius R carrying a longitudinal current I e.m.u., the circular field at a distance r from the axis is $2Ir/R^2$. If the cylinder is twisted through an angle ϕ radians per cm length, the shearing strain is $r\phi$, and the resulting stress of elastic origin is therefore $Gr\phi$. The observed twist is determined by the condition $-\eta' 2Ir/R^2 + Gr\phi = 0$, whence $\eta' = GR^2\phi/2I$. It is this quantity η' that must be known in order to evaluate the contribution of the eddy currents to the torsional decrement. It can be determined by measurement of the Wiedemann effect at currents sufficiently low that $H_\theta^2 \ll H_z^2$; under this condition ϕ is proportional to I.

An accurate measurement of η' is possible only when the specimen is a long thin wire. Preliminary calculation indicated, however, that the cylinders used in the decrement measurements would yield data adequate for the present purpose. The experimental method is as follows: The magnetizing coil assembly described in Part I is mounted on a rigid frame with its axis vertical. The specimen is clamped at the upper end, and to the lower end is attached a brass rod which terminates in mercury. The twist of the uniformly magnetized portion of the specimen is measured by observing the deflection of a light beam successively reflected from two mirrors, lying in approximately perpendicular planes through the axis of the rod, and at the same height, but mechanically attached to opposite ends of the portion of the rod under test. The deflection of the beam is thus unaffected by rotation of the system as a whole, and depends only on the difference of rotation of the two mirrors. To measure the small deflections (a few tenths of a mm at 1 m distance) a slit illuminated by monochromatic light is used as a source, of which a lens between the source and the mirrors forms an image about 1 m from the mirrors. The

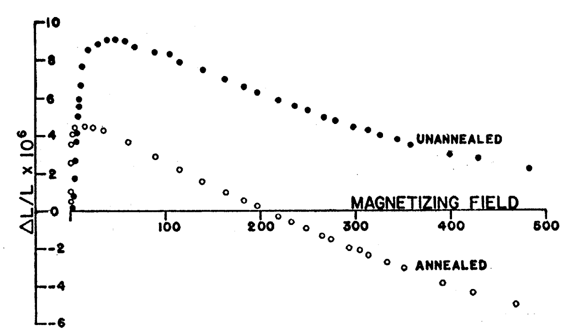

FIG. 4. The longitudinal magnetostriction of unannealed and annealed Armco iron.

[4] Becker, Zeits. f. Physik **87**, 547 (1933).
[5] Bitter, Phys. Rev. **42**, 705 (1932).
[6] Fromy, J. de phys. **7**, 13 (1926); Ann. d. physique **8**, 626 (1927).

image is viewed through a microscope equipped with a 32 mm objective and a filar micrometer eyepiece.

The deflection which accompanies a reversal of the current through the specimen is observed for different values of the current, the magnetization being held fixed. The initial slopes of the current deflection curves thus obtained are found by graphical extrapolation.

Because of the shortness and thickness of the cylinders used in the measurements, it is necessary to consider the effect of torques exerted on the ends of the specimen. It has been stated by Williams[7] that the end of the specimen, being a pole in a circular field, is subject to a torque of amount $2mI$, where m is the pole strength. This torque, if present, would introduce an error not greater than 1.5 percent of the maximum observed value of η'. Williams' formula, however, is obtained by attributing physical existence to the fictitious magnetic volume and surface distributions $\rho=-\nabla\cdot\mathbf{J}$ and $\sigma=\mathbf{n}\cdot\mathbf{J}$. Actually these distributions are obtained by a transformation of a volume integral, and lead to correct expressions for the total force and torque on a magnetized body, but not for the force and torque on a part of the body. Direct application of the formula $d\mathbf{F}=\mathbf{J}\cdot\nabla\mathbf{H}d\tau$ for the force $d\mathbf{F}$ on a volume element $d\tau$ shows at once that the only component of force exerting a torque about the cylinder axis is

$$dF_\theta=(J_z\partial H_\theta/\partial z+J_r\partial H_\theta/\partial r)d\tau.$$

Calculation of the resulting torque on the assumption of the worst possible conditions of nonuniformity of J and H at the ends of the specimen indicates that the error is less than 1 percent of the maximum value of η', and is therefore within the precision of the measurements, which is about 10 percent of this maximum value.

The variation of η' with intensity of magnetization in the annealed specimen is shown in Fig. 5. The relation between twist and current in the unannealed specimen is nonlinear, and shows much hysteresis. Hence it is possible only to set an upper limit of 0.5×10^5 on the value of η' in this material.

[7] Williams, Phys. Rev. **32**, 296 (1911).

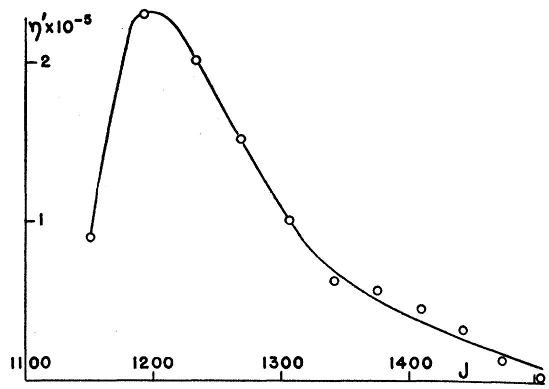

FIG. 5. The Wiedemann effect in annealed Armco iron.

THE DISSIPATION DUE TO EDDY CURRENTS

Kersten[2] has developed a theory which permits an evaluation of the contribution to Δ^E arising from eddy currents in a medium for which the simplifications of Becker's theory of ferromagnetism are valid, and which is in the remanent state of magnetization. For the present purpose a more complete theoretical analysis is necessary.

The specimen will be assumed to be an infinitely long, thin cylinder of radius R and resistivity ρ e.m.u., placed in a longitudinal magnetizing field H. Associated with the field are an induction B, and a longitudinal magnetostriction e. It will further be assumed that the changes resulting from a small variation of strain or of field are reversible. Under these assumptions, simple thermodynamic considerations[8] lead to relations between the alternating components of induction and stress, and those of field and strain.

Alternating longitudinal elongations, $\partial w/\partial z$, and associated lateral contractions (determined by the condition that the lateral tension shall vanish) produce an alternating induction B_z; the circular currents induced set up an alternating longitudinal field H_z, with consequent alternating longitudinal tension Z_z. The relations are

$$B_z=\mu H_z+4\pi\eta\partial w/\partial z, \qquad (1)$$

and

$$Z_z=-\eta H_z+E\partial w/\partial z, \qquad (2)$$

where μ is the incremental permeability, $\partial B/\partial H$,

[8] Cf. Houstoun, Phil Mag. **21**, 78 (1911); Becker, Zeits. f. Physik **87**, 547 (1933); Voigt, *Lehrbuch der Kristallphysik* (1928), pp. 223, 563, 939.

at constant strain,[9] E is Young's modulus at constant field, and $\eta = E(de/dH)$.

Alternating torsional strains, $\partial u_\theta/\partial z$, produce a circular induction B_θ; the longitudinal and radial currents induced set up an alternating circular field H_θ with consequent torsional stress θz.[10] The relations are

$$B_\theta = \mu' H_\theta + 4\pi\eta' \partial u_\theta/\partial z, \tag{3}$$

and

$$\theta z = -\eta' H_\theta + G\partial u_\theta/\partial z, \tag{4}$$

where μ' is the permeability, B/H (*not* $\partial B/\partial H$),[11] and G is the rigidity at constant field.

If the specimen is undergoing longitudinal displacements described by the formula

$$w = w_0 \exp i(\beta z + nt), \tag{5}$$

the components of magnetic field, magnetic induction, electric field and current density are determined by the electromagnetic field equations, in which the radiation terms may be neglected; by the auxiliary relations between magnetic field and induction, and between electric field and current density; and by the conditions of finiteness of the field vectors at $r=0$ and $r=\infty$, and of continuity of H_z and B_r at $r=R$. The usual relation, $B_z = \mu H_z$, is to be replaced by Eq. (1). Solution of these equations subject to these boundary conditions gives[12]

$$H_z = 4\pi i(n\beta w/\mu)[-1 + FI_0(hr)], \tag{6}$$

where

$$F = hK_1(\beta R)/[\mu\beta I_1(hR)K_0(\beta R) + hI_0(hR)K_1(\beta R)],$$
$$h^2 = \beta^2\mu/\mu' + 4\pi n\mu i/\rho,$$

and the I's and K's are Bessel functions as defined in reference 12. At the frequencies of vibration here considered the real part of h is negligibly small, so that the last equation reduces to $h = m\sqrt{i}$, where $m = (4\pi\mu n/\rho)^{\frac{1}{2}}$.

In accordance with Eq. (2) the field H_z produces a stress whose average value over the cross section of the rod is given by the expression

$$-\eta(H_z)_{\text{av}} = 4\pi(\eta^2/\mu) \times [1 - (2F/hR)I_1(hR)]\partial w/\partial z. \tag{7}$$

Like the stress of elastic origin, this contains $\partial w/\partial z$ as a factor, but the coefficient of $\partial w/\partial z$ is, in general, complex. The real part is equivalent to a change of Young's modulus, and measures the nondissipative reaction of the currents upon the mechanical vibration. The imaginary part is equivalent to a viscous coefficient, and measures the dissipation of energy by the currents. The changes in E and Δ^E thus produced by the currents are evaluated by comparing Eq. (2), in which the value of $-\eta H_z$ given by Eq. (7) has been substituted, with the usual stress-strain relation in a cylinder with elastic and viscous stresses. Thus

$$\frac{\Delta E}{E} = \frac{4\pi\eta^2}{E\mu}\left\{1 - \frac{2}{a}\frac{Q + (\mu b/a)(P^2+Q^2)}{(1+\mu bQ/a)^2 + (\mu bP/a)^2}\right\}.$$

and

$$\Delta_e{}^E = \frac{4\pi^2\eta^2}{E\mu}\frac{2}{a}\frac{P}{(1+\mu bQ/a)^2 + (\mu bP/a)^2},$$

where

$$P = (\text{ber}\, a\, \text{ber}'a + \text{bei}\, a\, \text{bei}'a)/(\text{ber}^2 a + \text{bei}^2 a),$$
$$Q = (\text{ber}\, a\, \text{bei}'a - \text{bei}\, a\, \text{ber}'a)/(\text{ber}^2 a + \text{bei}^2 a),$$
$$a = (f/f_c)^{\frac{1}{2}}, \quad b = \beta RK_0(\beta R)/K_1(\beta R),$$

and $f_c = \rho/8\pi^2\mu R^2$.

At frequencies far below or above f_c the following simplifications are possible:

When $f \ll f_c$, $\dfrac{\Delta E}{E} = 0$ and $\Delta_e{}^E = \dfrac{1}{2}\dfrac{\pi^2\eta^2}{E\mu}\dfrac{f}{f_c}$.

When $f \gg f_c$,

$$\frac{\Delta E}{E} = \frac{4\pi\eta^2}{E\mu}\left\{1 - \sqrt{2}\left(\frac{f_c}{f}\right)^{\frac{1}{2}}\frac{1+c\sqrt{2}}{(1+c)^2+c^2}\right\}, \tag{8}$$

and

$$\Delta_e{}^E = \frac{4\pi^2\eta^2}{E\mu}\sqrt{2}\left(\frac{f_c}{f}\right)^{\frac{1}{2}}\frac{1}{(1+c)^2+c^2}, \tag{9}$$

where

$$c = \mu b/(a\sqrt{2}) = \mu(f_c/2f)^{\frac{1}{2}}\beta RK_0(\beta R)/K_1(\beta R).$$

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[9] The μ obtained from the magnetization curve is the incremental permeability at zero stress, and exceeds the μ of Eq. (1) by $4\pi\eta^2/E$. This difference, as well as the differences between the adiabatic and isothermal values of μ, η, and E, is small enough to be neglected in this computation.

[10] The stress and strain notation is that of Love, *Treatise on the Mathematical Theory of Elasticity*, 4th edition, pp. 56, 90, 288.

[11] A small increment of H perpendicular to the original direction merely rotates the H and B vectors through the small angle $B_\theta/B = H_\theta/H$.

[12] See Gray, Mathews and Macrobert, *Bessel Functions*, 2nd edition, Chap. III; Russell, *Alternating Currents*, 2nd edition, Chap. VII.

Introduction of the values of η and μ at remanent magnetization, as given by Becker's theory,[13] and of a new reference frequency f_g, such that $f_c=(45\pi/512)f_g=0.2766f_g$, leads to formulae which agree with Kersten's[14] provided $c\ll 1$.[15]

If the specimen is undergoing torsional displacements described by the formula

$$u_\theta = r\phi = r\phi_0 \exp i(\beta' z + nt),$$

the quantities to be evaluated instead of H_z and $(Z_z)_{\text{av}}$ are H_θ and the total torque

$$2\pi\int_0^R \theta z r^2 dr$$

exerted across a surface normal to the cylinder axis. The analysis parallels that for longitudinal vibrations and leads to the following formulae:

$$\frac{\Delta G}{G} = \frac{4\pi\eta'^2}{G\mu'}\left\{1 - \frac{4}{a}P'\right\},$$

and

$$\Delta_e{}^G = \frac{4\pi^2\eta'^2}{G\mu'}\frac{4}{a}\left(Q' - \frac{2}{a}\right),$$

where

$$P' = (\text{ber}\, a\, \text{ber}' a + \text{bei}\, a\, \text{bei}' a)/(\text{ber}'^2 a + \text{bei}'^2 a),$$

$$Q' = (\text{ber}\, a\, \text{bei}' a - \text{bei}\, a\, \text{ber}' a)/(\text{ber}'^2 a + \text{bei}'^2 a),$$

$$a = (f/f_c')^{\frac{1}{2}} \quad \text{and} \quad f_c' = \rho/8\pi^2\mu' R^2.$$

When $f \ll f_c'$, $\dfrac{\Delta G}{G} = 0$ and $\Delta_e{}^G = \dfrac{1}{6}\dfrac{\eta'^2}{G\mu'}\dfrac{f}{f_c'}$.

When $f \gg f_c'$,

$$\frac{\Delta G}{G} = \frac{4\pi\eta'^2}{G\mu'}\left\{1 - 2\sqrt{2}\left(\frac{f_c'}{f}\right)^{\frac{1}{2}}\right\}, \tag{10}$$

and

$$\Delta_e{}^G = \frac{4\pi^2\eta'^2}{G\mu'}2\sqrt{2}\left(\frac{f_c'}{f}\right)^{\frac{1}{2}}. \tag{11}$$

Application of the Theory

In all the measurements under discussion the formulae for $f \gg f_c$ are valid. Since the second terms in Eqs. (8) and (10) are small compared with the first, the measured values of the elastic moduli are very nearly those at constant B rather than at constant H. For it follows immediately, on setting B_z and B_θ equal to zero in Eqs. (1), (2) and (3), (4), that

$$[E]_B - [E]_H = 4\pi\eta^2/\mu$$

and

$$[G]_B - [G]_H = 4\pi\eta'^2/\mu',$$

and these are precisely the differences given by the first terms of Eqs. (8) and (10). As was pointed out by Kersten, this means that the alternations of magnetic flux occur only near the surface of the specimen.

The effect of the eddy currents on the torsional decrement is small, being nowhere more than one percent of the measured value for either of the specimen materials.

In the measurements of the longitudinal decrement for the unannealed material, two cylinders of different diameter were used (cf. Part I, Fig. 6). The agreement between the results obtained on these two specimens at low magnetization indicates that here the eddy current contribution is negligible, since, by Eq. (9), it should vary approximately in inverse proportion to the diameter. The second rise in the curves at high magnetization, however, is entirely due to eddy currents, and, together with the difference in behavior of the two specimens, disappears when the contribution of the eddy currents to the decrement is subtracted. Similarly, the eddy currents are responsible for the change of slope observable at the highest magnetizations in the curve of Δ^E *vs.* J for the annealed specimen (Part I, Fig. 7). The corrected values of Δ^E for this substance are given below in Table I.

It should be noticed that the formula, Eq. (9), involves two factors, η and μ, which must be evaluated by graphical differentiation of experimental curves, and which, moreover, are raised to powers higher than the first. It must be remembered also that the specimen is not actually infinitely long and not uniformly magnetized. Consequently the correction for eddy currents cannot be computed with great precision; but the success of the theory in explaining the second rise of the curves in Fig. 6, and in removing the discrepancies between the observations on different specimens at high magnetization, leaves

[13] Kersten, Zeits. f. Physik **71**, 553 (1931); **82**, 723 (1933).
[14] Kersten, reference 2, Eqs. (3) and (4).
[15] The terms involving c result from the finite wave-length prescribed by the mechanical oscillation. Kersten's analysis assumes this wave-length to be infinite.

TABLE I. *Variation of the decrements and elastic moduli at constant B with magnetization in annealed Armco iron. Corrected for the effect of eddy currents. Normal magnetization.*

J gauss	$\Delta^E \times 10^4$	$(\Delta E/E) \times 10^2$	$\Delta^G \times 10^4$	$(\Delta G/G) \times 10^2$
0	25.91	0	25.75	0
40	25.90		25.79	−0.0020
80	26.13		25.88	−0.0023
119	26.30		26.06	−0.0017
159	26.28	−0.0037	25.97	−0.0003
199	26.45		25.84	+0.0003
239	26.75		25.66	0.0005
279	26.30		25.44	0.0013
318	26.34	+0.0058	25.26	0.0103
358			24.94	
398	25.88		24.76	0.0187
477	25.83	0.0105	24.09	0.0244
557	25.19		23.78	0.0307
637	24.55	0.0289	23.55	0.0366
716	23.98		23.51	0.0446
796	23.19	0.0360	23.51	0.0481
875	22.11		23.51	0.0546
955	21.43	0.0556	23.46	0.0595
1034	20.43		23.51	0.0650
1113	19.13	0.0779	23.55	0.0700
1193	17.09	0.1041	23.60	0.0776
1233	15.48		23.64	
1269	14.27	0.1532	23.45	0.0938
1342	11.60	0.1926	22.94	0.1167
1376			22.34	0.1295
1410	8.81	0.2323	21.73	0.1412
1444			20.98	0.1571
1475	6.84	0.2625	20.01	0.1743
1506			18.75	0.1914
1537	5.30	0.2910	17.27	0.2106
1565			15.66	0.2257
1593	4.14		13.95	0.2421
1619			11.98	0.2582
1623	3.65	0.3077	11.66	

little doubt that it is adequate for the present purpose, and that the corrected values of the decrement are characteristic of the material itself.

Correction of the observed variations of E and G to obtain the values at constant B is easily accomplished, since the correction terms are the second in Eqs. (8) and (10) and these are small. The correction to constant H includes the first terms in these formulae and is less reliable. $[\Delta E/E]_H$ for the unannealed specimen follows the observed curve (Part I, Fig. 5) to $J=1400$; beyond this point the corrected curve flattens out, reaching a maximum of 0.12 percent at about 1490 gauss and decreasing to 0.11 percent at 1630 gauss. For the annealed specimen, the flattening out begins at $J=1250$, and the curve rises more slowly, reaching about 0.17 percent at 1630 gauss. The effect of eddy currents on the rigidity is negligible for the unannealed specimen; for the annealed specimen it is negligible except between $J=1100$ gauss and $J=1500$ gauss, where $[\Delta G/G]_H$ is lower than the observed value, differing therefrom by perhaps 0.025 percent at $J=1300$ gauss.

The corrected values of the decrements and of the changes in the elastic moduli at constant B, for the annealed material in normal magnetization are given in Table I.

Theoretical Discussion

According to the domain theory of ferromagnetism,[16] a ferromagnetic crystal consists of a large number of "domains," each of which is permanently magnetized to saturation; the magnetization process for the crystal as a whole consists of a reorientation of the magnetization vectors in the domains. At high fields all the domains are magnetized in the field direction; in the demagnetized state their magnetization vectors are distributed at random among the various stable directions determined by crystalline forces and internal stresses. The transition from the demagnetized condition to technical saturation consists of two stages. In the steep part of the magnetization curve the random distribution changes to one in which most of the domains are magnetized along the stable direction nearest that of the field—this direction itself having meanwhile rotated toward the field. Above the knee of the magnetization curve the magnetization vectors are gradually rotated into the direction of the field. In the first stage, the redistribution occurs partly by a continuous and reversible growth of the more favorably oriented domains at the expense of the less favorably oriented, and partly by a sudden transition of the domain magnetization from a less stable to a more stable orientation, with dissipation of energy.

Stress plays a role similar to that of magnetic field in determining the possible directions of magnetization and the relative stability of these directions; and conversely, any change in the direction of magnetization of a domain (except a reversal) is accompanied by magnetostrictive changes in its dimensions. The application of a

[16] See especially Bozorth and Dillinger, Phys. Rev. **41**, 353 (1932); Becker, Physik. Zeits. **33**, 909 (1932).

tension or shearing stress to an unmagnetized ferromagnetic therefore produces redistributions and rotations similar to those produced by the field, and these give rise to a magnetostrictive component of strain in addition to the strain of purely elastic origin. At very high fields this is no longer true, because the direction of magnetization is that of the field and is unaffected by small stresses. There are two observable results of this magneto-mechanical process, both evident in the data on the elastic constants and decrements of *annealed* iron. First, the ratio of strain to corresponding stress is greater in the demagnetized state, and, in fact, throughout the steep part of the magnetization curve, than at high fields; and second, the energy dissipated per cycle of vibration is greater at low than at high magnetizations.[17]

It remains to explain why the internal stresses present in the *unannealed* material, which increase the magnetic hysteresis, nevertheless decrease the internal friction at zero magnetization. This is not difficult. For there is this important difference between the effect of field and stress: the former brings about a difference of stability between orientations 180 degrees apart and the latter does not. In relatively stress-free iron the stable directions are the six (positive and negative) crystal axes, and stress and field alike can reorient through 90 degrees, partly reversibly and partly irreversibly. In unannealed material, on the other hand, the stable directions are determined by the internal stresses, and are two in number, differing by 180 degrees. Transitions produced by the field are possible and are attended by considerable loss of energy, so that magnetic hysteresis is large. But transitions produced by stress cannot occur, and the only magneto-mechanical process which contributes to the ΔE and ΔG effects is a reversible rotation. As the field is increased, however, opposite orientations begin to differ in energy sufficiently that a small stress, acting on a domain magnetized in the less stable direction, can make this orientation unstable and produce an irreversible transition. Thus at fields sufficiently large to cause considerable difference in stability, yet not large enough to complete the transition process, a high mechanical decrement is observed.

A more detailed explanation of the variation of ferromagnetic internal friction with magnetization must await a more complete analysis of the magnetization process. In connection with the ΔE and ΔG effects, however, certain calculations are of interest.

Kersten's[18] theory of the ΔE effect for material under high internal stress is not applicable to iron, but Akulov's[19] theory for iron under low internal stress should be applicable to the annealed material. The theory contains formulae which describe the variation of E with magnetization at low magnetizations, and for the total change between demagnetization and saturation. The observed variation at low magnetizations is too small to yield any information, and even the highest fields used in this work are still considerably below that necessary for saturation. Nevertheless a comparison between the greatest observed change in E and G and the total change predicted by the theory should at least indicate whether the theory is correct as to order of magnitude.

The total changes in the elastic moduli between demagnetization and saturation in quasi-isotropic polycrystalline iron are given by the formulae

$$\Delta E/E=\tfrac{3}{5}\chi_0\lambda_{100}{}^2E/J_\infty{}^2, \tag{12}$$

and

$$\Delta G/G=(9/5)\chi_0\lambda_{100}{}^2G/J_\infty{}^2, \tag{13}$$

where J_∞ is the saturation magnetization, λ_{100} the longitudinal magnetostriction for a single crystal saturated in the (100) direction, and χ_0 the observed initial susceptibility of the material. The first of these formulae appears in Åkulov's paper. The second can be derived therefrom in the following manner: It is assumed that the compressibility is independent of magnetization (since no magneto-mechanical process follows the application of a uniform pressure), and in both the demagnetized and saturated state the material is isotropic. If the compressibility of an isotropic substance is to remain constant then

[17] Cf. also the experiments of Becker and Kornetzki, on torsional stress cycles in annealed iron, Zeits. f. Physik **88**, 634 (1934).

[18] Kersten, Zeits. f. Physik **85**, 708 (1933).

[19] Akulov and Kondorsky, Zeits. f. Physik **78**, 801 (1932); **85**, 661 (1933).

variations in E and G must be related by the formula $\Delta G/G=(3G/E)\Delta E/E$.

Eq. (12) has proved only moderately successful in accounting for the ΔE effect in polycrystalline nickel.[20] In this connection it should be noted that it has never been possible to obtain entirely satisfactory agreement between the measured values of the elastic constants of a polycrystalline material and those calculated from the properties of the constituent crystals. This problem has been thoroughly discussed by Bruggeman.[21] In Akulov's theory, uniform stress is assumed throughout the aggregate of crystals and the strains are averaged. The alternative method, originally used by Voigt,[22] is to assume uniform strain and to average the stresses. If this method of averaging is used in Akulov's theory, the factor 1/5 in Eq. (12) is replaced by $(1/20)(c_{11}-c_{12})^2/G^2$, where the c's are the elastic constants of the crystals.

TABLE II. *Comparison of theoretical and experimental values of the elastic moduli and of their variation with magnetization.*

		$E\times10^{-11}$	$G\times10^{-11}$	$k\times10^{-11}$	$\frac{\Delta E}{E}\times10^2$	$\frac{\Delta G}{G}\times10^2$
Theoretical	Uniform stress	19.45	7.42	17.3	0.309	0.393
	Uniform strain	22.80	8.90	17.3	0.101	0.128
Experimental		19.98	8.46	10.4	0.17	0.26

The experimental values of Young's modulus and the rigidity, and of their changes with magnetization up to the highest magnetizations attained, are compared with the values predicted by the theory on the alternative assumptions of uniform stress and uniform strain, in Table II. The following values of the constants were used in the calculation: $J_\infty=1710$; $\lambda_{100}=16.2\times10^{-6}$;[23] $\chi_0=28.7$;[24] $c_{11}=2.37\times10^{12}$, $c_{12}=1.41\times10^{12}$, $c_{44}=1.162\times10^{12}$.[25] The bulk modulus k is included to show the unreliability of the methods of averaging even when the question of uniform stress or uniform strain is not involved. The agreement between theory and experiment affords as good a confirmation of the theory as can be expected from measurements on polycrystalline materials. The desirability of measurements on single crystal specimens is obvious.

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[20] Siegel and Quimby, Phys. Rev. **49**, 663 (1936).
[21] Bruggeman, Zeits. f. Physik **92**, 561 (1934).
[22] Voigt, *Lehrbuch der Kristallphysik*, p. 962.
[23] Average value from measurements of Honda and Mashiyama, Sci. Rep. Tohoku Imp. Univ. **15**, 755 (1926).
[24] Part I of this paper.
[25] From measurements of Goens and Schmid, Naturwiss. **19**, 520 (1931).