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The Variation of the Internal Friction and Elastic Constants with Magnetization in Iron. Part I

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A new and precise method is described for measuring the elastic constants and coefficients of internal friction of solid substances. The method is applied to investigate the variation of Young's modulus and the longitudinal coefficient of internal friction of annealed and unannealed Armco iron with magnetization at room temperature. Data are given which describe the behavior of the iron in three different magnetic states, *viz.*, at different points on the normal induction curve and on a hysteresis cycle, and in the anhysteretic state obtained when an alternating field, the amplitude of which is gradually reduced to zero, is superposed on a steady magnetizing field. The observed coefficients of internal friction of the unannealed specimen require correction, and this, together with the torsional data and theoretical interpretation of the results, are given in Part II of this paper.

THE amplitude of vibration at all points of a free body which is vibrating in one of its normal modes diminishes with the time in a manner described by the formula

$$u=u_0 \exp(-\Delta f t), \quad (1)$$

in which u denotes the amplitude, f the frequency of vibration, t the time, and Δ a quantity whose value depends, in the first instance, upon the substance and upon the nature of the stress associated with the particular mode of vibration. The name "internal friction" has been given to the property of solid substances of which this attenuation in amplitude is a measure. An alternative view of the phenomenon follows from a consideration of the rate of transfer of organized vibrational energy into heat.[1] Thus, if W denotes the energy dissipated per cm^3 per cycle of vibration, then it can be shown that, in accordance with Eq. (1),

$$W=\xi S_0^2, \quad (2)$$

where S_0 is the stress amplitude, $\xi=\Delta/P$, and P is the appropriate elastic modulus.

The coefficients of internal friction, the ξ's, and the logarithmic decrements, the Δ's, of a substance resemble the elastic moduli in that their definition requires the specification of a stress. The types of stress here dealt with are the stretch and twist associated, respectively, with the longitudinal and torsional vibration of a slender bar. The corresponding decrements are hereafter denoted by the symbols Δ^E and Δ^G, and, conformably, the Young's and rigidity moduli by the symbols E and G.

The energy dissipated in a vibrating ferromagnetic body receives contributions from three distinct sources: first, internal friction of the sort present in nonferromagnetic materials; second, eddy currents induced in the medium by the change in magnetization produced by the varying stress; and third, internal friction of purely ferromagnetic origin. No theory of internal friction in nonferromagnetic solids has yet been written, nor has any attempt been made to lay an experimental basis for one. While it is hoped that the experimental method here described may encourage such an effort, the present research is concerned primarily with the two contributions to the internal friction which are associated with the magnetization. For, as will be shown in Part II of this paper, recent developments in ferromagnetic theory permit these to be correlated with other magneto-elastic properties of the material, and the success of such correlation lends added support to the theory.

Experimental Method

The experimental method utilizes the properties of a separately excited composite piezoelectric oscillator constructed by cementing a suitably cut right circular cylinder of crystalline quartz to one end of a cylinder of specimen material of identical cross section. The cement is hard shellac, softened by heating, and the join is made under pressure. The system is freely suspended, and a sinusoidally varying potential difference is applied between electrodes of gold leaf affixed in proper position to the quartz.[2]

[1] Kimball and Lovell, Phys. Rev. **30**, 948 (1927).

[2] Details of the method for preparing the quartz crystals are given in papers by Balamuth, Phys. Rev. **45**, 715 (1934), and Rose, Phys. Rev. **49**, 50 (1936).

In consequence of the piezoelectric stress which accompanies the electric field in the quartz, a stationary state of forced longitudinal or torsional vibration is established in the composite oscillator. A system of this sort offers two methods for measuring the elastic moduli and coefficients of internal friction of the specimen material.[3]

The first is that adopted by Zacharias,[4] who measured the variation of E and Δ^E with temperature in unmagnetized single and polycrystalline nickel over the temperature range 30°C to 400°C, and Siegel,[5] who measured the variation of E and Δ^E with magnetization and temperature in polycrystalline nickel over the same temperature range. In this method the amplitude of the applied potential difference is held constant, and means are provided for varying its frequency and for measuring the amplitude of the current which flows to the oscillator. The current amplitude varies critically with frequency in the neighborhood of certain "resonance frequencies" at which the amplitude of the forced vibration passes through a maximum. Values of the moduli and decrements are deduced from the observed nature of this variation. The method is generally applicable for the measurement of the moduli, but the precision of the decrement measurement is bad when the value of this quantity is less than about 10^{-3}. Accordingly it is not suitable for studying the internal friction of iron and most nonferromagnetic solids.

In the present method the composite oscillator forms one arm of an a.c. bridge which is excited by an alternating voltage of constant amplitude and variable frequency. Values of the moduli and decrements are deduced from the observed variation of the electrical impedance of the oscillator with frequency. The precision of the decrement measurement fails when this quantity is large of the order 10^{-3}, so that the two methods perfectly supplement each other.

The electro-mechanical theory of the single crystal oscillator,[6] and the mechanical theory of the two-part composite oscillator[7] have been set forth in detail by previous writers. The theoretical treatment of the present system follows so closely that of these others that it need here only briefly be outlined. The charge which must be placed on the gold leaf electrodes in order to establish a potential difference $\mathcal{E}$ between them can be separated into two parts: first, a part of the ordinary capacitative sort, equal to $C'\mathcal{E}$, where C' is the interelectrode capacity; and second, a part required to neutralize the piezoelectric charge produced by the vibrational strain in the quartz, and proportional to the space average value of this strain, taken along the cylinder. Accordingly, the current I, which flows to the oscillator, may be written in the form

$$I = C'd\mathcal{E}/dt - K'\epsilon dS_{av}/dt, \tag{3}$$

where K' is a geometrical constant, ϵ the appropriate piezoelectric coefficient, and S_{av} the average strain. Now S_{av} is proportional to $\mathcal{E}$, so that the ratio $Z = \mathcal{E}/I$ is independent of $\mathcal{E}$ and is the electrical impedance of the oscillator. The further development of the theory is concerned with the evaluation of S_{av}.

In order properly to estimate the effect of the layer of cement on the decrement measurements, the composite oscillator is initially regarded as composed of three coaxial cylinders of identical cross section and different lengths and materials. The actual piezoelectric stress in the quartz cylinder is represented by harmonically varying surface tractions over its end faces. The equation of motion of each medium is assumed to be of the form

$$\rho\partial^2 u/\partial t^2 = P(1 + T\partial/\partial t)\partial^2 u/\partial x^2, \tag{4}$$

where u is the particle displacement, ρ the density, P the appropriate elastic modulus, and T a dissipative coefficient which, in accordance with the theory of free vibrations in bars, is related to Δ by the formula

$$\Delta = 2\pi^2 fT. \tag{5}$$

The particle displacement in each medium is

[3] As a matter of fact, the composite piezoelectric oscillator has been used to measure the internal friction of solids by four different methods, of which the two here described are regarded as superior. For descriptions of the others see Quimby, Phys. Rev. **39**, 345 (1932), and reference 7.

[4] Zacharias, Phys. Rev. **44**, 116 (1933).

[5] Siegel and Quimby, Phys. Rev. **49**, 663 (1936).

[6] Dye, Proc. Lond. Phys. Soc. **38**, 399 and 457 (1926); Van Dyke, Proc. Inst. Radio Eng. **16**, 742 (1928).

[7] Quimby, Phys. Rev. **25**, 558 (1925); Balamuth, reference 2.

given by an expression of the form

$$u=\{A\exp(\alpha+i\beta)x+B\exp -(\alpha+i\beta)x\}\exp int, \quad (6)$$

where

$$n=2\pi f, \qquad \beta=n(\rho/P)^{\frac{1}{2}}, \qquad \alpha=\tfrac{1}{2}n\beta T,$$

and it is assumed that T is sufficiently small that $n^2T^2\ll 1$. The six amplitude coefficients (A and B for each medium) are evaluated by means of the six simultaneous equations which express, respectively, the continuity of stress and displacement at the two interfaces and of stress at the end faces of the oscillator. S_{av} is calculated from the resulting expression for u in the quartz, and the electrical impedance of the oscillator follows at once from Eq. (3).

The expression for Z can be arranged in the form $1/Z=inC'+1/Z_m$. Furthermore, in the neighborhood of a resonance frequency the form of the expression for Z_m is precisely the same as that for the electrical impedance of an inductance, capacity and resistance in series. It follows that, near resonance, a composite oscillator is electrically equivalent to a series resonant electrical circuit of impedance Z_m shunted by a capacity C'. If Z_m be written in the form $Z_m=R+iX$, then, for a three-part oscillator,

$$R=(Kn/D)\{(M_1z_1/y_1)(\sec.^2 y_1-p_1)\times[1-(M_2y_3^2/M_3)p_2p_3]+(M_2z_2/y_2)\times(\sec.^2 y_2-p_2)[1-(M_1y_3^2/M_3)p_1p_3]+(M_3z_3/y_3)(\sec.^2 y_3-p_3)\times[1-(M_1M_2y_3^2/M_3^2)p_1p_2]\}, \quad (7)$$

$$X=(Kn/D)\{M_1p_1+M_2p_2+M_3p_3-(M_1M_2y_3^2/M_3)p_1p_2p_3\}, \quad (8)$$

where

$$D=2(1-\sec.\,y_2)[1-(M_1y_3^2/M_3)p_1p_3]-(p_2y_2^2/M_2)(M_1p_1+M_3p_3),$$

K = a constant,[8]

M_i = mass of a cylinder, ($i=1, 2, 3$)

L_i = length of a cylinder,

$z_i=\alpha_iL_i$,

$y_i=\pi f/f_i$,

$f_i=(1/2L_i)(P_i/\rho_i)^{\frac{1}{2}}$,

$p_i=(\tan y_i)/y_i$, (9)

and the subscripts, 1, 2, 3, refer, respectively, to the specimen material, the quartz, and the cement. It will be noted that the quantities f_i are the fundamental frequencies of free vibration of the cylinders separately. The resonance frequencies of the oscillator are those at which $X=0$.

In practice the lengths of the quartz and specimen cylinders are so adjusted that f_2 is equal to an integral multiple of f_1 within ± 50 cycles at 50 kc. Furthermore, the thickness, L_3, of the layer of cement is small of the order 10^{-3} cm. A scrutiny of Eqs. (7) and (8) reveals that, under these circumstances and near a resonance frequency of the system, the presence of the cement is without appreciable effect upon the values of R and X. Accordingly, the resonance frequencies are the solutions for f of the equation

$$M_1p_1+M_2p_2=0. \quad (10)$$

Again, the variation of X with frequency near resonance is given by the formula

$$X=\tfrac{1}{2}\pi K(M_1+M_2)\delta f, \quad (11)$$

where δf is the departure from the resonance frequency, and the resistance of the oscillator at resonance by the formula

$$R=\tfrac{1}{4}K(M_1\Delta_1+M_2\Delta_2)f_0, \quad (12)$$

where f_0 is the resonance frequency.

The equivalent electrical inductance L, and capacity C, of the two-part oscillator are related to X by the formulae $L=1/n_0^2C=X/4\pi\delta f$, from which and Eq. (11),

$$L=\tfrac{1}{8}K(M_1+M_2), \quad (13)$$

and

$$C=8/Kn_0^2(M_1+M_2). \quad (14)$$

The experimental procedure for obtaining the elastic modulus and decrement of the specimen material is as follows: It will be noted that the behavior of the quartz crystal alone is described by Eqs. (10) to (14) with M_1 set equal to zero. Hence f_2 is the observed resonance frequency of the crystal vibrating alone, and Δ_2 is related to the observed resistance of the crystal by the equation $R=\frac{1}{4}KM_2\Delta_2f_2$. K is evaluated with Eq. (11) from the observed variation of the reactance of the composite oscillator with frequency. Finally, P_1 and Δ_1 are calculated with Eqs. (9), (10) and (12), from the observed resonance

[8] For longitudinal vibrations, $K=1/b^2\epsilon^2$, where b is the width of a gold leaf electrode.

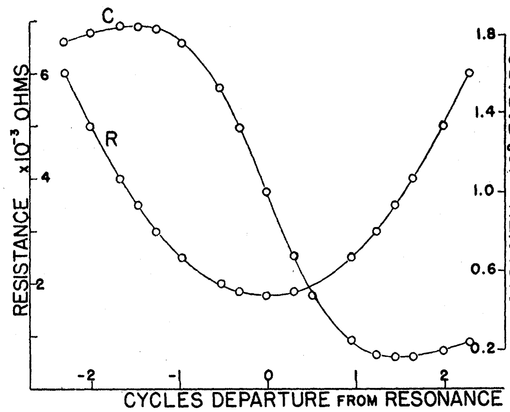

FIG. 1. Comparison of the observed and calculated values of the resistance and capacity required to balance an a.c. bridge in one arm of which is a composite piezoelectric oscillator.

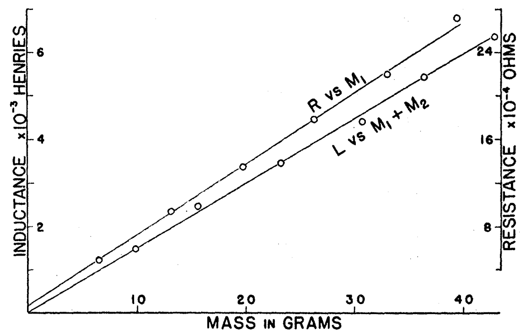

FIG. 2. The resistance of a composite oscillator varies linearly with the mass of the specimen cylinder, and its inductance with the sum of the masses of the quartz and specimen cylinders.

frequency and resistance of the oscillator at resonance.

The a.c. bridge employed is similar to that described by Stratton.[9] Essential features are adequate shielding and the use of a Wagner ground system. The ratio arms are those of a General Radio Co. type 210 unit, with an auxiliary variable condenser connected in parallel with the coil of lower resistance to compensate the distributed capacity of the other and the leakage resistance between the electrodes on the quartz. A condenser of about 30 cm capacity is connected in parallel with the oscillator in order that the reactance of this arm should be capacitative over the entire resonance range. The fourth arm of the bridge is a variable condenser connected in parallel with a Leeds and Northrup No. 4746 decade resistance box. Reliable measurements can be made of resistances as high as six megohms at 50 kc.

Power is supplied the bridge by a very stable vacuum tube oscillator[10] and amplifier. The frequency of the oscillator is controlled, in part, by a vernier condenser whose full scale represents a frequency change of 30 cycles at, roughly, 60 kc. Frequency differences can be measured with an accuracy of one-twentieth of a cycle. The vernier is frequency calibrated at intervals of 3 to 5 cycles by beating high harmonics of the oscillator against those of a piezoelectric clock.[11]

The apparatus functions remarkably well. The observations shown in Figs. 1 and 2 were made on a brass oscillator of square cross section 5 mm on edge. The brass cylinder was, initially, 18.64 cm long, and the resonance frequency 56,251 cycles. The points of Fig. 1 indicate the observed values of the resistance and capacity required to balance the bridge in the neighborhood of resonance. The curves, as drawn, represent the formulae of the theory. The agreement is noteworthy. The graphs of Fig. 2 show, in accordance with Eqs. (12) and (13), a linear variation of R with M_1 and L with (M_1+M_2). These data were secured with a succession of six oscillators, each obtained from its predecessor by sawing a half-wave of vibration from the brass cylinder, the resonance frequency thus remaining constant.

Fig. 3 is a cross section of the assembly for measuring the variation of the elastic moduli and internal friction of iron with magnetization. The oscillator is mounted in an evacuated glass tube on silk threads located at displacement nodes of vibration. The tube is surrounded, in turn, by a water jacket and by the magnetizing coils. The latter are designed to secure, as nearly as possible, a uniform magnetization over the length of the specimen. The end coils, B of Fig. 3, are each 190 turns of 5 mil copper strip 5 cm wide. The middle coil is six layers of No. 16 D.C.C. wire 21 cm long. Typical magnetic flux distribution curves are shown in Fig. 4.

Numerous experiments on iron oscillators of different lengths, oscillating at different harmonics of the fundamental mode, indicate that the experimental error in Δ_1 due to nonuniform

[9] Stratton, J. Opt. Soc. Am. **13**, 471 (1926).
[10] King, Bell Sys. Tech. J. **2**, No. 4, 31 (1923), Fig. 66.
[11] Dye, reference 6; Quimby, reference 3.

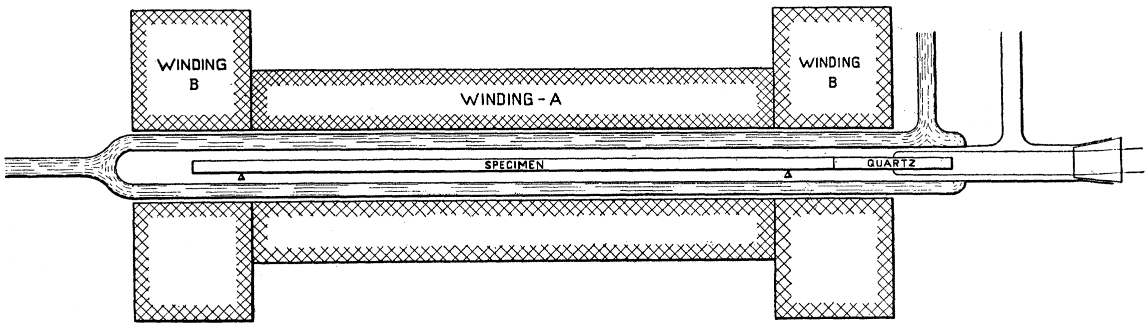

FIG. 3. Cross section of the assembly.

magnetization near the ends of the cylinder is never greater than 5 percent.[12] An error of about the same magnitude may occur in the measurement of the *percent change* of the elastic modulus with magnetization.

RESULTS

Specimen materials

The specimen materials upon which the observations here reported were made are cold-rolled Armco iron (hereafter designated "unannealed"), and the same substance after annealing at 930°C for 2 hours in hydrogen at atmospheric pressure and cooling to room temperature in 8 hours (hereafter designated "annealed"). The modulus, decrement and Wiedemann effect measurements were made on cylinders of circular cross section, 0.3 cm to 0.5 cm in diameter and about 30 cm long. The resonance frequencies at which the observations were taken lay in the neighborhood of 56 kc for the longitudinal vibrations and 39 kc for the torsional, so that there were seven half-waves of vibration in the specimen. The magnetic and magnetostriction measurements were made on an ellipsoid of revolution whose semi-axes are 0.197 cm and 15.24 cm.

Magnetic measurements

Modulus and decrement measurements were made subsequent to three different magnetic procedures: first, the iron was carried through many hysteresis cycles of magnetization at a chosen maximum value of the applied field, and left in the state corresponding to that maximum value (the data are hereafter designated "normal induction"); second, the iron was carried through a hysteresis cycle of magnetization and left at a chosen point on the cycle (the data are hereafter designated "hysteresis"); third, the iron was placed in a steady field of chosen value, and a 60-cycle alternating field was superposed thereon, the amplitude of which was gradually reduced to zero (the data are hereafter designated "anhysteresis").

The normal induction relations between magnetization J, and magnetizing field H, in the annealed and unannealed specimens are given in Table I. The hysteresis and anhysteresis data insofar as they differ from the normal induction, are given in Tables II and III. Values of J corresponding to values of H higher than those which appear in these two tables are in each case to be obtained from Table I.

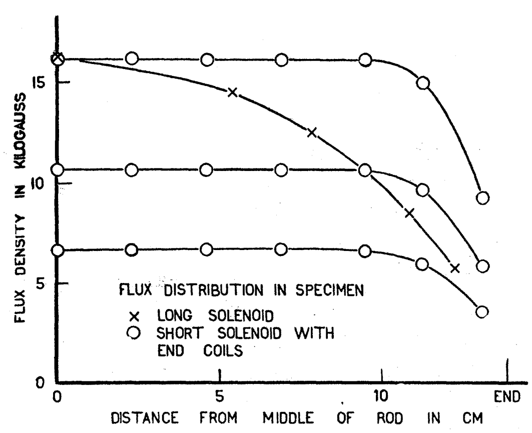

FIG. 4. Distribution of magnetic flux in the specimen. The curve with the crosses shows the distribution when the specimen is placed in a long solenoid.

[12] The same experiments reveal that, over the frequency range 36 kc to 72 kc, the coefficient of internal friction of unmagnetized, unannealed Armco iron is proportional to $f^{1/3}$. This is consistent with the data of Wegel and Walther (Physics **6**, 141 (1935)) who measured the frequency variation of the internal friction of many solids, and found values of the exponent of f ranging from $+0.54$ to -0.31.

TABLE I. *Normal magnetization data.*

Unannealed		Annealed	
H	J	H	J
1	19	0.2	20
2	117	.4	60
4	494	.6	470
6	797	.8	930
8	980	1	1070
10	1090	2	1144
12	1161	4	1188
15	1230	7	1219
20	1290	10	1238
30	1356	15	1254
40	1394	20	1267
60	1440	30	1292
80	1470	40	1313
100	1492	60	1349
150	1544	80	1380
200	1578	100	1406
250	1600	150	1462
300	1619	200	1506
350	1635	250	1539
400	1648	300	1567
450	1659	350	1590
500	1664	400	1609
550	1668	450	1626
		500	1641
		550	1656

TABLE II. *Hysteresis magnetization data.*

Unannealed			Annealed		
H	J		H	J	
0	570	570	0.0	1130	1130
5	1080	− 660	.2	1160	1000
10	1240	−1080	.5	1190	− 200
20	1340	−1290	1	1200	−1030
30	1380	−1350	2	1210	−1160
50	1430	−1420	5	1230	−1200
100	1500	−1500	10	1240	−1230
			15	1250	−1250

TABLE III. *Anhysteresis magnetization data.*

Unannealed		Annealed	
H	J	H	J
1	390	0.2	970
2	540	.4	1100
4	793	.6	1125
6	970	.8	1140
8	1085	1	1150
10	1160	2	1178
12	1206	4	1205
15	1249	7	1226
20	1301		

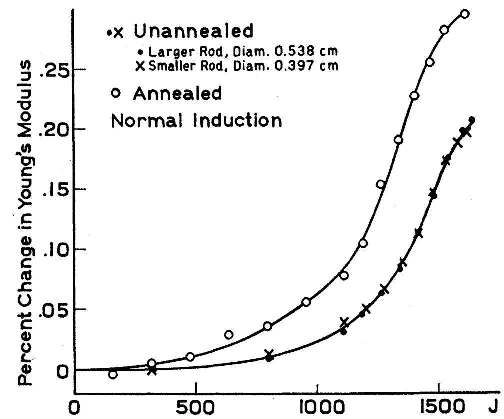

FIG. 5. The variation of Young's modulus with magnetization in unannealed and annealed Armco iron.

The coercive force, remanence, and initial susceptibility of the unannealed material are, respectively, 2.3, 570 and 7.4; and of the annealed material 0.46, 1130 and 28.7.

Young's modulus and longitudinal decrement

In order to test the validity of the theoretical expression for the contribution of the eddy currents to the internal friction, observations were made on two unannealed specimen bars of the same length and different diameters, *viz.*, 0.538 cm and 0.397 cm. Of these only the larger was subsequently annealed, since it was found that the internal friction in the annealed material is so large that the eddy current contribution is almost negligible.

The curves of Fig. 5 show the normal induction variation of Young's modulus E, with magnetization in unannealed and annealed Armco iron. The results obtained with the cylinders of different diameter are indistinguishable, as they should be, and supply a check on the method. The anhysteresis variation of E with J is the same as the normal induction. The value of Young's modulus is 18.6×10^{11} dynes/cm^2 for the unmagnetized unannealed material and 19.9×10^{11} dynes/cm^2 for the unmagnetized annealed material.

The points and circles of Fig. 6 show the observed variation of the longitudinal decrement Δ^E, with normal magnetization in the two unannealed specimens. The circles of which the right or left halves are blackened indicate the differences between the observed decrements of the two specimens and the contributions arising

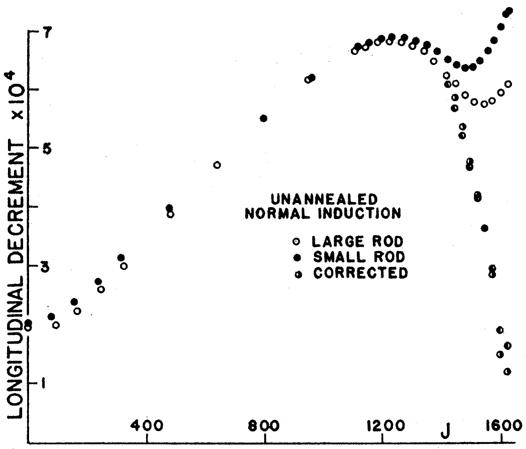

FIG. 6. The variation of the longitudinal internal friction with magnetization in unannealed Armco iron. The half-blackened circles indicate values of the decrement after the contribution arising from eddy currents has been subtracted.

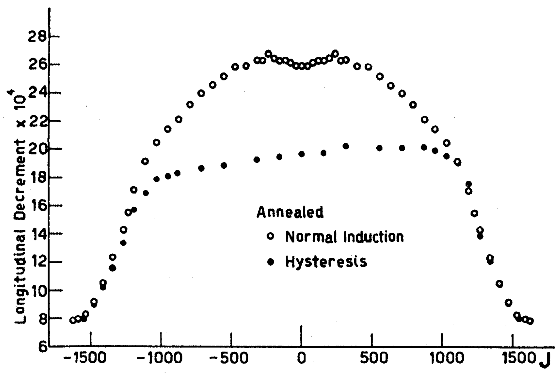

FIG. 7. The variation of the longitudinal decrement with magnetization in annealed Armco iron.

from eddy currents, when the latter are calculated in the manner described in Part II of this paper. It is clear that all effect of the form of the specimen is removed by this calculation, and that the course of the corrected curve in fact represents the variation with magnetization of that part of the internal friction which is of purely ferromagnetic origin.

The anhysteresis longitudinal decrement in the unannealed material differs inappreciably from the normal induction. The variation of the hysteresis decrement with magnetization is nearly the same at high magnetizations, but at low magnetizations the hysteresis values are all *larger* than the normal induction, and the variation does not show the marked asymmetry about the axis $J=0$ which characterizes the behavior of the torsional decrement (cf. Fig. 2 of Part II). The minimum value of the hysteresis decrement, which occurs at $J=0$, is about twice that of the normal induction.

The circles and points of Fig. 7 show, respectively, the normal induction and hysteresis variation of the longitudinal decrement in annealed Armco iron. It will be noted that here the effect of hysteresis is just the reverse of that in the unannealed material; the hysteresis values at low magnetizations are *smaller* than the normal induction. The eddy current contribution is responsible for most of the change in the slope of the curve at the highest magnetizations. Corrected values of Δ^E are given in Table I of Part II.