THE
PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893

VOL. 50, NO. 12 DECEMBER 15, 1936 SECOND SERIES

A Numerically Consistent Corpuscular Theory of Cosmic Rays*

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(Received October 2, 1936)

Theories for consideration divide themselves into two categories, those in which the primaries function as the measured entities, and those in which secondaries or higher order radiations play a significant or even a dominant role.

Primaries as measured entities. There exists a curve relating altitude and latitude, such that the rays which can just penetrate the earth's magnetic field at an assigned latitude can penetrate the atmosphere down to the altitude given by the curve. The curve cuts the sea-level axis at a latitude customarily associated with that at which the increase of intensity with latitude ceases for further increase of latitude. For all altitudes above the curve in question, and in particular for all altitudes above sea level, with latitude less than the sea-level critical latitude, there can be no variation of primary intensity with altitude; and, appeal must be made to secondaries for variation of intensity with altitude in these regions. In regions where the primary intensity varies with altitude, that variation must depend upon the energy spectrum of the incoming rays. The type of energy distribution necessary to secure an exponential law is determined. Its form depends upon the law of energy absorption in the atmosphere. Two cases are considered. They correspond to

$$-dE_x/dx=\alpha \quad (1); \qquad -dE_x/dx=\alpha+\lambda E_x \quad (2)$$

where E_x is the energy at the distance x from the top of the homogeneous water equivalent atmosphere, α corresponds to loss by ionization, and λE_x to loss by secondary production. Case (1) leads to an energy distribution which predicts at high altitudes a latitude variation which is enormous and out of all possible harmony with the facts. Case (2) leads to a reasonable high altitude latitude variation. However, the other difficulties concerned with a predicted absence of variation of intensity with altitude in equatorial regions remain.

Combination of primaries and secondaries as measured entities. The writer generalizes his former theory according to which the primaries produce secondaries in numbers which increase with increase of primary energy. The theory of the soft component is first considered, with (2) as a basis. The energy distribution function, $F(E)$ necessary to give an exponential law for the primaries is of the form $F(E)=B_p/(E+\alpha/\lambda)^p$; and, with this form the absorption coefficient μ_s is $\mu_s=(p-1)\lambda$. For mathematical convenience the quantity $W=(E+\alpha/\lambda)$, which is nearly the same as E is used throughout for purposes of discussion instead of the true energy E. It is shown that if the number of secondaries n_s accompanying a primary is given by $n_s=\gamma W_x^s$, then below the critical latitude $\mu_s=\lambda s$, and above the critical latitude $\mu_s=(p-1)\lambda$. If, in accordance with R. A. Millikan's observations we are to have μ_s the same above and below the critical latitude we must have $s=(p-1)$. If, moreover, μ_s has to have the value 0.5 per meter of water equivalent we must have $s=1.5$, and so $p=2.5$ when λ is assigned the value 0.33, which value is determined from the energy for penetration of the earth's magnetic field, 6×10^9 ev, at the critical latitude. The equality of μ_s above and below the critical latitude leads inevitably to an expression for the secondary intensity whose evaluation involves the integration of dW/W between a lower limit which is finite and an upper limit which is infinite unless we assign a finite upper limit E_M, beyond which $F(E)$ is zero. The concept of an upper limit E_M, and so an upper limit of W_M of W is consequently admitted; and, for purposes of illustration the limit taken is $E_M=5\times10^{10}$ ev. The intensity of the secondaries is proportional to $e^{-\mu_s x}\log_e W_M/W_\varphi$ below the critical latitude, where W_φ is the value of W for penetration of the earth's field at latitude φ. It thus shows a latitude variation. There is also a stronger latitude variation of the primary intensity which, however, does not vary with altitude below the critical latitude. The combined sea-level latitude variation is too great to correspond to the observations and it is necessary to look for the hard component to iron it out.

The hard component. A hard component showing no latitude effect when combined with the foregoing contri-

* A paper delivered at the Tercentenary Conference of Arts and Sciences at Harvard University, September, 1936.

butions to the intensity can be made to provide a satisfactory sea-level latitude variation, a satisfactory ratio of the intensities of the hard and soft components, and a satisfactory increase of latitude effect with altitude through the logarithmic term referred to above. A photon component can therefore fulfill the requirements; but, it is also shown that a charged particle component, operating mainly through its secondaries, will do equally well. Were it not for requirements of variation of latitude effect with altitude, it would be possible to tell the story of both the hard and the soft components as the outcome of the suitably chosen energy distribution in the incoming rays, which rays could then be all of one type. The increase of latitude effect with altitude demands, for the two types of primaries, properties which are different as regards production of secondaries.

Miscellaneous phenomena. It appears that the average energy of the hard component primaries is determined mainly by the upper limit of their energies while that of the soft component primaries is determined mainly by the lower limit for penetration of the earth's magnetic field. The conditions provided for the hard component secondaries are such that those secondaries completely outweigh the primaries in number; and, this explains why but few very high energy primaries are found in cloud chambers. The higher energy members of the soft component primaries are outweighed by their secondaries but this is not so for the lower energy members. The maximum in the intensity altitude curves shown by Regener's results and by those of the Bartol Research Foundation of the Franklin Institute is attributed to a range phenomena of the secondaries and results from the limitations of the atmosphere. The so-called secondary hump in the said curves represents the transition which occurs on crossing the critical-altitude critical-latitude curve. The theory provides for the fact that the east-west asymmetry increases with altitude more rapidly than does the total radiation. It provides for such apparently paradoxical conclusions as those resulting from the fact that there is very little asymmetry in shower production in lead, suggesting that showers are produced by hard rays, while shower production increases with altitude more rapidly than the cosmic radiation, suggesting that it is produced by soft rays. The theory in fact makes shower production in lead increase with the energy of the primaries, but provides nevertheless for its rapid increase with altitude in such a manner as to give, at first sight, the impression that it is produced by a soft radiation.

An alternative choice of constants. If one permits a value of λ as large as 0.5, it is possible to provide for $\mu_s=0.5$ by the assumption that the number of secondaries accompanying a soft component primary is proportional to W_x, or approximately to E_x as assumed in the writer's theory published earlier. This is attended with several advantages; but, necessitates relinquishment of the association between the sea-level critical latitude at 41 degrees and the critical latitude for the soft component. It becomes necessary to assume a second critical latitude phenomenon for the soft component primary, which critical latitude phenomenon does not show itself at all until an altitude of 0.6 meter above sea level is reached. It now becomes necessary to look to the hard component for the sea-level latitude effect. The soft component, however, provides the variation of latitude effect with altitude.

1. Introduction

THE evidence drawn from the variation of cosmic-ray intensity, with latitude at high altitudes, has created a school strongly in favor of the view that the entities which enter our atmosphere from outer space and are directly or indirectly responsible for cosmic-ray phenomena are charged particles.[1] It is true that the facts are unable to deny that an appreciable portion of the cosmic rays observed at sea level may result from uncharged entities entering the atmosphere, but they do not require this; and, according to A. H. Compton, practically a hundred percent of the rays observed at high altitudes owe their origin to charged particles. It is true that some of the high altitude data upon which Compton's conclusions were based have become subject to question as the result of observations secured since the time when he expressed the foregoing opinion. However, the likelihood of a purely corpuscular origin seems sufficiently great to make worth while an attempt to account for all of the phenomena in terms of charged particles.

There are three possibilities as to the relationship between the primary rays and the phenomena observed. The first visions the observed rays as the primary particles themselves; the second regards them for the most part as long range secondaries, initiated directly or indirectly by the primaries, and perpetuating in large measure the directional characteristics of the primaries. The third possibility represents a combination of the first and second. Presumably nobody would deny that even if the first possibility represented the main story, there would be, in the observed phenomena, many secondaries, since such are actually observed in cloud chamber and other experiments. In the second or third type of hypothesis cited above, however, I do not imply a condition in which the rôle of the secondaries is

[1] A. H. Compton, Rev. Sci. Inst. 7, 71 (1936).

limited to a mere complicating function which tends to obscure the story, but rather one in which the law of their production, as a function of the energies of the primaries, etc., plays a definite part in determining the main features characteristic of cosmic-ray phenomena and, in particular, the law of apparent absorption. Neither is it implied that the secondaries are necessarily produced from the primaries in single acts. Photons, for example, may function as intermediaries.

2. Nature of Primary Rays in Relation to the Hypothesis That They Constitute the Observed Entities

Electrons (positive and negative), alpha-particles, protons and other heavy particles have been suggested as primaries.[1] The investigations performed by the Bartol Foundation of the Franklin Institute both in the laboratory[2] and in the stratosphere,[3] have not given any evidence for the occurrence of protons, or other heavy particles functioning as the measured entities, with observable absorption,[4] however much such particles may participate in "silento" through the action of the secondaries which they may produce. Thus, in considering the case of primaries as *measured entities*, it would seem that we must confine our attention to positive and negative electrons, although much of the analysis which follows is applicable to other entities as well.

We shall confine attention throughout to the vertical intensity. The energy for vertical entry through the earth's magnetic field is 17×10^9 ev at the equator, 6×10^9 ev at 41 degrees magnetic latitude, and about 3×10^9 ev at 50° magnetic latitude for example.

At the equator, the rays which can penetrate the earth's magnetic field can also penetrate the atmosphere. There is, however, a magnetic latitude φ_{c0} sufficiently great that the energy of penetration of the field is only just sufficient to provide for penetration of the atmosphere. This latitude is customarily taken as the latitude at which the sea-level intensity fails to increase further with increase of latitude. It is 41° and corresponds, as has been stated, to an energy of penetration of 6×10^9 ev.

For each altitude there is a critical magnetic latitude φ_{cx}, such that the rays which can penetrate the magnetic field can also penetrate down to the altitude concerned. The form of the curve giving the relation between critical latitude and critical altitude is a function of the law governing the absorption of energy in the atmosphere. Two laws present themselves for our consideration in view of what will follow. They are

$$-dE_x/dx=\alpha, \tag{1}$$

$$-dE_x/dx=\lambda E_x+\alpha, \tag{2}$$

where E_x is the energy of a primary ray at a depth x below the top of the homogeneous water equivalent atmosphere, α is the loss per unit of path in ionization, and λ is a constant.[5]

On the basis of the first law we cannot account for an energy of atmospheric penetration anything like as high as 6×10^9 ev. The second form of law with $\lambda=0.333$ and $\alpha=0.77\times 10^8$ per meter of water equivalent[6] provides a satisfactory solution, giving a range equal to that of the vertical atmosphere for an energy of 6×10^9 ev. The solution of (2) is

$$E_x+\alpha/\lambda=(E+\alpha/\lambda)e^{-\lambda x}, \tag{3}$$

where E is the energy of the ray at vertical entry

[2] W. F. G. Swann, Phys. Rev. **49**, 478 (1936); also C. G. and D. D. Montgomery, W. E. Ramsey and W. F. G. Swann, Phys. Rev. **50**, 403 (1936).

[3] L. H. Rumbaugh and G. L. Locher, Phys. Rev. **49**, 855 (1936).

[4] The hypothesis that the primaries are the observed entities implies that a measurable fraction of them terminate their paths in the region of observation. The experiments cited in reference 2, were based upon an attempt to detect the protons near the ends of their ranges. High energy protons which passed right through the atmosphere without being stopped therein in measurable amount would be permissible in the light of reference 2. Such protons might be the basis of initiating secondary radiation which functioned as a large part of the observed radiation.

[5] The second of these laws has already been introduced by the writer in a former publication, Phys. Rev. **48**, 641 (1935). See also W. F. G. Swann, Phys. Rev. **46**, 828 (1934).

[6] This value of α is based upon A. Eisl's value 32.2 ev as the energy necessary to produce a pair of ions (see Ann. d. Physik **3**, 277 (1929)), and upon an assumption of 38 ion pairs per centimeter of path at N.T.P., as found by W. E. Ramsey and the writer, in an investigation as yet unpublished. C. D. Anderson and S. H. Neddermeyer (Int. Conf. Phys. **1**, 171 (1935)) quote data corresponding to values of α ranging between 10^8 ev and 1.5×10^8 ev, but it would seem that their values require division by the density of air at N.T.P. in relation to the density of water$\times 10^{-3}$.

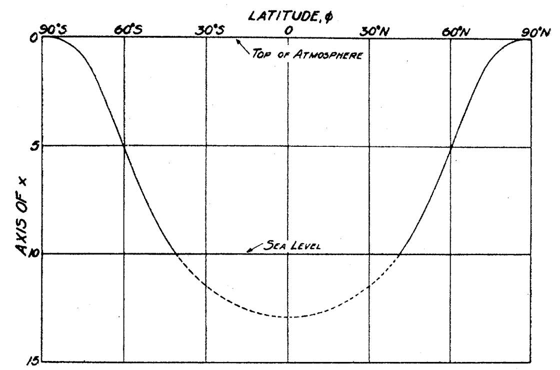

FIG. 1. The curve gives for any magnetic latitude the depth to which primary electrons, which can just penetrate the earth's magnetic field at that latitude, can reach in the homogeneous water equivalent atmosphere. The curve is calculated on the law of absorption specified in Eq. (2) with $\lambda=0.33$ and $\alpha=0.77\times10^{8}$ per meter of water equivalent.

into the atmosphere. The range R of the rays is given by putting $E_x=0$, so that

$$e^{\lambda R}=1+\lambda E/\alpha. \tag{4}$$

Fig. 1 shows the relation between the critical latitude and critical altitude as drawn from (4), with the foregoing quoted values of λ and α.

Now it is manifest that if we regard the primary rays as the measured entities there should be absolutely no variation of intensity with altitude in the regions lying above the curve in Fig. 1; and, in particular, there should be no variation with altitude for all latitudes below the sea-level critical latitude. Such a conclusion is contrary to the facts. In order therefore to secure an increase of intensity with altitude below the critical latitude *it is necessary to suppose that the primaries are accompanied by secondaries whose number increases with the altitude and so with the energy of the primary.*[7]

Quite apart from the foregoing consideration, however, other difficulties arise as a result of the law of distribution of incoming primary energies which it is necessary to assume in order to account for the variation of intensity with altitude in the regions where such variation of intensity is to be expected. For the elucidation of this matter, and also to provide the material for subsequent developments we shall proceed to determine the energy distribution necessary to provide for an exponential law or a combination of contributions obeying exponential laws.

3. Spectral Energy Distribution Necessary to Provide Exponential Law of Absorption of Primaries

In the remainder of this paper, whenever we speak of the "intensity of the rays," we shall refer to the coefficient of $dsd\omega$ in the expression for the number of rays passing per second through the horizontal element of area ds within the limits specified by the vertical solid angle $d\omega$.

Let us suppose that the range R in the homogeneous atmosphere is given as a function of its initial energy E by the relation

$$E=\psi(R). \tag{5}$$

Let N be the intensity at a depth x below the top of the homogeneous atmosphere. If dN is the change of N in a distance dx, then $-dN$ is the contribution to the intensity by those rays which, at the top of the atmosphere have energies between E and $E+dE$, where

$$E=\psi(x);\quad dE=(d/dx)[\psi(x)]dx. \tag{6}$$

Thus, if the energy distribution of the vertical rays at the top of the atmosphere is given by

$$dN_0=F(E)dE \tag{7}$$

at the poles, for example, where all of the rays can enter the atmosphere, we have

$$-dN=F[\psi(x)](d\psi/dx)dx. \tag{8}$$

If N is to be exponential in x, with an absorption coefficient μ, then

$$N=N_0e^{-\mu x} \tag{9}$$

and

$$-dN=\mu N_0e^{-\mu x}dx. \tag{10}$$

It is to be observed that N_0 is given by

$$N_0=\int_0^\infty F(E)dE. \tag{11}$$

It is further to be observed that, in accordance with the conclusions reached in Section 2, relation (9) holds only for depths x greater than x_φ, where

$$\psi(x_\varphi)=E_\varphi. \tag{12}$$

[7] I exclude here considerations founded upon "straggling," as such lines of escape do not seem to have been developed in sufficient detail to result in the correlation with all of the facts concerned.

In other words, it holds for $\varphi > \varphi_{cx}$. By comparison of (8) and (10), we thus have

$$F[\psi(x)](d\psi/dx) = \mu N_0 e^{-\mu x}.$$

The solution of this equation for F is

$$F(\xi) = \mu N_0 e^{-\mu\chi(\xi)}((d\chi(\xi))/d\xi), \tag{13}$$

where ξ is any variable, and $\chi(\xi)$ is the function which expresses R as a function of E through (5) in the form

$$R = \chi(E). \tag{14}$$

Thus
$$F(E) = \mu N_0 e^{-\mu R}(dR/dE), \tag{15}$$

where R is given as a function of E by (14), or if we prefer by (5). The energy spectrum thus becomes determined explicitly, directly we have assigned the range as a function of the energy through (5). It is to be observed, however, that the experimental existence of an exponential law of intensity up to any altitude h only determines the energy distribution for energies greater than a minimum value determined by $E = \psi(h)$, so that at any latitude φ, the energy distribution is only determined for values of E greater than the minimum energy of entry E_φ, as indeed might be concluded from fundamental considerations.

Case of constant energy loss per unit of path. This corresponds to Eq. (1) and yields

$$F(E) = (\mu/\alpha)N_0 e^{-\mu E/\alpha}, \tag{16}$$

or, starting with $F(E)$ as fundamentally assigned we see that if

$$F(E) = Ae^{-\beta E} \tag{17}$$

an exponential law will result for the intensity for $\varphi > \varphi_{cx}$ with

$$\mu = \alpha\beta; \quad N_0 = (\alpha/\mu)A = A/\beta. \tag{18}$$

With $\alpha = 0.77\times 10^8$; and, $\mu = 0.5$ for the soft component of the radiation, both expressed in terms of 1-meter water equivalent, we have $\beta = 0.65\times 10^{-8}$. The vertical intensity at the top of the atmosphere at magnetic latitude φ is given by integrating (17) from the entrance energy E_φ to infinity, and is proportional to exp $(-0.65\times 10^{-8}\,E)$. The ratio of the corresponding intensities at latitudes 41° and the equator, for example, is therefore exp $[0.65\times 10^{-8}(17-6)\times 10^9] = \exp(71.5)$, an enormous quantity out of all harmony with any possibility in relation to the facts.

Case where the energy loss per unit of path is composed of a constant part α, plus a part λE_x proportional to the energy of the primary. Using (15) in combination with (4), which was derived from (2) we find

$$F(E) = \frac{\mu\alpha^{\mu/\lambda}N_0}{\lambda^{(1+\mu/\lambda)}}\left[E + \frac{\alpha}{\lambda}\right]^{-(1+\mu/\lambda)}. \tag{19}$$

In terms of $F(E)$ regarded as fundamentally assigned, we conclude that provided that

$$F(E) = B_p/(E+a)^p, \tag{20}$$

where B_p and p are constants, an exponential law of the type (9) will result for the intensity, for $\varphi > \varphi_{cx}$, with

$$\mu = (p-1)\lambda, \tag{21}$$

$$(p-1)N_0 = (\lambda/\alpha)^{p-1}B_p, \tag{22}$$

provided that $a = \alpha/\lambda$.

It is of significance to note that both for this case and for that founded upon constant energy loss per unit of path, μ is not determined uniquely by the energy loss per centimeter of path, but depends also upon the law of energy distribution in the spectrum of the entering rays. In principle, the condition $a = \alpha/\lambda$ is illogical since it makes $F(E)$ depend upon processes of energy loss in the atmosphere. The condition $a = 0$ would remove the logical difficulty, but would destroy to some extent the exponential form for the intensity inherent in (20). However, since α/λ is of the order 4×10^8, and so is much smaller than even the energy necessary for penetration of the earth's magnetic field in all but the highest latitudes, the neglect of α/λ is not of fundamental importance. In other words, the α/λ contribution is not of fundamental importance one way or another. For these reasons we shall retain it, since its presence adds to the mathematical simplicity of our operations. In fact, in all that follows, we shall make our discussions in terms of $E+\alpha/\lambda$ instead of E, and shall use, in fact, a new quantity W, defined by

$$W = E + \alpha/\lambda. \tag{23}$$

Again, the vertical intensity at latitude φ is obtained by integrating $F(E)$ from E_φ, to infinity, and by use of (20) and (21) is seen to be proportional to $(E+\alpha/\lambda)^{(1-p)}$, where $(p-1) = \mu/\lambda$. Thus, in terms of the values already quoted, we find that the ratio of the intensities at 41° and at the equator is $(173/63)^{1.5}=4.3$. This value is quite reasonable in relation to the possibilities suggested by such experimental data as exist.

However, as already pointed out, such variation in the vertical intensity as is found at the top of the atmosphere should, on the present form of theory, be found also at all altitudes, including sea level, for $\varphi<\varphi_{c0}$. Moreover, no variation of any kind with altitude is provided for the regions $\varphi<\varphi_{cx}$ and no explanation is provided for the so-called hard component. We shall attack these matters in the next section.

4. Development of the View That the Observed Entities are Combinations of Primaries and Secondaries

Following the first analysis made by R. A. Millikan, various analyses of intensity-altitude cosmic-ray data have been made from time to time. W. Kramer[8] specified four components for the radiation with separate absorption coefficients, and with corresponding intensity coefficients J which correspond to the relative intensities for the different components when $x=0$. Kramer's values are as follows:

μ	0.52	0.16	0.075	0.021
J	92	4.3	3.3	0.4

We shall omit the small component with $\mu=0.021$. We shall assign $\mu=0.5$ for the soft component, and we shall lump the second and third components into one hard component with $\mu=0.12$. We shall assign the intensities of the hard and soft components so as to result in the same ratio for the sea-level values as is given by Kramer's soft component, and the combined value of his second and third components. Thus we have

μ	0.12	;	0.50
J	9.6	;	90.4

In former publications, the writer has emphasized the role played by secondaries and even rays from cosmic-ray bursts in contributing to the *observed* cosmic radiation;[9] and in particular he has developed a theory[10] in which it was supposed that the primaries passed, for the most part, right through the atmosphere and produced along their paths secondaries which perpetuated the directions of flight of the primaries. It was supposed, to a first approximation, that the product of the number of secondaries produced, per unit of path, and the range of the secondaries was proportional to the energy of the primary. These assumptions led to an exponential law for the measured intensity of the secondaries. It was not supposed that the primaries of necessity produced their secondaries in single acts. Photons, for example, might intervene as intermediaries. In the more detailed development of the theory, the law of loss of energy was generalized to the type (2), and it was shown that, in this form the theory possessed the power to correlate a large number of experimental phenomena. The primaries paid to the measured radiation a direct contribution whose story was not developed in any great detail. In the present section we propose to amplify the story in greater detail, particularly with regard to the mutual relationship of the primaries and higher order radiations, and we shall develop the theory to the point of harmonizing its numerical predictions with experiment.

The primary background. It will turn out that we shall have to use a primary background composed of contributions which, in the case of the background of the soft component, at any rate, obeys an exponential law. For this purpose, therefore, we have the form of $F(E)$ already developed in Eq. (20), which, as will readily be seen, is not limited to a single component but, in the form

$$F(E)=B_p/(E+\alpha/\lambda_p)^p+B_q/(E+\alpha/\lambda_q)^q, \quad (24)$$

for example, is capable of giving rise to two primary absorption coefficients of the form $(p-1)\lambda_p$ and $(q-1)\lambda_q$ as the result of a single type of primary entity. Unfortunately the desirable consummation inherent in such a hope will have to be modified for reasons which will develop later; but, in the meantime, it is of importance to realize that the different contributions to $F(E)$ pay independent contributions to the primary intensity, and therefore to the intensities of the

[8] W. Kramer, Zeits. f. Physik **85**, 411 (1933).

[9] W. F. G. Swann, Phys. Rev. **43**, 945 (1933); Phys. Rev. **44**, 1025 (1933); Phys. Rev. **46**, 432 (1934); also J. Frank. Inst. **220**, 373 (1935), in which the property of high degree of conservation of direction by secondaries is demonstrated.

[10] W. F. G. Swann, Phys. Rev. **46**, 828 (1934); Phys. Rev. **48**, 641 (1935).

secondaries arising from those primary contributions.

In what follows, it will become necessary to introduce an upper limit E_M to the energy of the primaries. This will destroy the rigorous realization of the exponential form for the component concerned, but it will turn out that no serious consequences result therefrom.

We confine our attention to a single component. We utilize the quantity W defined in (23), which quantity is approximately the energy of a ray; and, bearing in mind (4), we obtain the vertical intensity N_p of the primaries for $\varphi > \varphi_{cx}$ by integrating $F(E)$ up to $W_M = E_M + \alpha/\lambda$, from a lower limit $\alpha e^{\lambda x}/\lambda$, corresponding to $E_x = 0$. For $\varphi < \varphi_{cx}$, the lower limit of W is $W_\varphi = E_\varphi + \alpha/\lambda$, where E_φ is the energy for vertical entry at the latitude φ. Thus

$$N_p = \frac{B_p}{(p-1)}\left(\frac{\lambda}{\alpha}\right)^{p-1}\left[e^{-(p-1)\lambda x} - \left(\frac{\lambda W_M}{\alpha}\right)^{-(p-1)}\right] \quad \text{for } \varphi > \varphi_{cx}, \qquad (25)$$

$$N_p = \frac{B_p}{(p-1)}\left[W_\varphi^{-(p-1)} - W_M^{-(p-1)}\right] \quad \text{for } \varphi < \varphi_{cx}, \qquad (26)$$

where p is related to the absorption coefficient μ, by

$$(p-1)\lambda = \mu. \qquad (27)$$

For latitudes below the sea-level critical latitude, there is no variation with altitude; and for greater latitudes N_p changes from the form specified by (25) to that specified by (26) at the altitude given by Fig. 1. When W_M is infinite, (25) is purely exponential.

For the case where p is equal to or less than unity, an upper limit to E_M is necessary for the realization of a finite value for N_p. The case where $p=1$ is of interest; and, in that case

$$N_p = B_p \log_e [\lambda W_M/\alpha e^{\lambda x}] \text{ for } \varphi > \varphi_{cx}, \qquad (28)$$

$$N_p = B_p \log_e [W_M/W_\varphi] \quad \text{for } \varphi < \varphi_{cx}. \qquad (29)$$

When N_p does not obey a true exponential law, it is convenient to define a variable coefficient of absorption μ_x as

$$\mu_x = -(1/N_p)dN_p/dx.$$

We then have

$$\mu_x = (p-1)\lambda/[1-(\alpha e^{\lambda x}/\lambda W_M)^{p-1}], \qquad (30)$$

and, for $p=1$

$$\mu_x = \lambda/\log_e [\lambda W_M/\alpha e^{\lambda x}]. \qquad (31)$$

Both (30) and (31) approach their ideal values $\mu = (p-1)\lambda$, when $W_M = \infty$.

Theory of soft component

Certain guiding considerations control us in our choice of a theory at this stage. The first concerns the striking fact, which has been carefully established by R. A. Millikan and his collaborators,[11] that the coefficient of absorption for the soft component is 0.5 everywhere, i.e., for regions above and below the critical latitude.

Suppose now that the number n_s of secondaries accompanying a primary at the place where it has an energy E_x is a function of E_x, or rather of W_x of the form $n_s = f(W_x)$. Then, in view of (3) we have

$$n_s = f(We^{-\lambda x}), \qquad (32)$$

where W is the value of W_x at $x=0$. The number of primaries lying within the range W_x to $W_x + dW_x$ is equal to the number which at $x=0$ lie within the range W to $W+dW$, where W is related to W_x by (3). Thus, at places for which $\varphi < \varphi_{cx}$, and in particular for φ less than the sea-level critical latitude, we must have, for the intensity of the secondaries

$$N_s = \int_{W_\varphi}^{W_M} F(W) f(We^{-\lambda x}) dW. \qquad (33)$$

In order that N_s shall be exponential in x, with a coefficient μ_s, it is necessary and sufficient that $f(We^{-\lambda x})$, that is n_s shall be of the form

$$n_s = \gamma(We^{-\lambda x})^s,$$

where $s\lambda = \mu_s$, and γ is a constant.

In this case, with $F(W) = B_p/W^p$ (33) becomes

$$N_s = \gamma B_p e^{-\mu_s x}\int_{W_\varphi}^{W_M} W^{(s-p)} dW. \qquad (34)$$

Incidentally, with the value $\lambda = 0.33$ and $\mu_s = 0.5$ already assigned, the condition $s\lambda = \mu_s$ gives $s = 1.5$.

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[11] I. S. Bowen, R. A. Millikan and H. V. Neher, Phys. Rev. **44**, 246 (1933); also R. A. Millikan, H. V. Neher and S. Korff, Phys. Rev. **49**, 871 (1936).

For $\varphi > \varphi_{cx}$, i.e., for latitudes greater than the critical latitude, the primaries also suffer absorption. Now it has been shown elsewhere by the writer[12] that if the primaries obey a true exponential law, then over the range in which they obey that law there is a constancy of "quality" with altitude in the sense that, at all altitudes, the energy spectrum must be the same. All energies change in the same proportion with altitude. Under these conditions, regardless of the law of production of secondaries as a function of the primary energy, the factor of proportionality between intensities of secondaries and of primaries must be the same at all altitudes. *Under such conditions the secondary intensity must obey an exponential law with the same absorption coefficient as that of the primaries.* Thus if E_M is sufficiently large to guarantee a true exponential relation for N_p, in order to obtain a coefficient $s\lambda$ for the secondaries we must have, in accordance with (27) for E_M infinite,

$$(p-1)\lambda = s\lambda.$$

Hence
$$(p-1) = s. \qquad (35)$$

Thus (34) yields

$$N_s = \gamma B_p e^{-\mu_s x} \log_e (W_M/W_\varphi) \text{ for } \varphi < \varphi_{cx}. \qquad (36)$$

For $\varphi > \varphi_{cx}$, the only change necessitated is the replacement of the lower limit W_φ by $\alpha e^{\lambda x}/\lambda$. Thus

$$N_s = \gamma B_p e^{-\mu_s x} \log_e (\lambda W_M/\alpha e^{\lambda x}) \text{ for } \varphi > \varphi_{cx}. \qquad (37)$$

It is to be noted that regardless of the values of s and of λ the logarithmic form in (36) and (37) is inevitable.

With $s = 1.5$, necessitated by the values of μ_s and λ concerned, (35) gives $p = 2.5$, so that

$$F(E) = B_p/W^{2.5}. \qquad (38)$$

For the general case

$$p = 1 + \mu/\lambda. \qquad (39)$$

It will readily be seen that with the value $p = 2.5$, and with an upper limit as low even as 5×10^{10} ev the departure of (25) from the true exponential form is negligible.

It is of interest to observe that *the equality of μ_s for regions above and below the critical latitude leads inevitably to the form of dependence of secondary production upon primary energy and to the value of p for the primaries.*

Note on the significance of the relation between secondary production and primary energy. We have regarded the number n_s of secondaries accompanying a primary as a function of W_x, i.e.,

$$n_s = f(W_x). \qquad (40)$$

If r is the range of a secondary, and σ is the number of secondaries produced per unit of path, we have, approximately

$$n_s = \sigma r = f(W_x). \qquad (41)$$

If ϵ is the energy of a secondary, and if we regard the term λE_x in (2) as representative of the loss encountered per unit of path in producing these secondaries, then

$$\sigma\epsilon = \lambda E_x = \lambda W_x, \text{ approximately.} \qquad (42)$$

Thus, from (41) and (42)

$$\epsilon/r = \lambda W_x/(f(W_x)). \qquad (43)$$

For the special case to which we have been led, where $n_s = \gamma W_x^s$, we have

$$\epsilon/r = \lambda/\gamma W_x^{(s-1)}. \qquad (44)$$

The content of our assumption is comprised in the following: Suppose that theory or experiment has given us some relation between r and ϵ. Then (43) becomes an expression of an assumption as to how ϵ depends upon E_x. Next (42) becomes a statement of how σ is to depend upon E_x in order that the energy loss per unit of path shall be proportional to E_x. With (42) and (43) holding, (40) becomes provided for. In subsequent determination of the constants concerned, the constant γ, for example, our main care must be to see that ϵ/r as given by (44) does not come out smaller than would be reasonable for the average energy per unit range of a secondary. Thus, ϵ/r should not be less than 10^8 ev per meter of water equivalent, so that if the upper limit of the energies concerned is E_M (or W_M approximately), we must have

$$\gamma < \lambda/(W_M^{0.5} \times 10^8), \qquad (45)$$

for the case where $s = 1.5$.

The hard component

At this stage our theory has given us a satisfactory preliminary account of the soft compo-

[12] W. F. G. Swann, Phys. Rev. **47**, 575 (1935).

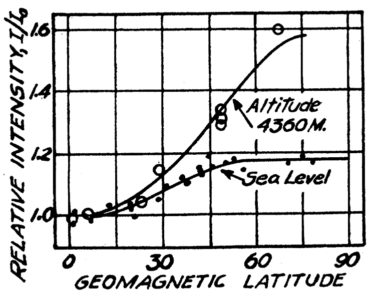

FIG. 2. Data, quoted from A. H. Compton, on the variation of cosmic-ray intensity with latitude at sea level and 4360 meters altitude.

nent. We must, however, examine it in relation to the latitude effect, and here the hard component will become involved. We shall denote by R_0 the ratio of the vertical sea-level intensity at the sea-level critical latitude to the corresponding sea-level intensity at the equator. R_x shall be the corresponding ratio for the same latitudes at the depth x below the top of the atmosphere.

For the secondaries, R is obtainable from (36). For an upper energy limit equal to 5×10^{10} ev $R_x=R_0=1.92$, and for an upper limit of 10^{11} ev it is 1.56.

The latitude ratio for the primaries is given by (26); and, for the above upper limits the ratios are approximately the same and are 5.5 and 4.9, respectively, since $p=2.5$. The resultant sea-level latitude ratio *for the soft component* cannot be less than the value given by the logarithmic term. It is then necessary to look to the hard component to iron out the sea-level ratio to the observed value, say 1.14,[13] leaving the soft component dominant at high altitudes, however.

An easy way to dispose of the hard component is to attribute it to photons, in which case it would have no latitude effect. On a preliminary survey such a procedure presents difficulties which tend to evaporate upon further scrutiny, however. The situation is worthy of discussion.

The latitude effect in the light of a hard component which itself shows no latitude effect. If we take Kramer's data, we find that the intensities of the hard and soft components at *sea level* are in the ratio 4.8 to unity. Taking these figures as applying to some place above the sea-level critical latitude, we find that if y is the ratio of the vertical intensity for the soft component at $\varphi=\varphi_{c0}$ to that at $\varphi=0$, and if 1.14 is the corresponding ratio for the total intensity, and if the hard component has no latitude effect

$$R_{c0}=(N_s+4.8N_s)/(N_s/y+4.8N_s)=1.14,$$

where N_s is the intensity of the soft component. Thus $y=3.5$. Incidentally this represents the maximum latitude effect which can be produced between φ_{c0} and $\varphi=0$ at any altitude apart from the invocation of circumstances of a type extraneous to those involved in the expression of the intensity in the form of the sum of two exponential contributions. Of course, the latitude effect extends to greater latitudes at higher altitudes. In fact, at any altitude, it extends to the latitudes given by Fig. 1, and naturally the intensity ratio goes on increasing up to this latitude. The value $y=3.5$ while theoretically attained only at the top of the atmosphere would be approached at altitudes for which the soft component dominates the hard component. Thus, for example, at an altitude corresponding to $x=6$ meters of water, we should have for the ratio of the intensity at $\varphi=42°$ to that at $\varphi=0$, a value R_6 given by

$$R_6=(4.52+4.67)/(4.58/3.5+4.8)=1.55,$$

for the case where $R_{c0}=1.14$ for sea level. The numbers 4.58 and 4.8 represent the relative vertical intensities at $x=6$ meters of water, calculated from Kramer's data cited earlier, in which the relative intensities of the soft and hard components at $x=0$ are 90.4 and 9.6, respectively, and in which the corresponding absorption coefficients are 0.5 and 0.12.

Now at an altitude of 4360 meters, which is not far from the equivalent of $x=6$ meters of water, A. H. Compton's data shown in Fig. 2 give a value of R_6 equal to 1.30. The excess of this ratio over unity, which is the thing which counts, is thus only a little more than one-half of the excess calculated on the assumption that the hard component shows no latitude effect. A latitude effect for the hard component would reduce the foregoing discrepancies, but would call for a charged corpuscular origin for the hard component, or at any rate for a portion of it.

While the foregoing considerations appear to mitigate against the claim of photons for the

[13] P. Auger and L. Leprince-Ringuet, Int. Cong. Phys. 1, 188 (1935).

position of the hard component, their force becomes annulled in the light of the conditions to which our theory of the soft component has led—conditions in which for $\varphi < \varphi_{c0}$ there is a primary contribution to the intensity, a contribution independent of altitude but dependent on latitude, and which possesses the power to provide at sea level the necessary latitude variation which a photon component would lack. In the light of these considerations, let us therefore examine the possibilities inherent in the assumption of a "non-field sensitive," a noncharged hard component, remembering of course that if such a component is to affect Geiger counters, it must act through its secondaries. Using (25), (26), (36) and (37) and recalling that $p=2.5$ we find for the vertical cosmic-ray intensity I,

$$I=B_p\left[\frac{2}{3}\left(\frac{\lambda}{\alpha}\right)^{1.5}+\gamma\log_e\left(\frac{W_M\lambda}{\alpha e^{\lambda x}}\right)\right]e^{-\mu_s x}+Ae^{-\mu_h x}-\frac{2}{3}B_p\left(\frac{1}{W_M}\right)^{1.5}\quad\text{for }\varphi>\varphi_{cx},\quad(46)$$

$$I=\frac{2}{3}B_p\left[\frac{1}{W_\varphi^{1.5}}-\frac{1}{W_M^{1.5}}\right]+B_p\gamma\left[\log_e\left(\frac{W_M}{W_\varphi}\right)\right]e^{-\mu_s x}+Ae^{-\mu_h x}\quad\text{for }\varphi<\varphi_{cx},\quad(47)$$

where A is the intensity of the hard component whose absorption coefficient is μ_h.

Only for $\varphi>\varphi_{cx}$ does our theory give a primary as well as a secondary variation with altitude for the soft component; and, it must be in this region that we regard Kramer's analysis as applicable. The contribution in (46) which is proportional to γ is not truly exponential,[14] but involves a slowly varying logarithmic coefficient. For the purpose of comparison with Kramer's analysis we shall regard the comparison as made at sea level, and shall evaluate the logarithmic coefficient in question there.

We assume $\lambda=0.33$ and $\alpha=0.77\times10^8$, as before, write $\gamma_0=\gamma\times10^{12}$, $A_0=A\times10^{12}$, and assume, for purposes of trial that $E_M=5\times10^{10}$ ev, which corresponds to $W_M=502\times10^8$ ev. Then Kramer's value 9.4 for the ratio of the intensities of the soft and hard components causes (46) to yield

$$0.190B_p+2.041\gamma_0B_p=9.4A_0.\quad(48)$$

Eq. (47), with $\mu_s=0.5$ and $\mu_h=0.12$ gives, for the sea-level critical-latitude ratio

$$\frac{1.27\times10^{-3}B_p+13.76\times10^{-3}\gamma_0B_p+0.301A_0}{2.88\times10^{-4}B_p+7.12\times10^{-3}\gamma_0B_p+0.301A_0}=R_0,\quad(49)$$

(48) and (49) yield

$$(7.35+79.1\gamma_0)/(6.37+72.4\gamma_0)=R_0.\quad(50)$$

If we assign $R_0=1.14$, we find $\gamma_0=0.043$ so that $\gamma=0.043\times10^{-12}$.

This value of γ leads to results which are not unreasonable. Thus, for the highest energy assigned, $W_M=502\times10^8$ ev (44) gives $\epsilon/r=0.3\times10^8$. This value, for the average energy per unit range of the secondary is dangerously small. However, in all places where observations are made which would have any significance in relation to our formulae, W would be much smaller than 5×10^{10} ev. Thus at $x=4$ meters, the maximum energy would be $5\times10^{10}\times\exp[-1.3]=1.2\times10^{10}$, and the corresponding value of ϵ/r would be about 0.75×10^8 ev. This represents the worst case, and for all lower values of W, the value of ϵ/r is greater.

The foregoing value of γ means that a 10^{10} ev primary would be accompanied by about 40 secondaries.

The ratio of the *total* number of secondaries to primaries for $\varphi>\varphi_{cx}$ is given by the ratio of the two contributions to the soft component in (46). It does not vary greatly with altitude (for $\varphi>\varphi_{cx}$) and amounts at sea level and at magnetic latitude 50° to 1 to 1.9. However, as already implied, the ratio is greater for the high energy primaries, being, as we have already shown, 40 for a 10^{10} ev primary. It is thus easy to see why cloud chamber photographs would not show many high energy particles. Such particles would be represented mainly by their low energy secondaries.

[14] No apology is needed for our departing from the true exponential forms. As a matter of fact the correct forms for the intensities above and below the critical latitude must fit together on the curve represented in Fig. 1. The sum of two exponentials, with constant absorption coefficients could not possibly fit together on this curve even with the best adjustment of the intensity coefficients. Thus the form of theory leading to true exponentials would be illogical. The departure from the true exponential forms in our theory provide for just those elements necessary to secure the fit along the curve Fig. 1.

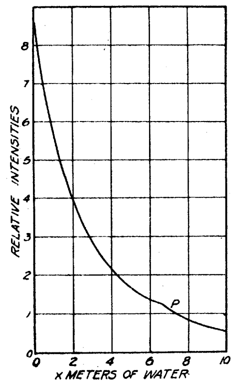

FIG. 3. Relative cosmic-ray intensities as a function of depth below the top of the homogeneous water equivalent atmosphere. The curve is drawn from the theoretical formulae given in Eqs. (46) and (47). The point P is representative of the secondary hump in the intensity altitude curves.

Variation of latitude effect with altitude. On account of the logarithmic term in (47), the latitude effect varies with altitude. With the value of γ chosen to give a sea-level latitude ratio of 1.14, the value of R_x at 6 meters of water equivalent depth comes out as 1.19; and, at $x=4$ meters it is 1.29. The maximum vertical current ratio for any altitude is the ratio of the intensity at the latitude and altitude determined by Fig. 1, to the intensity at the same altitude at the equator. The ratio in question comes out as 1.79 for $x=6$ meters and 2.66 for $x=4$ meters.

The increase of latitude effect with altitude is thus of the same order of magnitude as to correspond to Compton's data but is somewhat smaller. If the hard component possessed a sea-level latitude effect, the effect would be to weaken still further the increase of latitude effect with altitude. The increase of latitude effect with altitude is bound up with the logarithmic term in (47); and, as we have seen the logarithmic form is inevitable if μ_s is to have the same value above and below the critical latitude. The maximum latitude effect attainable at high altitudes is limited by the logarithmic form of this term. The increase of latitude ratio with altitude is very sensitive to the choice of the upper limit. Only when that upper limit approaches close to the limit necessary for vertical entry at the equator does one obtain any increase of latitude ratio with altitude at all. In fact, our theory makes the increase with altitude a symbol of the existence of an upper limit. If $E_M=1.7\times10^{10}$ ev, which is the energy for vertical entry at the equator, the latitude effect of the logarithmic term will be infinite. We are limited, however, in the degree of smallness of the value which we may assign to E_M, since we must permit rays to come in from the due east and due west; and, to secure entry in the equatorial plane this would necessitate an energy of about 6.0×10^{10} ev. Even if we should adopt as low a value as 2.5×10^{10} ev for E_M, the maximum latitude ratio for the logarithmic term for $x=4$ meters would only amount to 8.75, and the complete latitude ratio, including the hard component would be 3.0. All of the published data on variations of intensity with latitude give ratios less than 2, when the observations have been made with a single instrument at different latitudes. Such latitude ratios of the order of 2 to 1, and even 10 to 1 and more as have been quoted as applicable to the top of the atmosphere, depend upon comparison of observations taken with different instruments in different flights. Such comparisons are attended by the greatest difficulty. In the first place, the whole significance of the intensity is modified by the existence of a maximum. This maximum, on the theory developed in this paper, would simply be indicative of the fact that near the top of the atmosphere the range of the secondary electrons is greater than that corresponding to the whole of the air above the point of observation, so that the intensity suffers in relation to the value given by the theory on account of the absence of enough air to produce all of the secondaries required by theory. On this basis the observations of the Bartol Foundation lead to a secondary range of about 82 centimeters of water equivalent. However, our theory provides for a variation of the energy and so of the range of the secondaries with primary energy. There should, therefore, on this account be some variations in the altitude of the maximum with latitude. Then, matters are complicated considerably by the fact that, as the Bartol Research Foundation observations have shown,[15]

[15] See, for example, W. F. G. Swann and G. L. Locher, National Geographic Society, Contributed Technical Papers, Stratosphere Series, No. 1, 7 (1935), in which publication it appears that the horizontal intensity forms 20 percent of the vertical at 40,000 feet; see also W. F. G. Swann, J. Frank. Inst. **222**, 1–11 (1936) in which from our observations in the flight by Professor and Mrs. Jean Piccard we obtained more than 50 percent for the ratio

the horizontal intensity is comparable with the vertical intensity in the stratosphere on account of the bending of the paths of the secondary rays by the earth's magnetic field. Undoubtedly many of the secondary rays actually approach the apparatus from a direction below horizontal, so that the variation of shielding conditions in different experiments is apt to have an important bearing upon the results, particularly as it was formerly supposed that shields were only effective when placed *above* the ionization chambers.

The secondary hump in the intensity-altitude curve. Intimately bound up with the latitude effect is another having to do, possibly, with the secondary hump—the hump below the maximum—characteristic of intensity-altitude curves. At any latitude, there is a value of x determined by Fig. 1, for which the expression for the vertical intensity changes from the type characterized by (46), to that characterized by (47). Above this altitude the chief change is that resulting from the absence of the direct contribution to the soft component by its primaries. We should thus expect a hump in the ionization altitude curve at the value of x concerned. This hump should start at sea level for $\varphi=\varphi_{c0}=41°$, and should gradually climb up the curve with increasing latitude. Intensity-altitude curves obtained with ionization chambers must, of necessity, tend to obscure this hump, since there is a different critical latitude corresponding to each direction from the zenith. Fig. 3 represents the vertical intensity-altitude curve plotted from Eqs. (46) and (47), with the constants already determined and for magnetic latitude 55°. The curve does not, of course, show a maximum, since the maximum represents an added feature depending upon the limitations of the atmosphere as already stated. It does show the secondary hump at $x=6.6$ meters of water. This value of x is considerably greater than the value suggested by high altitude data. It is for this reason moreover that, in spite of our adjustment of the absorption coefficients and the relative sea-level intensities of the hard and soft components to their experimentally established values, the ratio of the total intensity even at $x=0$ to that at sea level is given by the curve as only about 17 instead of three or four times that value, as it should be. Above the secondary hump, the radiation becomes robbed not of its absorption coefficients but of part of its intensity factor; and, if the hump comes at too low an altitude, the ratios of the high altitude intensities to those at sea level became seriously affected. While the exact position of the secondary hump may be regarded at first sight as a matter of refinement which an approximate theory cannot be expected to reveal, it must be admitted that the defect inherent in a failure to make the right prediction becomes more vital on mature consideration. Fortunately, a way of overcoming the difficulty exists; but, since it involves a rather radical change in our views as to the significance of the critical latitude, and since it has been developed since this paper was presented at the Harvard Tercentenary Conference, it is here relegated to an Addendum at the end of the paper.

Possibilities inherent in a field sensitive hard component. If the hard component is of a charged particle type, and is to show an increase with altitude in equatorial regions, it also must be represented by secondaries. The coefficient of absorption is one-quarter that of the soft component, so that if the number of secondaries accompanying a hard component primary, which we shall call a q primary is to be proportional to $(E+\alpha/\lambda_q)^{s_q}$, i.e., $W_q^{s_q}$, we must again have $\mu_h=s_q\lambda_q$.

The most elegant solution of the problem would be one in which we had simply *one form of primary*, with an energy spectrum given by

$$F(W)=B_p/W^p+B_q/W^q, \qquad (51)$$

and with a law of production of secondaries such that the number n_s of secondaries accompanying a primary was

$$n_s=\gamma W^{1.5}+kW^{1.5/4}. \qquad (52)$$

The working out of the theory would then tell the story of *both the hard component and the soft component simply as the outcome of the form of the energy distribution in the incoming primaries.* The constants λ and α would, of course, be the same for the q primaries as for the p primaries. As a matter of fact, it is perfectly possible to fit up a

of the horizontal to the vertical intensity. In the last flight made by Captain A. W. Stevens and O. Anderson our apparatus gave practically as much horizontal as vertical intensity. It is of course to be observed that horizontal intensities as measured by Geiger counters correspond to the sum of the intensities for two diametrically opposite directions.

theory along these lines, and provide for the right ratio of the intensities of the components and for the sea-level latitude ratio. However, difficulty is encountered in the matter of the variation of latitude effect with altitude. The significant elements are these: If for the q primaries we proceed as we have done for the p primaries, we should conclude that $(q-1)=\mu_h/\lambda=0.38$; and, in line with our former demonstration, the contribution of the q secondaries to the intensity through the term $kW^{1.5/4}$ would come out proportional to $B_q \log_e (W_M/W_\varphi)$ below the critical latitude. However, there would be two additional "cross product terms" in the contribution to the secondaries, a term proportional to the integral of $(1/W^{2.5})(W^{1.5/4})kB_p$, and a term proportional to $(1/W^{1.38})(W^{1.5})\gamma B_q$. Below the critical latitude, for example, these two terms are proportional, respectively, to $B_pk(W_\varphi^{-1.88}-W_M^{-1.88})$ and $B_q\gamma(W_M^{0.12}-W_\varphi^{0.12})$. Of these two terms, the former contributes to the hard component and the latter to the soft component. There are thus two contributions to the soft component secondaries, one proportional to $B_p\gamma \log_e W_M/W_\varphi$, and one proportional to $B_q\gamma(W_M^{0.12}-W_\varphi^{0.12})$. Now when we make use of the intensity ratio of the soft and hard components, it becomes possible to show that the latter contribution to the secondaries dominates the former, and so determines in large measure the latitude effect at high altitudes since it is part of the soft component. However, the latitude effect of this term is weaker than that of the logarithmic term. It is, in fact weaker in this respect than all of the other contributions from the p primaries or secondaries or the q primaries or secondaries. Thus, its dominance at high altitudes tends to cause a decrease rather than an increase of latitude effect with altitude. *It is for this reason that it appears necessary to separate the p primaries and the q primaries into two distinct classes*, one whose law of production is given by $n_s=\gamma W_p^{1.5}$, and the other whose law of production is given by $kW_q^{1.5/4}$. As already stated, apart from the requirements of the increase of latitude effect with altitude, it would be possible to fit up a theory based upon (52), although even then it would be necessary, as indeed it is in any case, to assume for the q primaries a value of q less than 1.38, or even a negative value of q so as to avoid having too large a sea-level latitude effect for the hard component.

Let us then take for the q primaries a contribution to $F(W)$ of the form

$$F(W)=B_qW^n \qquad (53)$$

and assume that the number of secondaries n_s accompanying a primary is

$$n_s=kW^{s_q},$$

where

$$s_q\lambda_q=\mu_h.$$

We then have for the primary intensity

$$N_q=\frac{B_q}{(n+1)}\left[W^{n+1}-\left(\frac{\alpha e^{\lambda_q x}}{\lambda_q}\right)^{n+1}\right] \quad \text{for } \varphi>\varphi_{qcx}, \qquad (54)$$

$$N_q=\frac{B_q}{(n+1)}[(W_{M_q})^{n+1}-(W_{\varphi_q})^{n+1}] \quad \text{for } \varphi<\varphi_{qcx}, \qquad (55)$$

and for the secondary intensity

$$N_{sq}=\frac{kB_q}{(n+1+\mu_h/\lambda_q)}\left[(W_{M_q})^{n+1+\mu_h/\lambda_q}-(\alpha e^{\lambda_q x}/\lambda_q)^{n+1+\mu_h/\lambda_q}\right]e^{-\mu_h x} \quad \text{for } \varphi>\varphi_{qcx}, \qquad (56)$$

$$N_{sq}=\frac{kB_q}{(n+1+\mu_h/\lambda_q)}\left[(W_{M_q})^{n+1+\mu_h/\lambda_q}-(W_{\varphi_q})^{n+1+\mu_h/\lambda_q}\right]e^{-\mu_h x} \quad \text{for } \varphi<\varphi_{qcx}. \qquad (57)$$

Here we have recognized values of λW_M and critical latitudes which may be different for the q primaries than for the p primaries.

The essential thing to observe is that the greater n, the more does N_q approach a radiation with zero absorption coefficient, the more nearly does N_{sq} approach a true exponential with absorption coefficient μ_h, and the smaller the latitude effects of both N_q and N_{sq}, as shown by (55) and (57). Thus, for example, if $n=2$, and $W_{M_q}=5\times 10^{10}$ ev, the ratio of N_{sq} at $\varphi=41$ degrees to the value at the equator is less than $[1-(6/50)^3]/[1-(17/50)^3]$, i.e., less than 4 percent. For $n=3$, it is only about 1.3 percent. Moreover, there is no limitation upon k, and so upon the ratio of the secondaries to the primaries other than that imposed by the necessity of (44) leading to a reasonable value for ϵ/r where γ is replaced by k, λ by λ_q, and s by s_q. Remembering that n_s, the number of secondaries per primary is equal to kW^s, (44) becomes $\epsilon/r=\lambda_q W/n_s$. Hence, if ϵ/r is to be greater than 10^8 and if λ_q is comparable with λ for the p primaries, we have $W/n_s>3\times10^8$. Thus for a 5×10^{10} ev q primary, we can have n_s comparable with 200. This would correspond to $k=200/(5\times10^{10})^{0.36}$; and a 10^9 volt q primary would be accompanied by $200/(50)^{0.36}=50$ secondaries. Thus, under such conditions, the secondaries would dominate the primaries.

With a very small latitude effect provided for the hard component, there would be no marked critical latitude for it; and, its relationship to the soft component would be exactly the same as that already discussed in the case of the nonfield sensitive hard component. The net latitude effect would be calculated as before, and the variation of net latitude effect with altitude would follow as before.

Thus, it appears possible to supplement the story of the soft component equally well with a hard component which is field sensitive or with one which is not—a photon component, for example. The ratio of intensities of hard component to soft component can be provided for. The correct latitude effect can be provided for as can also the variation of latitude effect with altitude.

Relation of theory to shower production and to asymmetry

Shower production. It is a well established experimental fact that the rate of production of small showers from lead increases with altitude at a rate greater than the rate of increase of the total radiation. In fact, the shower production increases approximately with a coefficient equal to that of the soft component of the radiation. This has given rise to the statement that the shower producing radiation is a soft radiation. According to our theory, if the showers in lead are produced according to a law anything like that for the air secondaries, it is the *most energetic primary rays* which are the most efficient in the production of showers; but, in spite of this, they endow the shower production with a coefficient of increase with altitude which is large in proportion to the power of the energy which determines the shower production. It is true that, in the case of a photon hard component we could not predict the net increase of shower production with altitude without knowledge of the relative efficiency of the photons and the charged particles in producing showers (directly or through photon intermediaries); but, in the case of a hard component composed of particles which lose energy more or less continuously along their paths, so long as the shower production in lead by such particles depends upon a higher power of the energy than does the air secondary production by the hard component, shower production will increase with altitude more rapidly than does the cosmic-ray intensity. In the foregoing connections, it may be well to emphasize again, that the "softness" of the component, in the case of the p primaries, at any rate depends not merely upon the energy absorption but also upon the *energy distribution* of the incoming radiation.

East-west asymmetry. The view here taken removes a paradox which has existed between the variation of shower production with altitude, and the east-west asymmetry in shower production. According to the naive view that showers are produced by a soft radiation we might expect that they would show great asymmetry, since the east-west asymmetry is greater for the primary rays of smaller energy than for those of larger energy. Experimentally, however, very little asymmetry has been found in shower production. The reason on our present view, is very clear. It is the rays of high energy which produce the showers, so the showers show very little asymmetry. On the other hand, for the reasons stated, the intensity of shower production increases with altitude at a rate determined by a large coefficient.

Dependence of asymmetry upon altitude. The story of the dependence of asymmetry upon altitude is of interest. To fix our ideas let us imagine two similar cosmic-ray telescopes in a plane parallel to the equatorial plane and pointed at equal angles on the east and on the west side of the zenith. The asymmetry observed by these two instruments depends upon the rays which enter the atmosphere within certain energy limits, and which also succeed in penetrating down to the instruments. If the apparatus be now taken to a higher altitude, the asymmetry will alter mainly through the effect of decreased absorption permitting rays at the lower energy limit to enter, which rays would not have penetrated to the apparatus at the lower depths.

Now, according to the form of theory we have developed the average entrance energy of the p primaries is approximately $3E_\varphi$ and is determined by the *lower* limit of the energy for entry, while even for the case of a charged particle hard component the average entrance energy of the q primaries is approximately $(n+1)E_{M_q}/(n+2)$ and is determined by the *upper* limit of the energy. Thus, it is the p primaries which are mainly concerned in the variations of asymmetry with altitude, which variations are determined by variations in the numbers of rays entering the instruments at the lower energy limit. *We should thus expect that variation of asymmetry with altitude would be determined by the soft component absorption coefficient*, rather than by the variations with altitude of the cosmic radiation as a whole. A further conclusion derivable from the theory, and one in harmony with the results of T. H. Johnson, is the conclusion to the effect that the latitude effect for shower production is smaller than for the total radiation. Since, according to the theory, shower production depends upon the higher energy rays, its latitude effect will be determined largely by the latitude effect for those rays; but, it is only the rays lying below the upper limit for equatorial entry that show any latitude effect at all.

The nature of the hard component primaries and the soft component primaries

While the present theory does not lead to any very specific conclusions as to the actual *nature* of the particles concerned, the fact that the east-west asymmetry is determined by the low energy rays, points to its becoming associated with the p primaries; and, the fact that asymmetry measurements require positively charged rays suggest that the p primaries may be protons, while the q primaries are electrons. An assumption of protons for the p primaries would not be inconsistent with our failure to find such protons at sea level,[2] since the p primaries and their secondaries are associated with the soft component, and so play but little role at sea level.

Addendum

(Made since delivery of the paper at the Harvard Tercentenary)

The chief weakness in the theory as given in the foregoing lies in its prediction of too low an altitude for the "secondary hump" in the intensity-altitude curve. This difficulty could be alleviated if we could permit for the p primaries a larger value of λ. If, in fact we could take $\lambda=0.5$ instead of 0.32, the modified curve corresponding to Fig. 1 would show an altitude corresponding to $x=5.3$ meters for latitude 55°, so that the secondary hump would come at $x=5.3$ for this latitude, and would be better in agreement with such experimental data as exist. A still larger value of λ would improve matters still further. The value $\lambda=0.5$, would result in $s=1$, so that the number of secondaries accompanying a primary would be proportional to the first power of the energy as assumed in the first form of the theory which I published in 1934.[10] With $s=1$, Eq. (35) requires $p=2$, so that $F(W_p)$ would be inversely proportional to W_n^2. With $s=1$, moreover, requirement (44) now reverts to $\epsilon/r=\lambda/\gamma$ so that γ can be as large as $\lambda\times10^{-8}$ *viz.* 0.5×10^{-8} ev without causing ϵ/r to be less than 10^8 ev per meter of water. With this value of γ, a 10^{10} ev electron would be accompanied by 50 secondaries and there is no value of W_p, no matter how large, which would give trouble as regards condition (44). However, with $\lambda=0.5$, and $\alpha=0.77\times10^8$, Eq. (4) gives for a 1.7×10^{10} ev electron a range of only 9.4 meters of water equivalent. Thus, there would be no *sea-level* critical latitude for the soft component. It would be necessary to leave the sea-level latitude variation entirely to the hard component. Eqs. (55) and (57) provide the story. For $n=0.5$ in (53), and with $\mu_h/\lambda_q=0.36$, which corresponds to $\mu_h=0.12$, and the value $\lambda_q=0.33$ appropriate to the sea-level critical latitude we find 1.13 for R, the ratio of the sea-level vertical intensity of the secondaries at 41° to the corresponding value at the equator. This value of R

is quite satisfactory. The q primaries have a larger latitude effect than the secondaries as shown by (55); but, with the value of k already taken in our former illustrations, the q primaries are entirely masked by their secondaries.

Under the foregoing conditions there would be no variation of latitude effect with altitude until the altitude corresponding to $x=9.4$ already cited, i.e., the altitude 0.6 meter of water, was attained. At this altitude, a critical latitude would appear for the soft component, and would gradually work its way towards increasing latitude with increasing altitude. Thus, for altitudes above 0.6 meter of water, the latitude effect would increase with further increase of altitude as the soft component assumed more and more of the dominant role.

If only one is justified in assuming for the soft component a critical latitude different from the known sea-level critical latitudes, but one which only makes its appearance at altitudes greater than 0.6 meter, then I favor the story just given in preference to that developed in the previous part of this paper, although the only difference is in the choice of the constants. The fundamental theory remains the same for both; and is exhibited better in the general form in which we have presented it in the body of this paper.

A recent development by George Pfotzer. Since the foregoing paper was presented at the Harvard Tercentenary Conference, a paper by George Pfotzer in the August, 1936 issue of the Zeitschrift für Physik has come to my attention.[16] In this paper, the author following the lines indicated by B. Gross, develops the consequences of my 1934 theory,[9, 10] which theory, as already stated, resulted in an exponential law of apparent absorption and is based upon the law of energy loss specified in Eq. (2), combined with the assumption that the number of secondaries accompanying a primary increases with the energy of the primary. It is convenient to take this opportunity to discuss some of the points in Pfotzer's paper for purposes of comparison. In the development which I formerly published, the law of the increase was one of proportionality between secondaries and the first power of the primary energy; and, this is the form used by Pfotzer. Incidentally Pfotzer also gives the form of energy distribution stated here in Eq. (19) which I have derived as a particular case of the more general form developed in (13).[17]

In the development of the present paper I had started with the hope of utilizng the law in which secondary production was proportional to the primary energy, but was driven from the simplicity of this assumption by the necessity of providing for a soft component absorption coefficient equal to 0.5 per meter of water both above and below the critical latitude as demanded by Millikan's observations.[11] As already demonstrated, the necessity for this provision demanded that $s\lambda=\mu_s$, where s is the exponent of the primary energy in the expression of the number of secondaries accompanying a primary. With the value $\lambda=0.25$ which was derived from the energy 6×10^9 ev for penetration at the sea-level critical latitude and from a value of α falling within the limits specified by C. Anderson,[6] s became equal to 2; and, in accordance with (39) p became equal to 3 which gave rise to a law of approximately the inversed cube of the energy for the energy distribution function. Such a law is used by Pfotzer, but purely on an empirical basis. In this form the theory gave considerable difficulty in providing for the proper value of the latitude effect without invoking a situation in which the calculated energy per unit range of the secondaries was too small. I felt the necessity of assuming a smaller value of s, but the value of α stood in the way, since it controlled the value of λ. The situation became eased when it was pointed out to me by Dr. C. G. Montgomery that the values quoted by Anderson for the equivalent of α should really be divided by the density of air. It was now possible to choose between the experimentally allowable limits a value of α which, with 6×10^9 ev for the energy of penetration of the atmosphere gave $s=1.5$. This gave a much more satisfactory story as regards the secondary production in relation to the latitude effect and, incidentally, it led to an energy distribution function inversely proportional, approximately to the power 2.5 of the energy (see Eq. 38). The

[16] G. Pfotzer, Zeits. f. Physik **102**, 41 (1936).

[17] The special form (19), but not the general form (13) quoted by Pfotzer from B. Gross, Physik Zeits. **37**, 12 (1936). In Gross' reference to my paper he quotes J. Frank. Inst. **219**, 97 (1935). A more complete account of the theory concerned is, however, given in Phys. Rev. **48**, 641 (1935).

more radical step involving a return to $s=1$ through the choice of a value of λ as high as 0.5 has been referred to above, and necessitates the assumption of two critical latitudes, one of which does not appear, however, at sea level.

Pfotzer, on the other hand, uses a very large value for μ_s, a value equal to 0.85 per meter of water, and he chooses a very large value, 3×10^8 for α, which value he obtains by empirical adjustment to fit the experimental data.

Pfotzer does not place upon his development the requirement of equality of a soft component absorption coefficient above and below the critical latitude. Had he done so, then, quite apart from the magnitude of all of the constants concerned, the logarithmic forms (36) and (37) would have resulted inevitably, and would have demanded the assumption of an upper limit to the primary energy.

Another point of difference between the present paper and that of Pfotzer lies in the fact that in the former we have provided for a variation of intensity with latitude and for a variation of the latitude effect with altitude, which phenomena placed very stringent requirements upon the theory. Pfotzer obtains the sea-level altitude effect from the hard component, but does not provide for an increase of latitude effect with altitude.

Pfotzer attributes the maximum in the intensity-altitude curve to a range phenomenon of the secondaries and determines from it the secondary range at these altitudes. In this, his conclusions are substantially similar to those which I have reported here and on various former occasions,[18] the calculated range being equivalent to about 82 centimeters of water.

In conclusion, I wish to express my appreciation of the services of Dr. and Mrs. C. G. Montgomery who have given me considerable assistance, necessitating careful discrimination, in the numerical calculations, and who have checked the mathematical calculations.

[18] W. F. G. Swann, "Cosmic-Ray Measurements." Presented at Washington, May 1, 1935, as part of a symposium on the 1935 National Geographic U. S. Army Air Corps Stratosphere Flight. See also W. F. G. Swann, G. L. Locher, W. E. Danforth, C. G. and D. D. Montgomery, National Geographic Society Contributed Technical Papers, Stratosphere Series, No. 2 (1936).

DECEMBER 15, 1936 PHYSICAL REVIEW VOLUME 50

Cosmic Rays as Electrical Particles*

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(Received October 12, 1936)

Positive evidence that the primary cosmic rays consist of electrical particles is drawn from three types of experiments:

1. **Latitude and directional asymmetry effects.** Clay finds 83 percent as intense ionization at the equatorial minimum as in high latitudes. Of the remainder, Rossi's directional experiments show that about 12 percent at least is due to positively charged particles. Corresponding to the 73 percent "nonfield sensitive" remainder at sea level, at high altitudes the remainder must be less than 20 percent, perhaps no more than 2 percent, of that observable in polar regions. An energy distribution analysis, following the method of Zanstra, but using new latitude effect data collected on the Pacific Ocean in collaboration with R. N. Turner, shows a continuous energy distribution of the primary cosmic-ray particles between 0.9 and 1.9×10^{10} ev, and indicates the electrical particle origin of a large part, very possibly the whole of the ionization.

2. **Coincidence experiments.** Auger, Street and their collaborators have proved that most of the multiple coincidences observed with counter tubes are caused by single high energy ionizing (hence electrical) particles. Experiments by Rossi and Hsiung show that these coincidence producing particles are not secondaries, but originate beyond the atmosphere. Likewise latitude effect experiments by Johnson and absorption experiments by Rossi indicate that the shower producing radiation is produced by electrical primaries. Cloud chamber studies show that almost all of the observed cosmic-ray ionization is due either to particles of the coincidence type or to showers, and is hence ascribable to electrical primaries.

3. **Galactic rotation effect.** A directional asymmetry of cosmic rays ascribable to the motion of the earth with the rotation of the galaxy, seems to be established by sidereal time variations very recently reported for the northern hemisphere by Illing and for the southern hemisphere by Schonland, Delatizky and Gaskell, and by a new provisional observation by Compton and Turner of a difference of 0.6 percent between the northern and southern

* Based upon a paper delivered at the Tercentenary Conference of Arts and Sciences at Harvard University, September 8, 1936.