The Absorption of Ultra-Short X-Rays by Elements of High Atomic Number

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Employing a specially constructed two-crystal spectrometer and a 600-kilovolt x-ray machine, mass absorption coefficients have been measured, with an average probable error of 0.8 percent, for Pb(82), Ta(73), Sn(50), Ag(47), Mo(42), and Cb(41) in the wave-length range $30<\lambda<185$ X.U. Assuming the correctness of the Klein-Nishina formula for absorption due to scattering, the photoelectric absorption τ_a was calculated for the above elements. In the empirical equation $\tau_a=C_aZ^p\lambda^q$, p increases from 3.51 to 3.88 as λ decreases from 140 to 50 X.U., and q increases from 2.60 to 2.80 as Z decreases from 82 to 41. Agreements with data of other observers for Pb(82) are given. At 30 and 40 X.U., the present data for Pb(82) are about 10 percent higher than the theory developed by Hulme, McDougall, Buckingham and Fowler. New constants in Gray's empirical equation for τ_{Pb} are suggested of: $a=\bar{3}.7321$, $b=1.03$ and $c=0.44$.

INTRODUCTION

WHEN a beam of monochromatic x-rays passes through an absorbing material, the law

$$I=I_0e^{-\mu_a(N/A)(\rho x)}, \tag{1}$$

is obeyed, where I_0 is the intensity of the incident beam, I the intensity after passing through a thickness x of absorber of density ρ, μ_a the total atomic absorption coefficient, N Avagadro's number, and A the atomic weight.

Theoretical and empirical laws have been developed expressing the variation of μ_a with Z, the atomic number, and λ the wave-length of the x-rays; the empirical laws take the form

$$\mu_a=\tau_a(Z,\lambda)+\sigma_a(Z,\lambda), \text{ with } \tau_a(Z,\lambda)=C_aZ^p\lambda^q, \tag{2}$$

where $\tau_a(Z,\lambda)$ is the true or photoelectric atomic absorption coefficient, $\sigma_a(Z,\lambda)$ is the atomic absorption coefficient due to scattering, C_a is a constant over a given wave-length range determined by the absorption limits.

In the wave-length range $0.1<\lambda<0.7$A, Richtmyer[1] determined $p=4$, $q=3$, $0.1<\sigma/\rho<1.0$, and $C_a=22.4\times10^{-27}$ for λ less than the K limit and $6<Z<82$; $\sigma/\rho=\sigma_a(N/A)$. Allen[2] made a large number of measurements in the range $0.081<\lambda<2.0$A and as a result advocates a value of $q=2.92$ although some of his values came out as low as 2.6. Jaeger[3] measured μ/ρ, the mass absorption coefficient, for Pb(82) in the region $30<\lambda<127$ X.U., employing the end-radiation method. Later Alichanjan and Kosman[4] made similar measurements in the range $25<\lambda<60$ X.U., introducing a correction for the effective wave-length when using the end-radiation method. Hahn[5] made very careful absorption measurements on Pb(82), Au(79), W(74), Ta(73), Ag(47), Cu(29), Al(13), C(6) and paraffin in the range $139<\lambda<208.6$ X.U. The data at 208.6 X.U. were taken with a two-crystal spectrometer; the rest of the data with the second crystal removed. Recently, Read[6] measured absorption coefficients for Pb(82) and Sn(50) in the range $20.2<\lambda<52.5$ X.U. using a single rocksalt crystal; the width of the band of rays used by him was about 5 X.U. Read estimates his experimental error on the lead data to be 2 percent.

In the region $4.7<\lambda<47$ X.U., Chao[7] measured absorption coefficients in Pb(82), Sn(50), and Al(13) employing gamma-rays. Gray[8] summarized gamma-ray data for Pb(82) and determined the constants in an empirical equation suggested by Ellis, to fit this summary, using Allen's data at 100 X.U.; this equation for τ, the linear photoelectric absorption coefficient, is

$$\log_{10}\tau=a+\log_{10}\lambda+c(\log_{10}\lambda)^2, \tag{3}$$

[1] F. K. Richtmyer, Phys. Rev. **17**, 264 (1921); **18**, 13 (1921); **27**, 1 (1926); **30**, 755 (1927).
[2] S. J. M. Allen, Phys. Rev. **24**, 1 (1924); **27**, 266 (1926); **28**, 907 (1927).
[3] R. Jaeger, Zeits. f. Physik **69**, 565 (1931).
[4] A. Alichanjan and M. Kosman, Zeits. f. Physik **90**, 779 (1934).
[5] T. M. Hahn, Phys. Rev. **46**, 149 (1934).
[6] J. Read, Proc. Roy. Soc. **A152**, 402 (1935).
[7] C. Y. Chao, Proc. Nat. Acad. Sci. **16**, 431 (1930); Phys. Rev. **36**, 1519 (1930).
[8] L. H. Gray, Proc. Camb. Phil. Soc. **27**, 103 (1931).

where $a=\bar{3}.6505$, $b=1.0$ and $c=0.480$. This law fits gamma-ray data within about 10 percent.

A large number of theoretical calculations have been made for $\tau(Z, \lambda)$; the best of them, calculated by wave mechanics, gives values for τ lower than experiment in the x-ray region. The most recent calculation by Hulme, McDougall, Buckingham and Fowler[9] gives values agreeing with Gray's Eq. (3) above. Their formula is valid for all wave-lengths and atomic numbers but is impractical for all but very short wave-lengths.

Of the theoretical calculations for $\sigma_a(Z, \lambda)$, the absorption due to scattering, the Klein-Nishina formula has been checked experimentally by Chao,[7] by Read and Lauritsen,[10] and by Cuykendall;[11] it represents the results within experimental error. These checks were carried out on the elements of low atomic number, C(6) and Al(13), and at short wave-lengths ($\lambda<100$ X.U.), because the photoelectric absorption may be considered negligible in this range.

Since for gamma-rays ($\lambda<50$ X.U.) the absorption due to scattering plays the major role and for x-rays ($\lambda>100$ X.U.), the photoelectric absorption is the larger part of the total, at least for the heavier elements, the intermediate region $30<\lambda<150$ X.U. for x-rays is an important one for the investigation of the variation of σ and τ with the wave-length and atomic number. In order to study the photoelectric absorption in this region, it is better to use the elements of high atomic number for which σ is small compared to τ. There are almost no reliable data in this region for monochromatic rays. End-radiation measurements are not very accurate and the radiation employed is not monochromatic. Any measurements in this region with single crystal spectrometers as monochromators were taken with very low resolving power of the spectrometers. Therefore a study was undertaken of the photoelectric absorption in the range $30<\lambda<200$ X.U. for elements of high atomic number.

[9] H. R. Hulme, J. McDougall, R. A. Buckingham, and R. H. Fowler, Proc. Roy. Soc. **A149**, 131 (1935).
[10] J. Read and C. Lauritsen, Phys. Rev. **45**, 433 (1934).
[11] T. R. Cuykendall, Phys. Rev. this issue.

Apparatus

The apparatus used consists of a 600-kilovolt x-ray machine and a two-crystal spectrometer described by Cuykendall and Jones.[12] The source of the x-rays is a cascade-type Coolidge tube, with a tungsten target, which is operated on two 300-kilovolt induction coils in series. The monochromator is a two-crystal spectrometer specially designed for use in the wave-length region $30<\lambda<215$ X.U. Transmission of the x-rays through the body of the crystals and reflection from internal planes instead of the usual reflection from the surface, is employed. An ionization chamber containing argon at 80 atmospheres pressure, and a FP-54 electrometer-tube system is used to measure the intensity of the reflected radiation. The width of the wave-length band separated from the continuous spectrum produced by the x-ray tube, is of the order of $1.8<\delta\lambda<1.0$ X.U., for $50<\lambda<215$ X.U.[12]

Results

The elements investigated were Cb(41), Mo(42), Ag(47), Sn(50), Ta(73) and Pb(82) in the range $30<\lambda<185$ X.U. All were of high purity and known to be free of any impurities which would affect the results beyond experimental error. The silver samples were 99.98 percent pure containing no lead; most of the impurity was oxygen. Table I gives a summary of the observed mass absorption coefficients μ/ρ, atomic absorption coefficients μ_a, and photoelectric atomic absorption coefficients τ_a. Each value is obtained from 9 to 12 pairs of observations with an average probable error of 0.8 percent.

The observed values of the atomic absorption coefficients for Pb(82) are plotted in Fig. 1 along with the data of other observers. Richtmyer's values at wave-lengths less than the K limit agree with the observations. Richtmyer's and Hahn's data at 208.6 X.U. agree with the observed value within experimental error; this was taken as a check. Allen's values, in the range $50<\lambda<110$ X.U., are from 40 to 60

[12] T. R. Cuykendall and M. T. Jones, Rev. Sci. Inst. **6**, 356 (1935).

TABLE I. *Observed absorption coefficients.*

λ (X.U.)	Pb(82) μ/ρ	Pb(82) $\mu_a\times10^{22}$	Pb(82) $\tau_a\times10^{22}$	Ta(73) μ/ρ	Ta(73) $\mu_a\times10^{22}$	Ta(73) $\tau_a\times10^{22}$	Sn(50) μ/ρ	Sn(50) $\mu_a\times10^{22}$	Sn(50) $\tau_a\times10^{22}$
30	0.22	0.752	0.499						
40	0.394	1.346	1.064						
50	0.600	2.052	1.745	0.458	1.372	1.100	0.246	0.482	0.295
60	0.900	3.077	2.751	0.689	2.064	1.774	0.314	0.615	0.416
70	1.455	4.975	4.634	1.018	3.049	2.745	0.430	0.842	0.634
80	1.829	6.254	5.896	1.357	4.065	3.747	0.607	1.189	0.972
90	2.490	8.513	8.145	1.819	5.448	5.120	0.800	1.567	1.343
100	3.248	11.11	10.73	2.435	7.293	6.956	1.020	1.998	1.767
110	4.079	13.94	13.56	3.014	9.027	8.681	1.222	2.394	2.158
120	5.150	17.61	17.21	3.770	11.29	10.94	1.587	3.108	2.867
130	6.412	21.93	21.52	4.553	13.64	13.28	1.912	3.745	3.499
140				5.476	16.40	16.03	2.310	4.524	4.274
160									
184.5									
208.6	5.108								

	Ag(47) μ/ρ	Ag(47) $\mu_a\times10^{22}$	Ag(47) $\tau_a\times10^{22}$	Mo(42) μ/ρ	Mo(42) $\mu_a\times10^{22}$	Mo(42) $\tau_a\times10^{22}$	Cb(41) μ/ρ	Cb(41) $\mu_a\times10^{22}$	Cb(41) $\tau_a\times10^{22}$
50	0.229	0.407	0.232	0.177	0.280	0.124	0.181	0.278	0.125
60	0.283	0.504	0.317	0.227	0.360	0.193	0.232	0.356	0.194
70	0.400	0.712	0.516	0.319	0.505	0.330	0.305	0.469	0.298
80	0.544	0.969	0.764	0.412	0.653	0.470	0.402	0.617	0.440
90	0.715	1.273	1.062	0.535	0.847	0.658	0.530	0.814	0.631
100	0.900	1.602	1.386	0.650	1.029	0.836	0.667	1.025	0.836
110	1.081	1.925	1.702	0.821	1.300	1.102	0.805	1.237	1.043
120	1.362	2.425	2.198	1.036	1.643	1.440	1.065	1.636	1.439
130	1.656	2.948	2.717	1.243	1.969	1.762	1.290	1.982	1.781
140	2.038	3.628	3.392	1.513	2.397	2.185	1.555	2.389	2.184
160				2.108	3.339	3.123	2.226	3.420	3.210
184.5				3.193	5.057	4.837	3.092	4.750	4.534

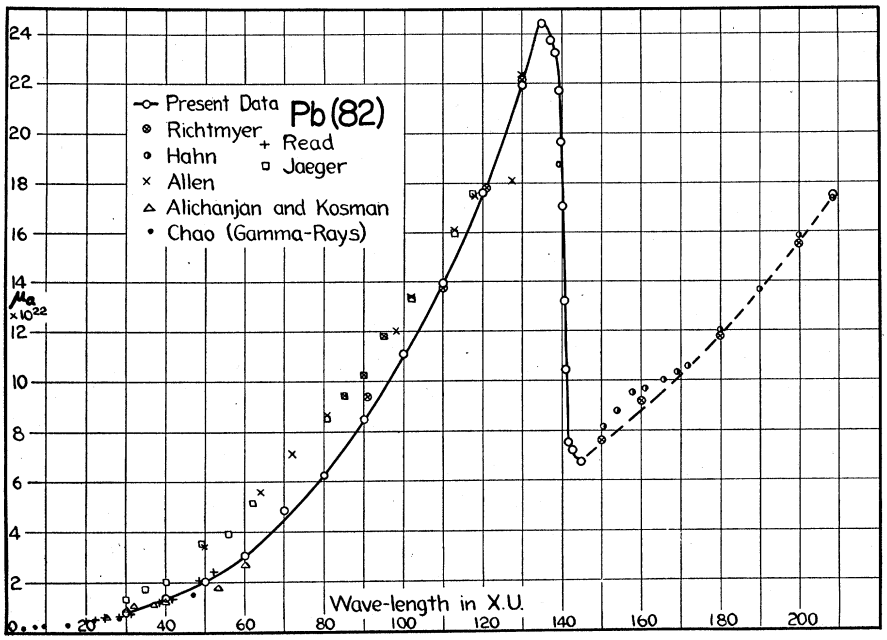

FIG. 1. Comparison of present values of μ_a for Pb(82) with those of other observers.

percent higher than the present data. This discrepancy could be accounted for by assuming that the radiation he employed was not monochromatic due to low resolving power of his apparatus in this region. Jaeger's data agree with Allen's but, as pointed out by Alichanjan and Kosman,[4] a correction should be made to the wave-length of the short-wave limit of their operating voltage, when employing the end-radiation method, to take into account the fact that a homogeneous beam of x-rays is never really reached by this method; the effective wave-length is longer than the short-wave limit. This would move Jaeger's data more in agreement with the present observations. Alichanjan and Kosman's data, for which this correction has been made, agree with the data herein reported. Read's data agree with the observed data within experimental error on the wave-length region covered by him. The gamma-ray measurements of Chao agree within 15 percent where they overlap with the present data.

Hahn's data for Ta(73) are lower than an extrapolation of the present data in the region $140<\lambda<160$ X.U. beyond the experimental error. His data for Ag(47) are higher in the same range. The data of Allen are consistently 10 to 50 percent higher than the present data. Read's data for Sn(50) fit on to an extrapolation of the observed data for $\lambda<50$ X.U.

The Klein-Nishina formula for the absorption due to scattering has been checked within experimental error;[7, 10, 11] therefore, if one assumes that the scattering per electron σ_e is independent of the atomic number, the photoelectric absorption τ_a per atom may be calculated from $\tau_a=\mu_a-Z\sigma_e$.* Plotting these values of τ_a against wave-length on a double logarithmic scale should give a straight line if q in Eq. (2) is constant; the slope of such a straight line gives the value of q. Fig. 2 is such a graph for the elements studied. The data for each element fall nearly on straight lines. Table II gives the values of q found from these curves.

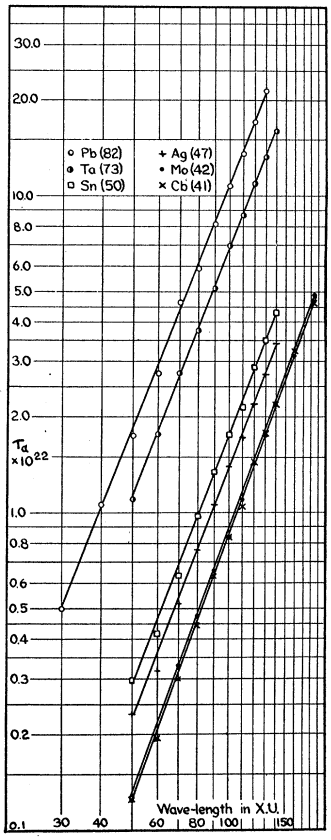

FIG. 2. τ_a plotted against wave-length for various elements.

In a similar manner, the exponent p, of Z in Eq. (2), may be determined from a double logarithmic plot of τ_a against Z. Fig. 3 is the graph obtained. Again the data fall nearly on straight lines. Table III gives the values of p. The variation of p agrees qualitatively with the theory[13] of photoelectric absorption which predict that $p\doteq 5$ as $\lambda\doteq 0$ and should decrease as the

TABLE II. *Experimental values of q in Eq. (2).*

Z	41	42	47	50	73	82
q	2.79	2.80	2.63	2.63	2.60	2.60

TABLE III. *Experimental values of p in Eq. (2).*

λ (X.U.)	p	λ (X.U.)	p
140	3.51	90	3.69
130	3.64	80	3.73
120	3.62	70	3.84
110	3.68	60	3.88
100	3.69	50	3.84

* As an extreme case, if, due to coherent scattering, σ_e were 5 percent greater for Pb(82) than calculated by the Klein-Nishina formula, at $\lambda=30$ X.U., τ_a would be less than 2 percent higher which is of the order of the experimental error and would not seriously affect the conclusions drawn herein.

[13] F. Sauter, Ann. d. Physik **9**, 217 (1931); **11**, 454 (1931).

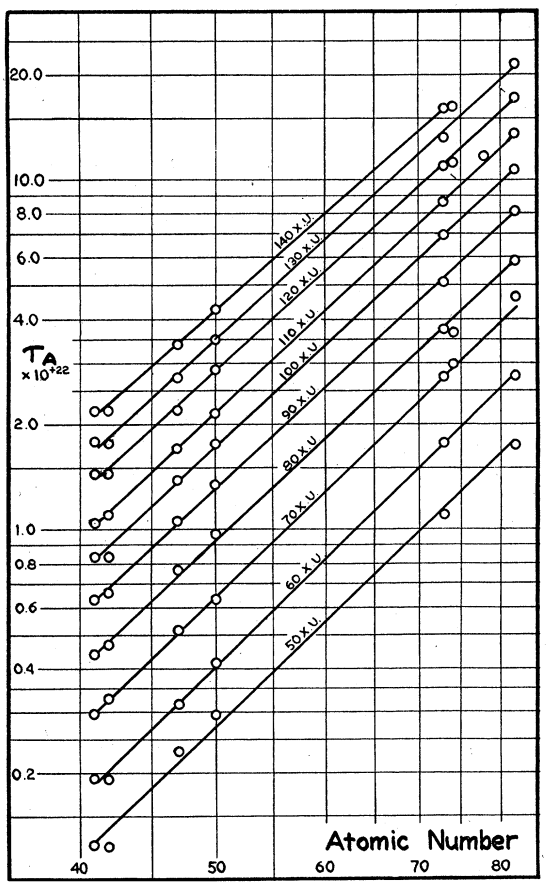

FIG. 3. τ_a plotted against atomic number for various wave-lengths.

wave-length increases. In the wave-length region here studied these data indicate that if there is a variation of q with λ, it is much less than that predicted by theory.

The agreement with Gray's empirical Eq. (3) is not good for the values of the constants given by him. Gray used Allen's data at 100 X.U. in the determination of his constants which partly explains the lack of agreement. Suggested values of the constants of Eq. (3) to fit the present data are: $a=\bar{3}.7321$, $b=1.03$ and $c=0.44$. Table IV gives the observed values, values calculated from Gray's equation, and values calculated using these new constants. Of course it is obvious that an equation in which q varies with wave-length can fit only approximately the present data for which q has been shown to be constant; therefore such an equation has no theoretical significance, but may be of practical use in irradiation work with hard x-rays.

TABLE IV. *Values of* $\tau_a \times 10^{22}$ *for Pb(82).*

λ (X.U.)	OBSERVED	CALCULATED FROM EQ. (3) using $a=\bar{3}.6505$, $b=1.0$, $c=0.48$	using $a=\bar{3}.7321$, $b=1.03$, $c=0.44$	CALC. FROM THEORY
30	0.50	0.45	0.49	0.47
40	1.06	0.92	0.98	0.91
50	1.74	1.64	1.70	
60	2.75	2.62	2.69	
70	4.63	4.06	4.07	
80	5.90	5.90	5.82	
90	8.14	8.26	8.02	
100	10.73	11.20	10.74	
110	13.56	15.51	14.63	
120	17.21	19.26	17.98	
130	21.52	24.45	22.63	

Calculations taken from the theoretical photo-electric absorption curve given by Hulme, McDougall, Buckingham and Fowler[9] for Pb(82) are given in the last column of Table IV. The calculated values are about 10 percent lower than the observations.

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