heating to dull red in air. He therefore ascribed this surface loss, in part at least, to a layer of some easily volatilized substance left by the cleaning agent. In order to see whether a similar effect existed for reflection, measurements of the reflecting power of lithium fluoride at 45° and λ1433A of a preliminary nature have been made by the author using a crystal obtained from Dr. Schneider and polished by Mr. David Mann. The specimen was arranged so that it could be heated to dull red between reflecting power measurements, in vacuum all the while. Before heating it was found that the reflection coefficient was about 15 percent greater than the value computed from the index of refraction as measured dioptrically by Schneider, a discrepancy nearly the same as was observed for fluorite. After heating, however, the reflection coefficient decreased to the computed value to within experimental error. It seems probable, then, on account of this similarity between surface losses and reflecting powers, respectively, that the surface layers on fluorite and lithium fluoride were of a similar character. It is thought, therefore, that had it been possible to heat fluorite without shattering it, indices of refraction in much closer agreement with those of Handke would have been obtained.

It is possible, however, that the surface layer was not entirely one of a foreign substance, and that it included a region of fluorite in which polishing had disturbed the crystal structure and so altered the optical constants. No direct evidence bearing on an effect of this kind has been obtained in the present work. However, the work of Margenau[13] in the ultraviolet and of Hilsch and Pohl[14] down through λ1600A lends some support to this point of view. They have shown that the condition of the surface can cause a shift in the center of an absorption band amounting to as much as 44A. However, in the case of fluorite a shift of 125A, which is greater than they have reported, would be required to account for the whole discrepancy between the dispersion curves.

The author takes great pleasure in thanking Professor Lyman for constant encouragement and valuable advice given during the entire course of this research. He wishes also to express his appreciation of Mr. David Mann, instrument maker of the Harvard Physics Laboratories, whose skill and ingenuity helped make this work possible.

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[13] Margenau, Phys. Rev. **33**, 1035 (1929).
[14] Hilsch and Pohl, Zeits. f. Physik **59**, 812 (1930).

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A Very Accurate Test of Coulomb's Law of Force Between Charges

S. J. Plimpton and W. E. Lawton, *Worcester Polytechnic Institute, Worcester, Massachusetts*

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The exponent 2 in Coulomb's inverse square law of force between charges in empty space has been found experimentally to be correct to within 1 part in 10^9. The well-known electrostatic experiment of Cavendish and Maxwell with concentric metal globes was replaced by a quasi-static method in which the difficulties due to spontaneous ionization and contact potentials were avoided. A "resonance electrometer" (undamped galvanometer with amplifier) was placed within the globes, the input resistor of the amplifier forming a permanent link connecting them, so as to measure any variable potential difference between them. It was shown theoretically that the presence of the resonance electrometer would have no effect on the result and that it could replace electrically a part of the inner globe. The galvanometer was observed through a "conducting window" at the top, made so by covering it with salt water. No effect was observed when a harmonically alternating high potential V (>3000 volts), from a specially designed "condenser generator" operating at the low resonance frequency of the galvanometer, was applied to the outer globe. The sensitivity was such that a voltage $v=10^{-6}$ volt was easily observable above the small fluctuations due to Brownian motion.

If the exponent in the law of force were not exactly 2 but rather $2\pm q$ then $q<v/VF(a, b)$ where $F(a, b)=0.169$, a and b being the radii of the globes. This gives $q<2\times 10^{-9}$ in space remote from matter. The formula for $F(a, b)$ was derived by Maxwell's theory in which the effect of gravity is assumed negligible. Reasons are given for believing that this assumption does not invalidate the result.

INTRODUCTION

THE inverse square law of force between electric charges was tested by Coulomb with his torsion balance and more precisely by Maxwell using a modification of a method invented by Cavendish. Maxwell used a spherical air condenser consisting of two insulated spherical shells ("globes"), the outer having a small hole in it so that the inner could be tested for charge by inserting an electrode from a Thomson electrometer. This hole was closed by a small lid carrying an inward projection which simultaneously connected the two shells together. The outer shell was then charged to a high potential V, the lid and connector removed by a silk thread, and the outer shell earthed. Maxwell showed[1] that if the exponent in Coulomb's law is not 2 but $2\pm q$ the magnitude of the potential of the inner shell (neglecting higher orders of q) would then be $VqF(a, b)$, where $F(a, b)$ is a known function of the ratio of the radii a and b of the outer and inner spheres, namely:

$$F(a, b) = \frac{n}{2}\log\frac{n+1}{n-1} - \tfrac{1}{2}\log\frac{4n^2}{n^2-1}, \quad n = \frac{a}{b}.$$

Since the potential remaining on the inner shell was found by the experiment to be less in magnitude than a small detectable potential v, then $q < v/VF(a, b)$. In this way Maxwell obtained the result still generally quoted in text-books, $q < 1/21{,}600$.

Professor A. Wilmer Duff directed the attention of the writers to the problem of improving the accuracy of this test, i.e., of reducing this upper limit for the value of q, with the aid of the more sensitive electrometers now available.

PRELIMINARY INVESTIGATIONS

In the derivation of the expression $VqF(a, b)$ it is assumed that the globes are exact spheres and that the charges are distributed uniformly over them. These assumptions are made merely to facilitate the computation, and the order of magnitude of the result is unaffected if the globes are only approximately spherical and the distribution only approximately uniform. The smallness of the result for the upper limit for q depends not upon a close approach to sphericity, but rather upon the great difference in the magnitudes of the applied voltage V and the smallest detectable voltage v.

At first sight one would suppose that an enormous improvement in the accuracy of the test by Maxwell's method would be possible, due to recent advances in the production of high voltages and in the detection of extremely small voltages. Curiously enough, however, Maxwell apparently reached about the limit attainable by his method even with modern equipment. In the first place with what is now known about the disturbing effect of unavoidable *spontaneous ionization*[2] it is a question how long a charge on the inner globe, even as small as that involved in Maxwell's experiment, would remain unaffected. The most serious difficulty in going to great sensitivities in this method is due, however, to *contact potentials*. It is the experience of the writers that in the measurement of *an isolated charge* 0.001 volt is about the practical limit. This is entirely different from the use of contacts in the measurement of potentials associated with small currents, as in ionization measurements, where the energies involved are greater and initial effects become smoothed over with time. The following statement of Hoffman[3] in reference to his electrometer is interesting in this connection: "The contact potential difference of the duants (entirely platinized) with respect to each other proved to be of the order of magnitude of a thousandth of a volt; the drop to the needle is greater."

The possibility of applying the Maxwell method with a vacuum between the globes and no contacts was studied, the potential of the inner globe being measured by its effect on a small inductor which was connected to an amplifier and moved in the vicinity of the charge. Even here there seemed to exist somewhat variable space gradients of potential of the order of 0.0001 volt.

DETECTOR INSIDE GLOBE

In the method finally adopted the effects of contacts were entirely eliminated by placing the detector inside the inner globe, and connecting it

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[1] Maxwell, *Electricity and Magnetism*, I, p. 83.

[2] See, e.g., J. A. Bearden, Rev. Sci. Inst. **4**, 271 (1933).

[3] G. Hoffman, Ann. d. Physik **52**, 701 (1917).

permanently so as to indicate any change in the potential of the inner globe relative to the outer one. This modification of Maxwell's method, in particular the presence of the conducting mass of the detector within the inner globe, changes somewhat the conditions under which the inequality $q<v/VF(a, b)$ was derived. In order to make sure that this formula would still be applicable we extended the theory as given by Maxwell to the case of three concentric spherical shells of radii a, b, c (where $a>b>c$), the two inner ones being connected together but insulated from the outer one. It was thus found that charging the outer shell to the potential V would cause a potential difference between it and the inner shells given by the same expression $VqF(a, b)$ if the exponent is $2\pm q$, where $F(a, b)$ is the function already stated. If this potential difference is shown experimentally to be less than v we have as before

$$q<v/VF(a, b).$$

Calculation also showed that the magnitude of the charge necessary to neutralize this potential difference is found by multiplying it by the capacity $ab/(a-b)$, whether or not the innermost shell is present.

Since, then, the presence of a conducting body inside the inner globe and connected to it has no effect on the results, it was possible to arrange dimensions so that the detector, which consisted of a tube electrometer and amplifier, together with its shield was approximately equivalent to the lower half of the inner globe. This lower half could then be replaced by the detector.

CONSTRUCTION OF GLOBES AND DETECTOR

To serve as the outer globe A (Fig. 1) two approximately hemispherical shells, five feet in diameter, were constructed of sheet iron gores soldered together. One of these was mounted on a cylindrical porcelain insulator to form the lower half of A. The copper boxes containing the detector D were properly located in it, supported by porcelain insulators on a slat floor. The detector D consisted of a five-stage, resistance-capacity coupled amplifier designed for a frequency of about 2 cycles per second so as to operate a panel galvanometer G having this frequency and low damping. The first tube was an FP 54 with an input grid resistor of 10^{10} ohms. This was followed by three stages using PJ 11 tubes, which were enclosed in cast iron boxes suspended on rubber to avoid microphonic effects. The final stage of the amplifier was of conventional design. The sensitivity was adjusted so that the Johnson effect (Brownian heat motion) in the input resistor caused an average motion of the galvanometer of about one small division, corresponding to about $\frac{1}{2}$ microvolt.

The inner hemispherical dome B of four feet diameter, constructed of sheet iron gores similar to those of A, was mounted on Pyrex glass insulators and connected to the amplifier. Tests were made with the outer shell A connected to the grid of the first tube, the amplifier shield being connected to the dome B, and also with the grid connected to B and the amplifier shield connected to A. In neither case could any change in the potential of B relative to A be detected when A was properly charged and discharged.

The galvanometer G was illuminated by a lamp drawing current from the amplifier batteries, and after the upper half of A was in place it could be viewed through holes at the top of A and B. It was important that there be no break in the conducting surface of the outer globe A. The problem of a *conducting window* was solved by using a glass-bottomed vessel threaded into the hole in A, which contained a solution of ordinary salt in water with its surface flush with the outer surface of A. Submerged in the solution was a disk of fine wire gauze, covering the glass and soldered to the threaded rim of the vessel. The presence of the salt solution was essential to the success of the experiment.

QUASI-STATIC METHOD

When the outer globe was charged and discharged simply by opening and closing circuits, the galvanometer gave deflections which were due to electromagnetic effects ($Mdi/dt=Md^2Q/dt^2$ where Q is the charge on A and M is the mutual conductance between A and B). It was necessary to devise a method for changing the charge Q on A in such a manner that these electromagnetic effects would never be large enough to be detectable. In other words, di/dt had to be kept

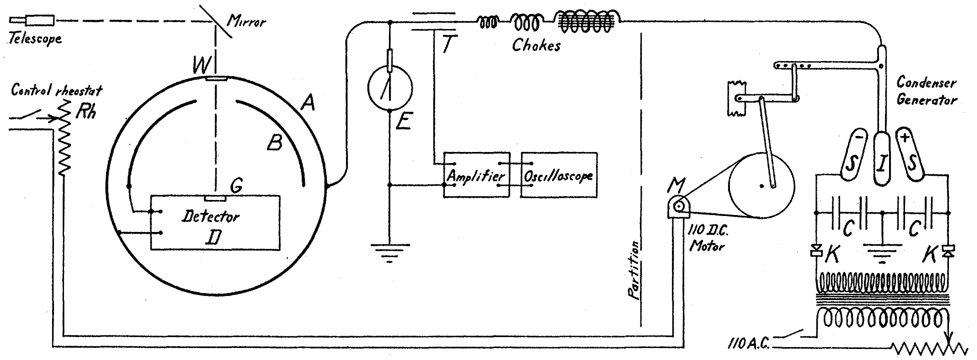

FIG. 1. Apparatus for testing the inverse square law of force between charges.

small. For this purpose the use of a resistor alone was not satisfactory, on account of the large value of *di/dt* during a very short time at the beginning of the charge or discharge, and it was not feasible to construct a sufficiently large choke. A quasi-static method was therefore adopted.

In this method the detector was employed as a *resonance electrometer*. The high voltage was applied to the outer globe as a smooth sinusoidal wave of very low frequency, so as to build up a resonant vibration of the galvanometer. This had great advantages apart from that of sensitivity. The chief advantage may be explained as follows. The sensitivity of the amplifier is limited by Brownian heat fluctuations. These consist of all frequencies with equal energies, according to the equipartition law. The galvanometer responds in a resonant manner to the components among these with frequencies approximately equal to its own. The disturbance to be measured (signal) is at a relative disadvantage (by a factor depending on the damping of the galvanometer) as compared with this heat motion unless it too has a frequency equal to that of the galvanometer.

As indicated above, a low frequency was necessary to reduce electromagnetic effects. Calculation showed that with a frequency of about 2 cycles per second the inductive effect (*Mdi/dt*) would be entirely submerged in the fluctuations (about $\frac{1}{2}$ microvolt) due to heat motion in the detector.

CONDENSER GENERATOR

There appears to be no commercially available source of such low frequency high voltage with smooth wave form. Transformers and filters are not practical at such low frequencies, nor is high voltage directly from an ordinary generator. What might be called a *condenser generator* was developed for the purpose. It may be described as consisting of three large metal combs: two stationary (stators) nearly parallel to each other with their teeth upward, and the third (the inductor) held with teeth downward and swung harmonically in and out of mesh with the other two alternately. It is shown diagrammatically at the right of Fig. 1 and disassembled in Fig. 2.

The maximum voltage provided by this generator is determined by the spark-over potential between the inductor I and the stators SS. No attempt was made to use extremely high voltages because of the disturbing effect of the radiation associated with brush discharge. The peculiar design of the generator was the result of an attempt to realize the two following conditions: a smooth sinusoidal wave form and a rapid increase of the gap between combs as the inductor moved away from either stator, so that the rising potential difference would not cause spark-over at any point.

The stators SS (Fig. 1) were charged by a transformer from the lighting circuit with the aid of the kenetrons KK. Their capacity to ground, which was increased by adding mica condensers CC in parallel, was large compared with that of the globe A, so that their potentials were not affected appreciably by the variation of their inductive relation with A as the inductor I swung in and out of mesh. Since these condensers retained their charges for hours, this arrangement also made it possible to turn off the kenetron charging system during the experiment, thus avoiding the difficulty of filtering out disturbances from this system.

ARRANGEMENT OF APPARATUS

From Fig. 1 it will be seen that the generator was connected to the globe A through graduated chokes as a further protection against high frequencies. The applied voltage could be read directly on the Braun electrometer E for any position of the inductor I. For observing the wave form a cathode-ray oscilloscope was also coupled to the line through a tubular air condenser T. The amplifier shown in front of this oscilloscope was required on account of the high impedance of the condenser. The oscilloscope was provided with a 60-cycle horizontal sweep, which became a very slender ellipse owing to pick-up from the surroundings. This ellipse moved up and down with the applied voltage, retaining its smooth shape and thus proving the absence of high frequency disturbances.

If the variation of the voltage applied to the globe A produced an appreciable potential difference between A and the dome B, it would be indicated by resonant motion of the galvanometer G in the detector D connected between A and B as already described. G was observed through the conducting window W by means of a mirror and telescope as shown. The observer at the telescope could start and stop the motor M which drove the generator and, by means of the rheostat Rh and meters not shown, he could control the speed of M and thus keep the frequency of the applied voltage at the resonance value (130 per minute). This frequency adjustment could be checked at any time by blowing enough salt water out of the window W so that the galvanometer would respond.

METHOD OF CALIBRATION

The connections for the calibration are shown in Fig. 3. The outer shell A was grounded, and a small known fraction $r/(R+r)$ of the voltage from the condenser generator was applied to the dome B by means of the high resistance potentiometer R+r, the frequency being kept at the resonance value. The amplitude of the voltage from the generator was read directly on the oscilloscope O (without sweep) which, being independent of frequency, could be calibrated by applying voltages from a battery as indicated. In this way a very nearly linear calibration for the resonance electrometer D was obtained.

In determining the smallest voltage v detectable by D it was desirable to keep both the relaxation time and the impedance of the input of the amplifier as nearly as possible the same as under working conditions. The presence of the resistance r reduced slightly the fluctuations of the galvanometer due to heat motion. These were restored when a small air condenser c (with shield not shown) was inserted in the lead to the dome. By making allowance for the known impedance of this condenser the calibration was found to be as before. The minimum voltage v observable above the usual heat fluctuation was thus determined with the calibrating device so loosely coupled that it had practically no effect on the sensitivity of the resonance electrometer.

RESULT

With the apparatus operating properly at the resonance frequency, many trials with the amplitude of the applied voltage V never less than 3000 volts and v remaining very consistently equal to 1 microvolt convinced various observers that no change in the small heat motion of the galvanometer could be detected when the generator was started and stopped at random. For our apparatus $a=2.5$ ft., $b=2.0$ ft., and $F(a, b)=0.169$. Substitution in the inequality $q<v/VF(a, b)$ then yields $q<10^{-6}/3000\times0.169$, whence

$$q<2\times10^{-9}.$$

The exponent 2 in the inverse square law of force

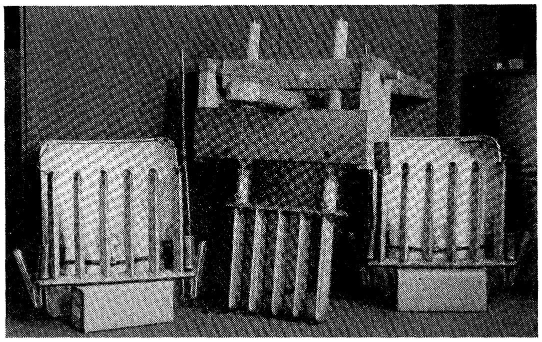

FIG. 2. Condenser generator disassembled.

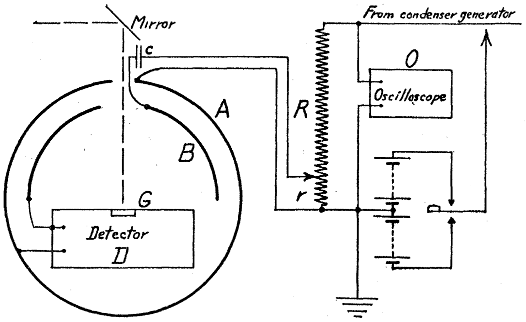

FIG. 3. Method of calibration.

between charges in space remote from matter is thus shown to be correct to within one part in 10^9.

EFFECT OF GRAVITY

In view of the accuracy of the test some statements as to the possible effect of gravity are desirable. The expression for $F(a, b)$ was derived by means of classical electrodynamics in which the effect of gravity is neglected. It was assumed, for instance, that the charge density on the surface of a spherical conductor is the same everywhere. If electrons (inertial mass m_ϵ, charge $-\epsilon$) have weight $m_\epsilon g$ the electron density on the conductor must be unsymmetrical, being greater near the bottom, in such a way that at any point of its surface the force of gravity on an electron is balanced by the electrical force due to the vertical potential gradient dV/dh, that is, $\epsilon dV/dh = m_\epsilon g$. This gives by integration a maximum potential difference over globe A of not more than 10^{-10} volt. Since this is far less than the minimum detectable voltage $v = 10^{-6}$ volt, such an effect is too small to cast any doubt on the result. Moreover, the force due to gravity may be thought of as neutralized by the electrical force due to the unsymmetrical electron density before any charge is applied to the globe. The applied charge would then be distributed symmetrically. Looked at in this way, gravity should have no effect on the experiment however accurately performed. We can then say that what we have tested is the law of force between charges in space remote from matter, that is, not in a gravitational field.

An objection to these statements might be raised on the ground that it should not be assumed that there is no interrelation between gravitational and electrical fields, and that it is conceivable that the gravitational field might, by virtue of this interrelation, affect the electrical field in just such a way as to give no variable potential differences inside the globes even if q for empty space were greater than 2×10^{-9}. The possibility of such a coincidence seems remote, to say the least, especially in view of the magnitudes involved. However, we have considered the shielding action of the globe from the point of view of the general theory of relativity and have arrived at a conclusion which is even more reassuring. The reasoning on which this conclusion is based involves considerable mathematics and is left to a later paper.

The authors wish to express their appreciation to Professor A. W. Duff at whose suggestion this investigation was undertaken, to Professor H. H. Newell for his stimulating interest and advice, and to Mr. R. F. Field of the General Radio Company for helpful discussions regarding the action of amplifiers.

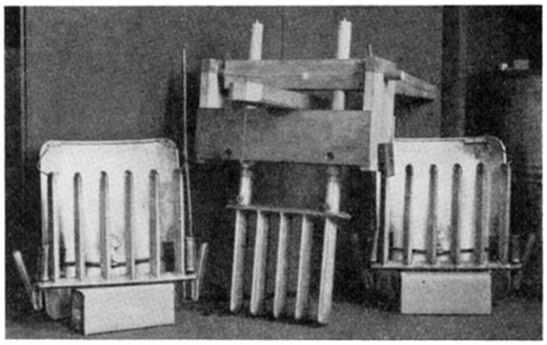

FIG. 2. Condenser generator disassembled.