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The Absorption of X-Rays of Wave-Length $50 \leq \lambda < 150$ X.U. by Elements of Low Atomic Number

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With a 600 kv x-ray source and a two-crystal spectrometer especially designed for short wave-length x-rays, the mass absorption coefficients for C(6), Na(11), Al(13), S(16), K(19), Ni(28), Cu(29) were determined for wave-lengths from 50 to 209 X.U. A small $\delta\lambda$ interval was used. The results are estimated to be in error by no more than two percent. The present results are compared with the few overlapping measurements of other observers. The values of the electronic scattering coefficient predicted by the Klein-Nishina formula are in excellent agreement with the experimental values of the scattering by carbon in the region $40 \leq \lambda \leq 100$ X.U., which confirms the Klein-Nishina formula for this wave-length region. The photoelectric absorption in copper was found to be proportional to $\lambda^{2.83}$.

INTRODUCTION

THE determination of the absorption coefficients of monochromatic x-rays in the region of wave-lengths shorter than 150 X.U. is of importance (1) in testing the Klein-Nishina formula[1] for the intensity lost by scattering, and (2) in providing additional information which may be used to check the validity of recent formulae for the photoelectric absorption. Furthermore, absorption measurements for monochromatic radiation are of value to the medical profession in the use of high voltage x-rays for therapeutic purposes.

No systematic study of absorption coefficients has been made for the present wave-length region. Absorption measurements at wave-lengths less than 120 X.U. require a powerful, high voltage, x-ray source of constant intensity, and an analyzing spectrometer or other device which effects a satisfactory compromise between intensity and monochromaticity of the emergent beam. It is to be noted that there exists no *practical* target material, except possibly uranium, whose characteristic x-radiation is shorter than 120 X.U., and therefore the continuous spectrum must be used.

Early determinations[2–5] of absorption coefficients include a few measurements in the present wave-length region. However the low intensity of the continuous spectrum produced by the x-ray sources then available, combined with the difficulties encountered in measuring very small ionization currents with the electrometer as compared with the FP-54 vacuum tube, favored intensity at the expense of resolving power. Thus, although a single crystal spectrometer was used as monochromator, the $\delta\lambda$ interval (as calculated from the spectrometrical geometry) was a large fraction of λ, which means that the uncertainty in the effective wave-length was large. A further disadvantage of the single crystal spectrometer, resulting from the small Bragg angles encountered at short wave-lengths, is the problem of adequately shielding the ionization chamber from scattered radiation.

The filtered end-radiation from a heterogeneous beam has been used recently in the determination of absorption coefficients.[6] The results obtained with this method are discussed elsewhere.[7]

In consideration of the low resolving power used in previous work, and the small number of elements studied, the present investigation in the wave-length region $50 \leq \lambda < 150$ X.U. was undertaken with a 600 kv, 2000-watt, x-ray source, and with a two-crystal spectrometer especially designed for this wave-length region. The results of the absorption measurements for a series of elements of low atomic number are given herein, and for elements of high atomic number are given in the following paper by M. T. Jones.[7]

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[1] Klein and Nishina, Zeits. f. Physik **52**, 853 (1928).

[2] Compton and Allison, *X-Rays in Theory and Experiment*, p. 534.

[3] Richtmyer, Phys. Rev. **18**, 13 (1921).

[4] Richtmyer and Warburton, Phys. Rev. **22**, 539 (1923).

[5] Allen, Phys. Rev. **27**, 266 (1926).

[6] Alichanjan and Kosman, Zeits. f. Physik **90**, 799 (1934).

[7] Jones, Phys. Rev. **50**, 110 (1936).

Apparatus and Method of Observation

The apparatus used in this investigation has been previously described.[8] Because of the greater practical resolving power without undue sacrifice of intensity, a two-crystal, rather than a single-crystal, spectrometer was used as monochromator. The particular design of the present instrument, which utilizes internal reflection of the x-rays as they are transmitted through the body of the crystals, permitted adequate shielding of the ionization chamber from scattered radiation. The vertical limiting slits were so adjusted that the geometrical resolving power was many times greater than that due to the crystals. The horizontal divergence was limited by the size of the focal spot. The $\delta\lambda$ interval, as determined from considerations previously discussed,[8] was $1.8 < \delta\lambda < 1.0$ X.U. for $50 < \lambda < 215$ X.U., respectively.

The absorbers, placed between the x-ray tube and the first crystal, were mounted as described in reference 8. Lead shields having holes $3/4 \times 1/2$ inches in the path of the x-ray beam protected the absorbers from radiation scattered by other objects in the vicinity. It was thought possible that an error might be introduced in the observations taken at wave-length λ_1, due to radiation of $\lambda < \lambda_1$, present in the beam, undergoing a wave-length shift by multiple Compton scattering within the absorber in such a way as to be reflected by the crystals as radiation of wave-length λ_1. The error due to this effect was calculated and found to be entirely negligible.

Each measurement of an absorption coefficient consisted of 9 to 12 values of $(I+I_z)$ and of (I_0+I_z), where I and I_0 represent, respectively, the ionization current with the absorber in the path of the beam, and removed, and where I_z represents the background or residual ion current. At intervals during the recording of the other ion currents, 4 or 5 values of I_z were determined by stopping the x-ray beam with a lead shutter, 3/4 inch thick, placed between the target and the first crystal. That this method gave the true residual current is shown by the fact that for the shutter open or closed no difference was observed in the residual ion current (I_z) when one crystal had been turned a few degrees (about 5°) from its position previously determined by the setting of the wave-length screw (see reference 8) for a given value of λ. This test was made for several wave-lengths, including 50 X.U.

At each wave-length λ the x-ray tube was operated at a voltage such that there was not present in the beam second-order energy of wave-length $\lambda/2$. Especial care was taken to keep the tube voltage constant, but due to the type of voltage source employed it is probable that occasionally there were random fluctuations of as much as 3 percent. It is believed that this fluctuation was the greatest source of error. In order to reduce errors due to this cause, a large number of observations were taken.

The Absorbers Investigated

Particular care was taken to obtain absorbers of high purity. The effect of such impurities as were present in the samples is less than the experimental error of the results. The carbon absorbers (99.9 percent pure) were regraphitized graphite made especially for spectroscopic purposes. The aluminum used was 99.97 percent pure and contained 0.008 percent copper and 0.008 percent iron as the principal impurities. Chile copper, 99.96 percent pure, was used. The nickel absorbers were the same high purity absorbers used in a previous investigation.[4] The sulfur absorbers were carefully cast from the highest grade C.P. sulfur obtainable.

The sodium and potassium absorbers were prepared by the R. and H. Chemicals Department of the E. I. duPont de Nemours & Company. The metals were cast in blocks of suitable dimensions and each block was wrapped in sheet of aluminum foil whose edges were sealed with adhesive tape and paraffine. The purity of the samples was reported to be as follows: The sodium was 99.7 percent pure and contained no heavy metals and not over 0.04 percent calcium. The potassium was 99.0 percent pure with sodium as the other main constituent. Samples of both metals contained as other impurities traces of the oxide, hydroxide, and carbonate.

Results

The atomic absorption coefficient, μ_A, is defined by the relation

$$\mu_A = \tau_A - Z\sigma_e, \tag{1}$$

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[8] Cuykendall and Jones, Rev. Sci. Inst. **6**, 356 (1935).

where τ_A is the photoelectric absorption coefficient, Z the atomic number, and σ_e the total scattering per electron. As discussed presently, for elements of low atomic number and for the present wave-length region, the scattering, $Z\sigma_e$, is the greater part of the observed absorption. Several theoretical formulae have been proposed which express the variation of σ_e with wave-length. Of these the Klein-Nishina formula[1] predicts values of σ_e which differ considerably from those of the earlier theories, and the Klein-Nishina values have been shown to agree with the observed scattering at wave-lengths from 20 to 50 X.U.[9] The present report is largely concerned with the comparison of $Z\sigma_e$ as predicted by theory and as experimentally observed in the wave-length region $40<\lambda<100$ X.U.

The variation of τ_A with Z and λ may be written as

$$\tau_A = KZ^p\lambda^q, \tag{2}$$

where K is a constant for $\lambda<\lambda_{K\ \text{limit}}$, and p and q are constants, approximately 4 and 3, respectively. [More precisely, as discussed elsewhere,[7] p and q may be functions of Z and λ.] It has been shown that for gamma-rays the photoelectric absorption in elements $Z=1$ to $Z=16$ is less than experimental error.[10] Hence, because of the relation in Eq. (2), it may be expected that the photoelectric absorption in carbon, $Z=6$, is less than the experimental error of μ_A at wave-lengths considerably greater than those of gamma-rays. Therefore, the absorption measurements for carbon provide a test of the validity of the Klein-Nishina formula.

The experimental results for carbon are given in Table I. In Fig. 1 are plotted the experimental values of μ_A/Z and also the theoretical values of the total scattering per electron, σ_e, as calculated from the Klein-Nishina formula. The values of μ_A/Z and σ_e are in excellent agreement from 40 X.U. to the wave-length at which the photoelectric absorption becomes appreciable, about 100 X.U., which confirms within experimental error the Klein-Nishina formula for this wave-length region.

TABLE I. *Measured values of the mass absorption coefficient, μ/ρ, and the electronic absorption coefficient, μ_e, for carbon; and comparison with the Klein-Nishina values of the scattering per electron, σ_e.*

λ (X.U.)	μ/ρ (cm^2 g^{-1})	PROBABLE ERROR	EXP. VALUE[1] $\mu_e\times10^{25}$	KLEIN-NISHINA VALUE[2] $\sigma_e\times10^{25}$	$\frac{\mu_e-\sigma_e}{\mu_e}\times100$
40	0.105	0.001	3.46	3.447	+0.5
50	0.114	0.001	3.76	3.734	0.7
60	0.120	0.001	3.96	3.969	−0.2
70	0.126	0.002	4.16	4.167	−0.2
80	0.132	0.002	4.36	4.336	+0.5
85	0.134	0.001	4.42	4.405	0.4
90	0.137	0.001	4.52	4.483	0.8
100	0.142	0.001	4.69	4.610	1.6
110	0.147	0.002	4.85	4.723	2.7
115	0.149	0.001	4.92	4.778	3.0
120	0.151	0.002	4.98	4.824	3.2
130	0.154	0.001	5.08	4.915	3.4
140	0.158	0.003	5.21	4.995	4.4
160	0.163	0.002	5.38	5.132	4.8
184	0.171	0.002	5.64	5.280	6.9
209	0.177	0.001	5.84	5.395	8.3

[1] $\mu_e=\mu_A/Z=\mu/\rho\cdot(A/ZN)$: A, atomic weight: Z, atomic number: N, Avogadro's number.

[2] In the calculation of the Klein-Nishina values of σ_e, given in the above table, the following values of the general physical constants were employed:

$e=4.767\times10^{-10}$ e.s.u. $\quad h=6.542\times10^{-27}$ erg·sec.

$m=0.9038\times10^{-27}$ g $\quad c=2.998\times10^{10}$ cm/sec.

Recently Bäcklin and Flemberg (Nature **137**, 655 (1936)) have reported a new value for e (4.800×10^{-10}) which is in good agreement with that utilized by Birge (Phys. Rev. **48**, 918 (1935)) (4.8036) in obtaining a new value of h (6.6286). Using $e=4.800\times10^{-10}$ and $h=6.6286\times10^{-27}$, an evaluation of the Klein-Nishina formula for σ_e gives 3.821 and 4.724($\times10^{-25}$) at 50 and 100 X.U., respectively. These are each some 2 percent greater than the values in Table I.

The absorption coefficients determined for Na, Al, K, Ni, and Cu are given in Table II, and for S are given in Table III. The mass absorption curves are shown in Fig. 2. The values of τ_A were calculated from the absorption Eq. (1) in which were used the values of σ_e given in Table I. The variation of τ_A with λ is shown in Fig. 3, from which $q=2.83$ and 2.6, for copper and aluminum, respectively. However the actual

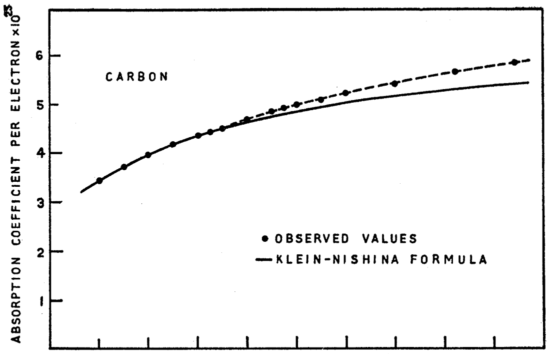

FIG. 1. The experimental values of μ_A/Z for carbon and the theoretical values of σ_e as calculated from the Klein-Nishina formula. The agreement in the region $40\leq\lambda\leq100$ X.U. is within experimental error, which confirms the Klein-Nishina formula for this wave-length region.

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[9] Read and Lauritsen, Phys. Rev. **45**, 433 (1934).

[10] Rutherford, Chadwick and Ellis, *Radioactive Radiations*, p. 473.

TABLE II. *The observed mass absorption coefficients; and the atomic and photoelectric coefficients* ($\times 10^{24}$).[1]

λ (X.U.)	Na(11)			Al(13)			K(19)			Ni(28)			Cu(29)		
	μ/ρ	μ_A	τ_A	μ/ρ	μ_A	τ_A	μ/ρ	μ_A	τ_A	μ/ρ	μ_A	τ_A	μ/ρ	μ_A	τ_A
50				0.113	5.02	0.17	0.115	7.41	0.32	0.126	12.2	1.7	0.129	13.5	2.7
55				0.117	5.20	0.17				0.148	14.3	3.5			
60	0.119	4.52	0.16	0.121	5.38	0.22	0.136	8.76	1.22	0.163	15.8	4.7	0.160	16.7	5.2
65										0.168	16.3	4.9			
70	0.126	4.78	0.20	0.131	5.79	0.37	0.144	9.27	1.35	0.187	18.1	6.4	0.182	19.1	7.0
75				0.135	6.00	0.48				0.195	18.9	7.0			
80	0.134	5.08	0.31	0.138	6.14	0.50	0.162	10.43	2.19	0.218	21.1	9.0	0.215	22.5	9.9
85				0.142	6.31	0.59									
90	0.140	5.30	0.37	0.144	6.40	0.57	0.180	11.59	3.07	0.250	24.2	11.7	0.259	27.2	14.2
95				0.147	6.54	0.64									
100	0.147	5.58	0.51	0.150	6.67	0.68	0.194	12.49	3.73	0.297	28.7	15.8	0.306	32.1	18.7
110	0.154	5.84	0.65	0.158	7.03	0.89	0.202	13.01	4.04	0.349	33.8	20.6	0.371	38.9	25.2
115				0.162	7.20	0.99									
120	0.161	6.11	0.80	0.168	7.47	1.20	0.228	14.68	5.51	0.420	40.6	27.1	0.434	45.5	31.5
125				0.175	7.78	1.46									
130	0.168	6.38	0.97	0.176	7.83	1.44	0.256	16.49	7.15	0.490	47.4	33.6	0.513	53.8	39.5
140	0.174	6.60	1.10	0.191	8.50	2.01	0.297	19.13	9.64	0.594	57.5	43.5	0.606	63.5	49.0
160	0.186	7.06	1.41	0.213	9.47	2.80	0.366	23.57	13.82	0.771	74.6	60.2	0.830	87.0	72.1
184[2]	0.208	7.89	2.09	0.240	10.67	3.81	0.459	29.56	19.53	1.10	106.5	91.7	1.17	122.6	107.3
209[3]	0.231	8.78	2.84	0.279	12.37	5.36	0.606	39.03	28.78	1.56	151.	136.	1.63	170.9	153.3

[1] $\tau_A = \mu_A - Z\sigma_e$. Values of σ_e are given in Table I.
[2] W $K\beta$ radiation.
[3] W $K\alpha_1$ radiation.

values of q are of little significance since the photoelectric absorption is, in the case of aluminum at 100 X.U., only 10 percent of the observed absorption, and consequently contains an unavoidably large experimental error.

The present experimental values of μ/ρ for carbon and aluminum are within one percent of those recently reported by Hahn[11] for $\lambda > 140$ X.U., but the present values differ from Allen's[5] in some cases by as much as 10 percent. It is to be noted, however, that Allen's value for carbon at 209 X.U. is but 2 percent greater than the scattering predicted by the Klein-Nishina formula, at 140 X.U. is about 1/2 percent greater, and at 81 to 90 X.U. is more than 8 percent greater than the predicted scattering. In view of the present measurements it is unlikely that the photoelectric absorption varies with wave-length in this manner.

In Fig. 4 the author's results for aluminum are plotted with the results of other investigators.[9, 11, 12] From the τ_A curve for aluminum, Fig. 3, it appears that at 50 X.U. the value of τ_A is greater than 0.12×10^{-24}. Assuming this minimum value, we have $\mu_A = 4.98\times10^{-24}$, which is approximately 2 percent greater than the

TABLE III. *The absorption coefficients determined for sulfur.*

λ (X.U.)	μ/ρ	$\mu_A\times10^{24}$	$\tau_A\times10^{24}$
75	0.142	7.51	0.72
100	0.172	9.09	1.71
120	0.200	10.57	2.85
130	0.219	11.58	3.72

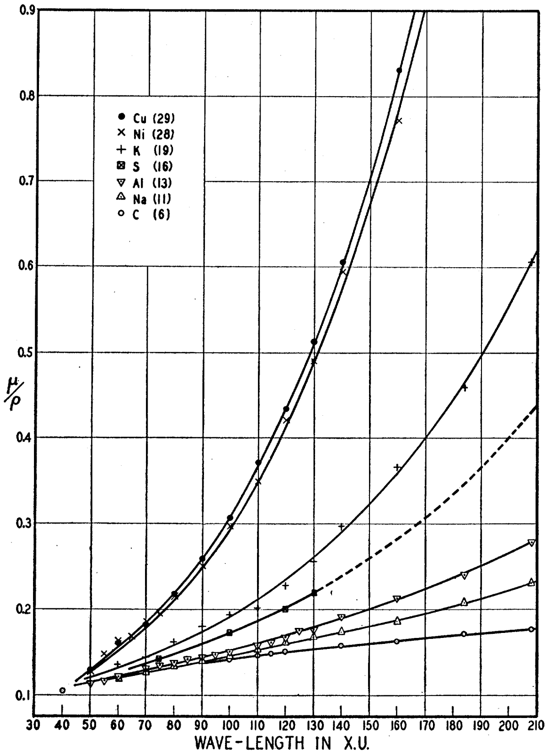

FIG. 2. Mass absorption curves for the elements studied.

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[11] Hahn, Phys. Rev. **46**, 149 (1934).
[12] Chao, Phys. Rev. **36**, 1519 (1930).

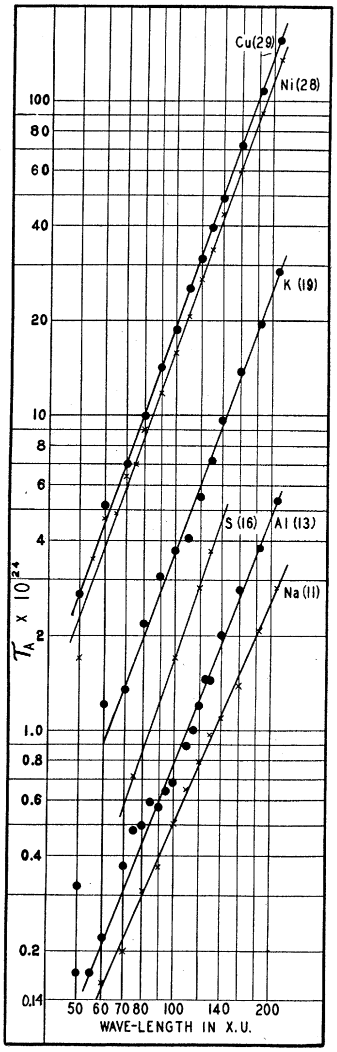

FIG. 3. The variation of the photoelectric absorption with wave-length.

value reported by Read and Lauritsen.[9] At the same wave-length their value for carbon is about 3 percent less than that predicted by the Klein-Nishina formula. These discrepancies may be due to the fact that Read and Lauritsen employed a much smaller resolving power than was used by the author.

Discrepancies exist between the previously reported results[3, 4, 5, 11] for nickel and copper and the present measurements in the region $140 < \lambda < 209$ X.U. Three independent determinations of μ/ρ for copper at 140 X.U. gave values within 2 percent of each other, but these values are 8 percent less than the value reported by Hahn.[11]

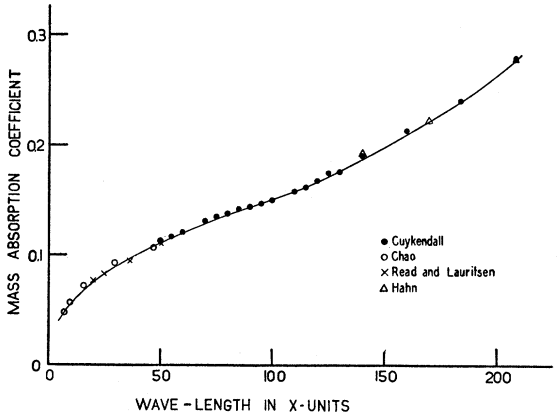

FIG. 4. The present results for aluminum plotted with those of Read and Lauritsen,[9] Hahn[11] and Chao[12] show the natural extension of the absorption measurements to the gamma-ray region. The form of the curve is explained as follows: At wave-lengths less than 75 X.U. the absorption is practically all scattering; at longer wave-lengths the photoelectric absorption becomes important, increasing to about 60 percent of the total absorption at 200 X.U.

Read[13] has measured the absorption in copper in the region 20 to 47 X.U.: a smooth curve through his results gives at 50 X.U. a value within 2 percent of the present result.

The probable errors given in Table I were calculated in the usual manner from the variations of the individual readings obtained during the determination of the mass absorption coefficient. After an interval of more than two months many of the values of μ/ρ were redetermined showing good agreement with the earlier results. Considering all the sources of error, the writer estimates that the values of μ/ρ given in Tables I, II, and III are in error by no more than 2 percent.

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[13] Read, Proc. Roy. Soc. **A152**, 402 (1935).