TOTAL RADIATION FROM METALS.

By Vernon A. Suydam.

FROM a consideration of the experimental results thus far obtained the statement is justified that with many substances total radiation may be expressed with fair accuracy by a simple relation of the form

$$E = c'T^n. \tag{1}$$

In the case of some metals and alloys, however, this equation does not express the radiation accurately owing to irregularities in emission. In those cases where an equation of this form does hold n is not equal to 4, as Stefan supposed, except perhaps in exceptional cases. The character of the surface of the body and its internal structure are factors which affect radiation, and, as these vary with the temperature, it is to be expected that the emission cannot always be expressed so simply. From an application of Kirchhoff's law, which may be written

$$\frac{f(T)}{cT^4} = a(T), \tag{2}$$

coupled with the fact that $a(T)$ is a function of the temperature in most cases, it is easily seen that $f(T)$ is not equal to $c'T^4$. $f(T)$ may be of the form $c'T^n$, but n cannot be equal to 4 unless $a(T)$ is constant. The work of E. Hagen and H. Rubens shows $a(T)$ in the case of metals to be a function of the temperature for infra-red radiation, as it is made to depend upon the electrical resistance.[1] The coefficient of electrical resistance for metals is positive when the metal is in the solid state, and hence $a(T)$ increases with T, approaching unity as a limit. Hence metals approach black-body radiation with increase in temperature. The present investigation shows that glass, brass and lamp-black are better absorbers at high than at low temperatures. In the case of all the metals tested n was found to be greater than 4. This result agrees with the observations of F. Paschen and also E. Aschkinass.[2]

Method of Observation.

The metal to be tested was in the form of a wire which was stretched coaxially in a cylindrical enclosure, a light spring being used to keep it

[1] Phil. Mag., 7, p. 157, 1904. Preuss, Akad. Wiss. Berlin Sitz. Ber., 23, p. 467, 1910.

[2] Drude Ann., 17, p. 960, 1905.

taut when heated. Fine platinum potential-wires were fused to the test-wire at points well removed from its points of entrance to the enclosure. In the case of silver the potential wires were silver soldered. The watt-input was calculated from the fall in potential between the points p_1p_2, Fig. 1 (measured with a low-resistance Leeds and Northrup potentiometer), and the current flowing. The current was calculated

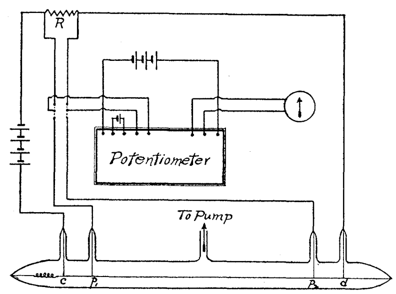

Fig. 1.

from the fall in potential over a known standard resistance, R, placed in series with the test-wire. The arrangement of the apparatus is indicated in Fig. 1.

The temperature of the wire, for any given steady current, was determined from its resistance, which was calculated from the known current and fall in potential between the points p_1p_2. In order that the resistance

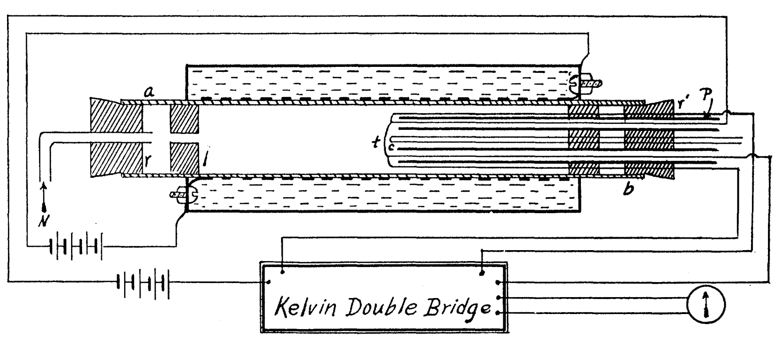

Fig. 2.

of the wire might serve as a measure of its temperature it was necessary to obtain data for a temperature-resistance curve, from which the temperature corresponding to any given resistance, could be determined. The arrangement of the apparatus for obtaining such data is indicated in Fig. 2.

A long quartz tube, *ab*, wound with nichrome ribbon and packed in sand served as a heating furnace. The nichrome was protected from oxidation by being covered with a layer of alundum cement. With this arrangement the furnace could be heated to 1500° K. repeatedly. The ends of the quartz tube were sealed with rubber corks, *rr'*. To protect the rubber corks from the heat of the furnace lavite plugs, *l*, were inserted at a short distance from the corks. This gave sufficient protection for the temperature attained. Through the cork *r'* and the corresponding lavite plug were passed two porcelain tubes, *P*, and a Pt *vs.* Pt + 10 per cent. Rh thermocouple. This thermocouple was calibrated by comparing it with a standard thermocouple which had been calibrated at the Bureau of Standards. The wire, *t*, whose resistance was to be measured was placed in the center of the furnace with the hot junction, *c*, of the thermocouple close against it. The current terminals entered the furnace through the porcelain tubes, the potential terminals just outside a Leeds and Northrup Kelvin-double-bridge being used to measure the resistance. The wire was protected from oxidation by passing nitrogen through the furnace. The nitrogen was obtained after the method of G. A. Hulett.[1] This method consists in heating copper and copper oxide held in a hard-glass tube, and passing air and hydrogen through the tube. The hydrogen combines with the oxygen of the air and the emergent gas is pure nitrogen.

The metals tested were silver, platinum, nickel and iron, and the alloy nichrome. When platinum was tested the enclosure was made of brass blackened within with lamp-black. For all the other samples a glass enclosure was used. In all cases the surface of the test-wire was carefully cleaned and polished. Before taking a set of observations the wire was heated, by sending a strong current through it, in order to drive off accluded gases.

When the test-wire was platinum the temperature of the enclosure was first held at 90° K., by immersing it in liquid air, then at 273° K., by packing it in melting ice, and then at 373° K., by immersing it in boiling water. In other cases only the temperatures 273° K. and 373° K. were used as fixed temperatures. It was found that the temperature of the enclosure had an influence upon the watt-input. In every case the energy-temperature curve obtained by holding the enclosure at a low temperature was steeper than when it was held at a higher temperature, indicating that the reflecting power of the enclosure increased with decrease in temperature, or $a(T)$ increased with increase in temperature. A. Schleiermacher[2] in his report on radiation from platinum (his method of experimentation was the same as the one here used) observes that the

[1] Jour. of Am. Chem. Soc., 27, p. 1415.

[2] Wied. Ann., 26, p. 287, 1885.

curves meet at a temperature somewhat above that of the highest temperature of the enclosure and states that they coincide above this temperature. The author found in his work that the curves cross but do not coincide at any temperature above the crossing point although they do not separate widely. Data are given here for only two cases, platinum and silver, where the temperature of the enclosure was other than 273° K., as it was found that the influence of the enclosure was negligible for temperatures of the test-wire large in comparison with that of the enclosure.

DISCUSSION.

The watt-input, which is measured, is the difference between the true emission from the metal when at a given temperature and the energy which it receives from the enclosure. This latter is made up of two parts: energy emitted by the enclosure and reflected from it. This action may be represented by the following functional equation:

$$f(T) = \omega(T) + a(T)\{r(T, T_e) + e(T_e)\}, \tag{3}$$

where $f(T)$ is the total radiation from the metal, $\omega(T)$ the watt-input, $a(T)$ the absorption coefficient, $r(T, T_e)$ the reflected energy, which depends upon the temperature, T, of the test-wire and the temperature T_e, of the enclosure, $e(T_e)$ the total radiation from the enclosure, constant for any given set of observations.

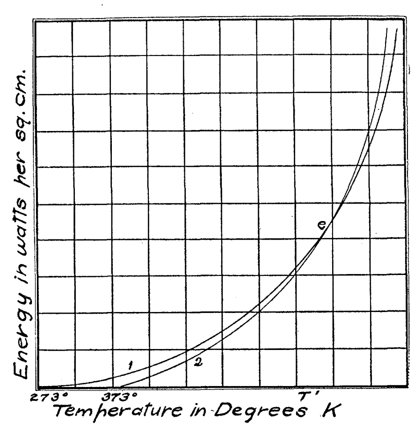

Fig. 3.

Referring to the accompanying diagram, Fig. 3, which is a representation of the curves obtained in this investigation, it is seen that at the point e, where the curves cross at temperature T'

$$\omega_1(T') = \omega_2(T'), \tag{4}$$

where the subscripts refer to the curves obtained when the enclosure was at 273° K. and 373° K. respectively. From equations (3) and (4) we obtain

$$r(T', T_{e_1}) - r(T', T_{e_2}) = e(T_{e_2}) - e(T_{e_1}). \quad (5)$$

The right-hand member of this equation is constant and depends only upon the substance and temperature of the enclosure. Since

$$e(T_{e_2}) > e(T_{e_1})$$

it follows that

$$r(T', T_{e_1}) > r(T', T_{e_2}).$$

That is, the quantity of energy reflected per second per unit area from the enclosure is greater when it is held at 273° K. than when held at 373° K. Thus in the case of all the enclosures used $a(T)$ increased with increase in temperature over the temperature range used in this investigation.

If there were no energy reflected from the enclosure the slope of the energy-temperature curve would be greater at every point with the

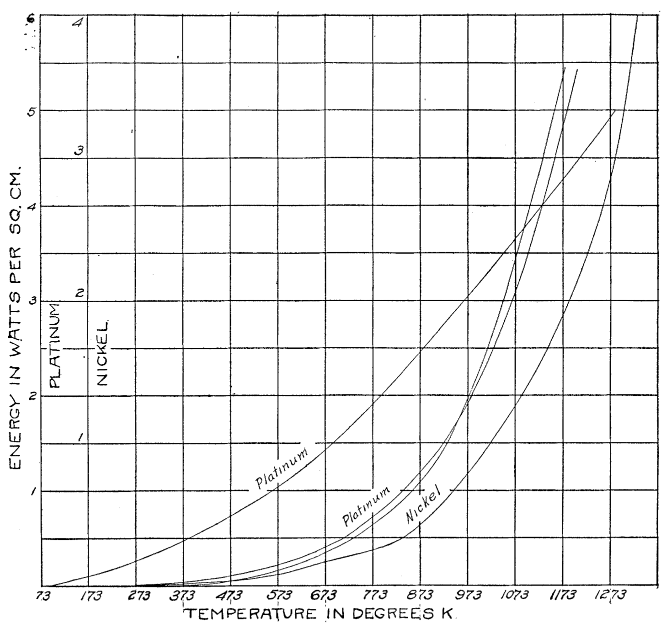

Fig. 4.

Energy-Temperature Curves.

temperature of the enclosure at 273° K. than at 373° K., because less energy would be sent to the test-wire from the enclosure. Since the reverse is the case, it follows that the enclosure is a better reflector at a low than at a high temperature, and hence that the reflected energy plays a larger part in maintaining the temperature of the test-wire than does the emitted energy. This accounts for the crossing of the curves. That the curves do not separate more widely above the point of crossing seems to indicate a falling off of the percentage of reflected energy for higher temperatures of the test-wire. This may be accounted for from the fact that the energy emitted from the test-wire contains an ever increasing amount in short waves, and as the short waves are more readily absorbed and transmitted there will be a decrease in the percentage of reflected energy.

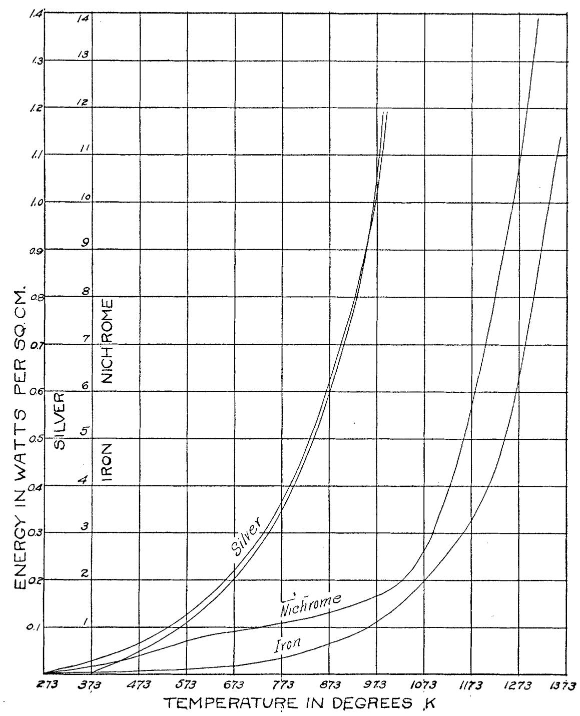

Fig. 5.

Energy-Temperature Curves.

Fig. 4 shows the energy-temperature curves obtained by holding the temperature of the enclosure at 90° K., 273° K. and 373° K. The test-wire was platinum and the enclosure was brass blackened within with lamp-black. It will be noticed that the slope of the curve in each case is greater when the temperature of the enclosure was maintained at a high temperature than when at a lower temperature. It requires less energy to maintain the temperature of the test-wire at, say, 1,133° K., with the enclosure at 90° K. than with the enclosure at 373° K. In the former case the energy required was 4.02 watts per sq. cm. and in the latter 4.51 watts per sq. cm.

From equations (2) and (3) we obtain

$$\omega(T) = f(T) \left\{ 1 - \frac{r(T, T_e) + e(T_e)}{cT^4} \right\}. \tag{7}$$

This equation shows that when $T = Te$

$$cT^4 = r(T, T_e) + e(T_e),$$

since $\omega(T) = 0$ and $f(T) \neq 0$. This is a statement for black-body radiation in an enclosure in thermodynamic equilibrium. In the development of this equation no account has been taken of extraneous radiation that might enter the enclosure from without, and it has been

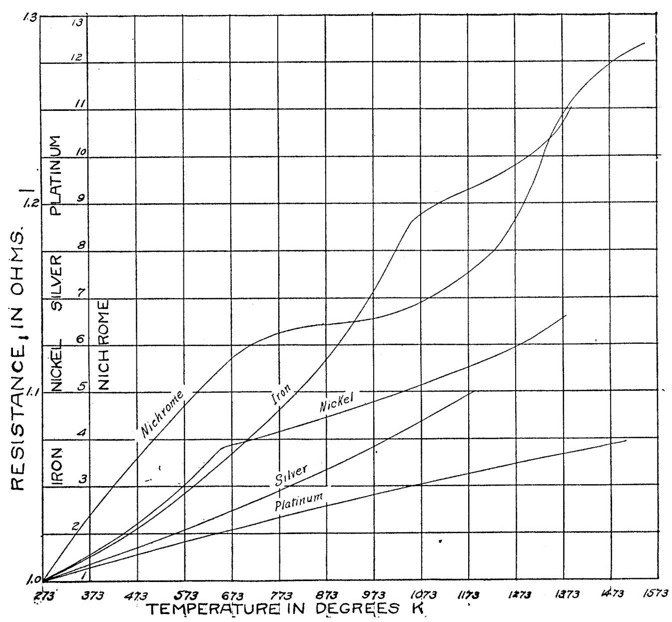

Fig. 6.

Resistance-Temperature Curves; reduced to one ohm at 273° K.

assumed that the enclosure reflects and emits energy as though equal in area to that of the test-wire.

In order to calculate approximately the relative magnitudes of $\omega(T)$ and $f(T)$ in equation (7) it is necessary to evaluate the terms in brackets. L. Graetz[1] found that glass conforms very nearly to black-body radiation for temperatures lying between 273° K. and 456° K., and gives as the average value for c

[1] Wied. Ann., 11, p. 913, 1880.

$$4.52 \times 10^{-12} \frac{\text{watt}}{\text{cm.}^2 \text{ sec. temp.}^4}.$$

For a black-body c is

$$5.65 \times 10^{-12} \frac{\text{watt}}{\text{cm.}^2 \text{ sec. temp.}^4}.$$

Using these values we obtain, by substituting in the last term of the right-hand member of equation (7),

$$\frac{e(T_e)}{cT^4} = \frac{4.52 \times 10^{-12} \times 273^4}{5.65 \times 10^{-12} \times 950^4} = 0.00545,$$

the temperature 950° K. being taken from Table II. Applying Kirchhoff's law to this case we find

$$\frac{e(T)}{cT^4} = \frac{4.52 \times 10^{-12} T^4}{5.65 \times 10^{-12} T^4} = 0.80 = a(T).$$

This gives 20 per cent. for the reflecting power of glass in the temperature interval here used. As a first approximation we may take $\omega(T)$ as equal to $f(T)$. Referring to Table II., the watt-input for silver at

TABLE I.

Platinum.

$T_e = 90°$ K.		$T_e = 273°$ K.		$T_e = 373°$ K.	
Temp. in Deg. K.	Emission in Watts per Sq. Cm.	Temp. in Deg. K.	Emission in Watts per Sq. Cm.	Temp. in Deg. K.	Emission in Watts per Sq. Cm.
115	0.0291	541	0.1540	534	0.1271
126	0.0472	596	0.2323	572	0.1721
144	0.0734	647	0.3293	642	0.2586
201	0.1682	698	0.4562	699	0.3954
283	0.2650	772	0.7104	725	0.4886
351	0.4500	834	0.9776	802	0.7233
487	0.7557	879	1.2477	870	1.0522
567	1.0000	914	1.4726	917	1.3845
632	1.2527	981	2.0338	944	1.6741
723	1.6490	991	2.1597	979	2.0582
786	1.9905	1,018	2.4129	1,007	2.3975
863	2.4250	1,046	2.7412	1,044	2.8930
914	2.7746	1,091	3.3427	1,048	2.9482
1,134	4.0079	1,118	3.7820	1,100	3.7880
1,166	4.2512	1,148	4.2532	1,135	4.4696

$T = 950°$ $K.$ is found to be 0.4903 watt per sq. cm. 20 per cent. of this is 0.098, which is the amount returned to the wire, $r(T, Te)$. Substituting this value in the term next to the last in the right-hand member of equation (7) we find

$$\frac{r(T,\ T_e)}{cT^4} = \frac{0.098}{5.65 \times 10^{-12} \times 950^4} = 0.0218.$$

The substitution of these values in equation (7) gives a first approximate value for $f(T)$. Thus

$$\omega(T) = 0.973\,f(T).$$

Other errors for which no corrections have been made would change this relation slightly. There is a small loss of energy, due to conduction along the potential wires and in the residual gas and volitalization of the metal, which tends to make $\omega(T)$ too large.

To a close approximation, therefore, $\omega(T)$ may be taken as equal to $f(T)$ for values of T large in comparison with Te. Thus for high temperatures the watt-input, $\omega(T)$, is a function of the same form and of the same order of magnitude as $f(T)$. A. Schleiermacher concludes from the fact that the curves nearly coincide for large values of T that the reflection and radiation from the walls of the enclosure are relatively unimportant.

Results.

Silver.—The diameter of the wire used was 0.0128 cm., and the length between potential points was 44 cm. Silver was found to conform to

Table II.

Silver.			Silver.		
$T_e = 273°$ K. Average Pressure = 0.0002 Mm.			$T_e = 273°$ K. Average Pressure = 0.0002 Mm.		
Temp. in Deg. K.	Emission in Watts per Sq. Cm.	c'.	Temp. in Deg. K.	Emission in Watts per Sq. Cm.	c'.
610	0.0859	3.26×10^{-13}	625	0.0875	3.01×10^{-13}
686	0.1313	3.09×10^{-13}	697	0.1306	2.88×10^{-13}
738	0.1735	3.02×10^{-13}	749	0.1741	2.99×10^{-13}
805	0.2435	2.97×10^{-13}	805	0.2595	3.16×10^{-13}
885	0.3693	3.05×10^{-13}	869	0.3279	2.92×10^{-13}
950	0.4903	3.03×10^{-13}	901	0.3927	3.01×10^{-13}
			945	0.4739	3.00×10^{-13}
			981	0.5691	3.08×10^{-13}

the fourth-power law more nearly than any of the other metals tested. Its emission may be represented very accurately by the equation

$$E = c'T^{4\cdot 1},$$

where c has the mean value of 2.91×10^{-13} when the temperature of the enclosure was 373° K., and 3.07×10^{-13} when it was 273° K. The enclosure was glass. Only one sample of sliver was tested, but many sets of observations on several wires cut from the same coil gave very consistent results.

TABLE III.

Platinum. $T_e = 273°$ K. Av. Pressure = 0.0014 Mm.			Platinum. $T_e = 373°$ K. Av. Pressure = 0.003 Mm.		
Temp. in Degrees K.	Emission in Watts per Sq. Cm.	c'.	Temp. in Degrees K.	Emission in Watts per Sq. Cm.	c'.
647	0.3293	2.90×10^{-15}	642	0.2586	2.43×10^{-15}
698	0.4562	2.75×10^{-15}	699	0.3954	2.37×10^{-15}
772	0.7104	2.59×10^{-15}	725	0.4886	2.44×10^{-15}
834	0.9776	2.42×10^{-15}	802	0.7233	2.18×10^{-15}
879	1.2477	2.38×10^{-15}	870	1.0522	2.11×10^{-15}
914	1.4726	2.31×10^{-15}	917	1.3845	2.14×10^{-15}
981	2.0338	2.24×10^{-15}	944	1.6741	2.23×10^{-15}
991	2.1597	2.26×10^{-15}	979	2.0582	2.29×10^{-15}
1,018	2.4129	2.21×10^{-15}	1,007	2.3975	2.31×10^{-15}
1,046	2.7412	2.19×10^{-15}	1,014	2.8930	2.33×10^{-15}
1,091	3.3427	2.16×10^{-15}	1,048	2.9482	2.33×10^{-15}
1,118	3.7820	2.16×10^{-15}	1,100	3.7880	2.35×10^{-15}
1,148	4.2532	2.13×10^{-15}	1,135	4.4696	2.37×10^{-15}

TABLE IV.

Nichrome. $T_e = 273°$ K. Av. Pressure = 0.0002 Mm.			Nickel. $T_e = 273°$ K. Av. Pressure = 0.0002 Mm.		
Temp. in Degrees K.	Emission in Watts per Sq. Cm.	n.	Temp. in Degrees K.	Emission in Watts per Sq. Cm.	n.
325	0.0417	4.13	463	0.0258	4.65
392	0.1168	4.17	518	0.0462	4.66
483	0.3259	4.19	603	0.1000	4.67
540	0.5973	4.18	643	0.1343	4.70
673	0.9080	4.14	661	0.1526	4.67
823	1.2349	4.06	711	0.1935	4.68
1,013	1.8913	4.00	773	0.2408	4.62
1,073	2.6900	4.02	803	0.2808	4.62
1,113	3.6500	4.05	883	0.4498	4.63
1,163	5.3282	4.07	923	0.5852	4.63
1,178	5.9170	4.07	951	0.7012	4.64
1,228	8.2590	4.10	1,028	1.0484	4.64
1,258	9.8000	4.11	1,071	1.2553	4.64
1,283	11.5910	4.12	1,123	1.5499	4.64
1,308	13.3820	4.13	1,181	1.9324	4.64
			1,283	2.9572	4.65

Platinum.—Two specimens of platinum were tested with good agreement in results. Many sets of observations were taken on each specimen. The temperature-resistance curve for the specimen contained in this report was plotted from the equation

$$\rho_t = \rho_0(1 + \alpha t + \beta t^2),$$

where $\rho_0 = 1$ ohm, $\alpha = 0.0028$, $\beta = -0.00000033$. The enclosure was brass coated within with lamp-black. The diameter of the wire was 0.0159 cm. and its length between potential points was 40.3 cm. The emission was found to conform to the equation

$$E = c'T^5,$$

Table V.

Iron.		
$T_e = 273°$ K. Av. Pressure = 0.0002 Mm.		
Temp. in Degrees K.	Emission in Watts per Sq. Cm.	c'.
696	0.2230	3.54×10^{-17}
753	0.3103	3.35×10^{-17}
805	0.4362	3.25×10^{-17}
853	0.5661	3.06×10^{-17}
915	0.7952	2.89×10^{-17}
975	1.1604	2.99×10^{-17}
1,083	2.1810	3.13×10^{-17}
1,133	2.7135	3.04×10^{-17}
1,180	3.3628	3.00×10^{-17}
1,192	3.5813	3.02×10^{-17}
1,233	4.7000	3.29×10^{-17}
1,240	4.9420	3.35×10^{-17}
1,303	7.8800	3.99×10^{-17}

Table VI.

Platinum.		Silver.		Nichrome.	
Temp. in Degrees K.	Resistance in Ohms.	Temp. in Degrees K.	Resistance in Ohms.	Temp. in Degrees K.	Resistance in Ohms.
273	1.000	273	1.000	273	1.000
373	1.280	323	1.145	317	1.020
473	1.553	373	1.350	339	1.033
573	1.820	423	1.510	398	1.045
673	2.079	477	1.720	448	1.058
773	2.332	523	1.889	503	1.073
873	2.579	574	2.070	556	1.088
973	2.816	623	2.275	606	1.103
1,073	3.049	683	2.520	668	1.118
1,173	3.275	723	2.685	735	1.128
1,273	3.494	792	3.000	867	1.133
1,373	3.706	823	3.105	945	1.138
1,473	3.911	854	3.240	1,018	1.143
1,573	4.109	913	3.545	1,058	1.149
1,673	4.300	953	3.730	1,103	1.154
		1,073	4.355	1,188	1.169
				1,238	1.179
				1,285	1.199
				1,320	1.219
				1,352	1.239
				1,375	1.249
				1,503	1.279

where c has the mean value of 2.36×10^{-15} when the enclosure was at 273° K. and 2.29×10^{-15} when it was at 373° K.

Nickel.—The diameter of the wire for which data are here given was 0.0455 cm. and its length between potential points was 29.8 cm. The energy-temperature cuvre suffers a slight variation in the neighborhood of the temperature at which recalescence occurs. In this neighborhood the emission increases less rapidly with rising temperature than either above or below. The variation in emitting power occurs at about the same temperature as that at which there is a pronounced variation in the coefficient of electrical resistance but does not persist over as great a range of temperature. The equation $E = c'T^n$ does not hold for nickel as well as for platinum. By holding c' constant — $c' = 1.04 \times 10^{-14}$— the variations in n are exhibited. The data for nickel are given in Table IV. n has the average value 4.648. Since nickel showed a

TABLE VII.

Iron.		Nickel.	
Temperature in Degrees K.	Resistance in Ohms.	Temperature in Degrees K.	Resistance in Ohms.
273	1.000	273	1.000
308	1.202	348	1.400
376	1.568	448	1.970
411	1.777	512	2.649
481	2.196	555	2.868
511	2.406	591	3.200
588	2.945	625	3.542
611	3.138	651	3.808
669	3.644	704	3.945
694	3.870	730	4.028
763	4.541	766	4.138
813	5.037	790	4.243
851	5.491	824	4.359
904	6.020	900	4.621
948	6.799	980	4.801
983	7.322	1,009	4.966
1,028	8.211	1,070	5.132
1,093	8.891	1,113	5.298
1,177	9.361	1,153	5.463
1,231	9.623	1,193	5.610
1,271	9.806	1,273	5.920
1,290	9.937		
1,328	10.198		
1,363	10.632		

variation from the Stefan equation at about the temperature 650° K. several different samples were tested with many sets of observations. The results were uniformly the same. With any given sample the curves could be reproduced with great accuracy.

Iron.—The diameter of the wire for which data are here given was 0.0476 cm., and its length between potential points was 29.9 cm. There is a pronounced change in the coefficient of electrical resistance in iron at about the temperature 1050° K. However, contrary to expectation, no variation in its emitting power was detected at this temperature. The energy-temperature curve was regular and conformed to Stefan's equation very well. This point was examined carefully. Different samples of iron were tested with great care but none of them showed any variation in emitting power at this temperature. c' has the average value 3.23×10^{-17} when n is taken equal to 5.55.

Nichrome.—The diameter of the wire was 0.046 cm. and its length between potential points was 24.6 cm. The temperature-resistance curve for nichrome suffers a pronounced change in the temperature interval 743° K. to 1043° K. In this same temperature range the energy-temperature curve varies. The emission from nichrome does not conform well to the equation $E = c'T^n$. The variations from this equation are shown by holding c' constant—$c' = 1.76 \times 10^{-12}$—and calculating n, Table IV. The samples of nichrome which were tested were all cut from the same coil. Many sets of observations were taken with uniformly consistent results.

This investigation was undertaken at the suggestion of Professor Augustus Trowbridge. The author takes this occasion to express his appreciation for the interest taken in this work by Professor Trowbridge and for the many helpful suggestions which were offered by him during its progress.

Palmer Physical Laboratory,
Princeton University,
June, 1914.