THE CATHODE FALL IN GASES.

By C. A. Skinner.

THIS article is the result of an attempt to develop a theory of the so-called cathode fall in gases, at least to an extent sufficient to suggest intelligent experimental investigation. From the nature of the problem the theory as developed is rather crude. Since, however, it has led to the discovery of certain interesting and quite general relations between the magnitudes involved, which corroborate in part the theoretical deductions, its presentation seemed a logical introduction to these experimental results and necessary to their interpretation.

The theory is based on the generally accepted principles governing ionization by impact and the mobility of the ions, on the one hand, combined on the other with the suggestion made by the writer that the accumulation of the positive ions at the cathode face arises from an elastic rebound of these ions when they impinge on the cathode.[1]

For the sake of clearness we will give first a brief summary of the principal features connected with this cathode fall. Take for example a cylindrical glass tube containing gas at a pressure such that a glow current can be maintained between two disk electrodes placed one in each end of the tube and perpendicular to its axis. For simplicity let the current be confined to the front face of the cathode.

Potential measurements show an extremely low gradient in the negative glow as compared with that at any other portion of the conducting gas. Passing toward the cathode from this region of minimum gradient, one finds that the gradient begins to increase relatively rapidly at a certain point and climbs gradually higher as the cathode is approached.[2]

The negative glow is marked usually by its hazy blue color. The point where the abrupt rise in the gradient begins, is, especially at the higher gas pressures or higher current densities, very distinctly marked as the outer boundary of a bright luminous striation (much whiter than the rest of the negative glow). Towards the cathode this bright striation shades off rapidly to darkness, in the so-called cathode dark space, un-

[1] C. A. Skinner, Phil. Mag., VI., Vol. 4, p. 490, 1902.

[2] This characterization does not include the cases at very low gas pressure in which Graham (Ann. d. Physik, 64, p. 49) observed the gradient to climb in a sinuous manner in passing from the negative glow to the cathode.

accompanied by any correspondingly marked change in the gradient except its gradually increasing magnitude. On the face of the cathode is again a very bright and relatively thin striation.

The potential difference between the cathode and the very low gradient region of the negative glow is known as the "cathode fall." This cathode fall is remarkable in that its magnitude is the same over a wide range of gas pressures, provided the conditions admit of an automatic adjustment of the cathode current density to its so-called normal value.[1] The cathode fall thus obtained is known as the "normal cathode fall."

The potential difference between the cathode and a very fine wire placed only a fraction of a millimeter from it, is, in hydrogen, one half or more of the cathode fall; in nitrogen, it may be a large part of the fall.

If, by means of auxiliary electrodes connected with a constant potential source, one sends a current across the main stream, this cross-current is found to be distinctly larger in the negative glow than at any other part. It decreases especially rapidly as one moves from the negative glow to the cathode. This fact, together with the results of various other experiments which point to the same conclusion, has led to the generally accepted view that the negative glow is very largely the source of the ions which carry the current between that point and the cathode. Thus we have on the cathode side of the negative glow the current carried very largely by the less mobile positive ions moving to the cathode, while to the anode side we have the more mobile electrons. Consequently, on the cathode side we have a greater density of charge, hence a more rapidly increasing potential gradient than on the anode side.

The relatively large potential difference between the cathode and the gas immediately adjacent necessitates the assumption of a greatly decreased mobility of the ions at this point. Many investigators explain this decreased mobility by assuming the existence of a high resistance film on the face of the cathode.

We are interested here however in deducing the results which should arise if the impinging positive ions rebound from the cathode with a definite fraction of their incident energy and only communicate their charge to the cathode (or receive a neutralizing charge from it) when they finally come to rest at its surface. A still more acceptable view would allow for a part of the kinetic energy of the rebounding ion being absorbed by friction in the gas, but the theoretical application of this view appears impossible.

[1] At low gas pressures the minimum cathode fall rises distinctly above this "normal" value, as repeatedly observed during the present investigation. The phenomenon of an automatic adjustment of current density also disappears.

Theory.

Fall of Potential in the Region between the Negative Glow and Apparent Cathode Film.—Consider a plane cathode perpendicular to the x-axis having a discharge area sufficiently large so that the equipotential surfaces between it and the negative glow are plane and parallel—also the current density uniform.

Throughout that region in which the fall of potential per mean free path, $\bar{\lambda}$, of the electron, is greater than the ionizing potential, we should have, on the average, each electron which leaves the cathode resulting in two electrons at the end of the first mean free path; four at the end of the second; eight, at the third; and so on. With a current density of electrons leaving the cathode equal to $\bar{j}_k$, there should be under a stationary current, at a distance from the cathode $(x_k - x)$, a current density of electrons

$$\bar{j} = \bar{j}_k \cdot 2^{\left(\frac{x_k - x}{\bar{\lambda}}\right)} \qquad (1)$$

provided the potential gradient is large enough to cause ionization to take place at each mean free path; and provided, as we may safely assume with such high field strength, the disappearance of electrons by recombination is negligible.[1] Beyond this region of ionization at every mean free path we are under the necessity of assuming another law of ionization. For reasons which will become evident we have assumed the upper limit, namely: that, in the remainder of the path to the point of minimum gradient in the negative glow, ionization occurs on the average whenever an electron traverses a distance over which the fall of potential is equal to the ionizing potential. This is the maximum ionization which could occur under the most favorable conditions of perfectly elastic impact and no recombination.

Between the position x_a where the electrons cease to ionize at each impact and that of the minimum gradient in the negative glow x_0, the number of electrons doubles on moving through the ionizing potential ϵ. Then, since from (1) the negative current density at x_a is

$$\bar{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\bar{\lambda}}\right)}$$

that at any potential V between x_a and x_0 is

$$\bar{j} = \bar{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\bar{\lambda}} + \frac{V - V_a}{\epsilon}\right)} \qquad (2)$$

[1] From our measurements of the gradient in hydrogen and Franck and Hertz's value (Verh. d. D. Phys. Ges., *15*, p. 34, 1913) of 11 volts for its ionizing potential, this region of ionization at every mean free path extends nearly to the outer boundary of the cathode dark space.

and the positive current density at the same point necessarily

$$\overset{+}{j} = j - \overset{-}{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\lambda} + \frac{V - V_a}{\epsilon}\right)} \tag{3}$$

where j, the total current density, is the same at all parts of the path. We shall use these two equations for obtaining the desired relation between potential and current density, but first it is necessary to obtain $\overset{-}{j}$, $\overset{+}{j}$ and $\overset{-}{j}_k$ in terms of j.

At the position x_0, having potential V_0, (2) gives

$$\overset{-}{j}_0 = \overset{-}{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\lambda} + \frac{V_0 - V_a}{\epsilon}\right)}. \tag{4}$$

Poisson's equation,

$$\frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2} = -4\pi\rho,$$

(ρ being the volume density of the charge at the point considered)—applied to the present case, where the equipotential surfaces are plane and parallel, gives

$$\frac{d^2 V}{dx^2} = -4\pi\rho. \tag{5}$$

At x_0, dV/dx being a minimum, we have from this equation

$$\rho_0 = 0.$$

That is

$$\overset{+}{\rho}_0 + \overset{-}{\rho}_0 = 0. \tag{6}$$

But

$$\overset{+}{\rho}_0 = -\frac{\overset{+}{j}_0}{\overset{+}{\mu}\left(\frac{dV}{dx}\right)_0} \quad \text{and} \quad \overset{-}{\rho}_0 = \frac{\overset{-}{j}_0}{\overset{-}{\mu}\left(\frac{dV}{dx}\right)_0}, \tag{7}$$

where $\overset{+}{\mu}$ and $\overset{-}{\mu}$ are the mobilities (velocity in unit field) of the positive and negative ions respectively, hence $-\overset{+}{\mu}(dV/dx)_0$ and $\overset{-}{\mu}(dV/dx)_0$ respectively their velocities at x_0.[1] Substituting from (7) in (6) and applying also the necessary condition that

$$\overset{+}{j}_0 + \overset{-}{j}_0 = j$$

we obtain

$$\overset{-}{j}_0 = \frac{\overset{-}{\mu}}{\overset{+}{\mu} + \overset{-}{\mu}} j. \tag{8}$$

Using this last equation to obtain the value of $\overset{-}{j}_k$ from (4) and then substituting that value in (2) and (3)

[1] The signs as used here enable both $\overset{-}{j}_0$ and $\overset{-}{\mu}$ to be treated as positive quantities, $(dV/dx)_0$ being negative.

$$\bar{j} = \frac{\bar{\mu} + \overset{+}{\mu}}{\bar{\mu}} \cdot j \cdot 2^{-((V_0 - V)/\epsilon)},$$
$$\overset{+}{j} = j\left\{ 1 - \frac{\bar{\mu}}{\bar{\mu} + \overset{+}{\mu}} 2^{-((V_0 - V)/\epsilon)} \right\}. \tag{9}$$

We now proceed to obtain the desired solution. At the position x, having a potential V, equation (5) gives

$$\frac{d^2 V}{dx^2} = -4\pi\rho$$
$$= -4\pi(\overset{+}{\rho} + \bar{\rho})$$
$$= 4\pi\left(\frac{\overset{+}{j}}{\overset{+}{\mu}\dfrac{dV}{dx}} - \frac{\bar{j}}{\bar{\mu}\dfrac{dV}{dx}} \right). \tag{10}$$

Substituting now in 10 the values of $\bar{j}$ and $\overset{+}{j}$ as given in 9 we obtain the differential equation for the region comprised between the point x_0 of minimum gradient and x_a where the ions cease to ionize at every mean free path:

$$\frac{dV}{dx}\frac{d^2 V}{dx^2} = \frac{4\pi j}{\overset{+}{\mu}}\left\{ 1 - 2^{-((V_0 - V)/\epsilon)} \right\}. \tag{11}$$

Integrating this

$$\frac{dV}{dx} = -\left\{ \frac{12\pi j}{\overset{+}{\mu}}[(V_0 - V) - \epsilon(1 - 2^{-((V_0 - V)/\epsilon)})] \right\}^{1/3}, \tag{12}$$

where the condition is introduced that, as shown by the experiment, the gradient dV/dx is practically zero at the position x_0, in the negative glow. A general solution of this equation for V is impossible. Since however the exponential term diminishes rapidly with increasing values of $[(V_0 - V)/\epsilon]$ one may find by trial a value of $(V_0 - V) = q$ such that, if the upper limit of $(V_0 - V)$ be sufficiently high,

$$\int_{V_0}^{V} \frac{dV}{dx} dx = \left(\frac{12\pi j}{\overset{+}{\mu}}\right)^{1/3} \left\{ \int_{V_0}^{V_0 - q} (V_0 - V)^{1/3} dx + \int_{V_0 - q}^{V} (V_0 - V - \epsilon)^{1/3} dx \right\}. \tag{13}$$

The following less accurate approximation, however, lends itself more readily to an experimental test of the theory.

From (12) it is obvious that

$$\frac{d}{dx}(V_0 - V) < \left(\frac{12\pi j}{\overset{+}{\mu}}\right)^{1/3} (V_0 - V)^{1/3},$$

also (14)

$$\frac{d}{dx}(V_0 - V) > \left(\frac{12\pi j}{\overset{+}{\mu}}\right)^{1/3}(V_0 - V - \epsilon)^{1/3}.$$

Solving these

$$V_0 - V < \left\{\left(\frac{16\pi j}{\overset{+}{\mu}}\right)^{1/3}(x - x_0)\right\}^{3/2} \tag{15}$$

and

$$V_0 - V > \left\{\left(\frac{16}{9}\cdot\frac{\pi j}{\overset{+}{\mu}}\right)^{1/3}(x - x_0) - \epsilon^{2/3}\right\}^{3/2} + \epsilon \tag{16}$$

for the region between x_0 and x_a.

The right hand side of (15) may be shown to be the value of $(V_0 - V)$ in case x_0 were the sole source of the ions. Then (15) holds for the minimum possible (zero) ionization in the region considered. Hence it holds also for the entire region between negative glow and apparent cathode film.

On the other hand (16) holds, as already stated, for the maximum possible ionization in the region between x_0 and x_a. This same law of ionization, if applied in the region between x_a and the apparent cathode film, would produce a larger ionization than that at every mean free path. It follows then that (16) also holds between the negative glow and the apparent cathode film.

Letting then x_b locate the outer edge of the apparent cathode film and V_b its potential

$$\begin{aligned} V_0 - V_b &< \left\{\left(\frac{16}{9}\frac{\pi j}{\overset{+}{\mu}}\right)^{1/3}(x_b - x_0)\right\}^{3/2} \\ &> \left\{\left(\frac{16}{9}\frac{\pi j}{\overset{+}{\mu}}\right)^{1/3}(x_b - x_0) - \epsilon^{2/3}\right\}^{3/2} + \epsilon. \end{aligned} \tag{17}$$

For convenience of application this expression may be written in another form (in which the mobility $\overset{+}{\mu}$ is assumed proportional to the m.f.p., $\bar{\lambda}$, of the electrons, that is

$$\overset{+}{\mu} = g\bar{\lambda} \tag{17a}$$

g being a constant)

$$\begin{aligned} V_0 - V_b &< \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right)\right\}^{3/2} \\ &> \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right) - \epsilon^{2/3}\right\}^{3/2} + \epsilon. \end{aligned} \tag{18}$$

The variable quantities are here enclosed in parentheses ( ).

Fall of Potential in the Region of Rebound.—We shall assume that the positive ions in their last mean free path to the cathode are freely accelerated under the action of the electric field. Also that they rebound from the cathode with a fraction κ^2 of their incident energy, and so on with each return, finally transmitting their charge to the cathode simultan-eously with the disappearance of their kinetic energy of rebound. To just what extent the presence of the gas should alter the results derived from these simpler assumptions, we have not attempted to solve.

The problem is, to find the effectual velocity of the positive ions in the region of rebound and then apply Poisson's equation as in the preceding part.

On its first journey inward each positive ion, having a charge e and mass m, reaches the cathode with a kinetic energy equal to $V_b e$, the potential of the cathode for convenience being assumed zero.

On the first excursion outward the ion rebounds to x_1 at a distance $(x_k - x_1)$ from the cathode such that all of its kinetic energy is converted into potential energy, then it is driven again to the cathode. The potential V_1 of the first turning point is given therefore by the equation

$$V_1 e = \kappa^2 V_b e.$$

Likewise for the second turning point at x_2

$$\begin{aligned} V_2 e &= \kappa^2 V_1 e \\ &= \kappa^4 V_b e, \end{aligned}$$

and so on for the others at x_3, x_4, $\cdots$.

The effectual velocity of the ion at any point x is obtained from the total length of time dt it is between the planes x and $x + dx$.

On its first journey inward (the potential at x being V) the velocity $v_{b,\,k}$ at x is given by the equation

$$\tfrac{1}{2} m v_{b,\,k}^2 = (V_b - V)e,$$

from which

$$v_{b,\,k} = + \sqrt{\frac{2e}{m}}\,(V_b - V)^{1/2}.$$

On its first excursion out and back it passes the point x twice with a velocity $v_{1,\,k}$ given by the similar equation

$$\begin{aligned} \tfrac{1}{2} m v_{1,\,k}^2 &= (V_1 - V)e \\ &= (\kappa^2 V_b - V)e, \end{aligned}$$

from which

$$v_{1,\,k} = \pm \sqrt{\frac{2e}{m}}\,(\kappa^2 V_b - V)^{1/2}.$$

Likewise we have

$$v_{2,k} = \pm\sqrt{\frac{2e}{m}}(\kappa^4 V_b - V)^{1/2}$$
$$\vdots \qquad \vdots$$

and finally

$$v_{p,k} = \pm\sqrt{\frac{2e}{m}}(\kappa^{2p} V_b - V)^{1/2},$$

in which we consider

$$x_p < x < x_{p+1},$$

so that on the pth excursion the ion passes between the planes x and $x + dx$ its last time.

The total length of time dt during which the ion is within the given space dx is then

$$dt = \frac{dx}{v_{b,k}} + \frac{2dx}{v_{1,k}} + \frac{2dx}{v_{2,k}} + \cdots + \frac{2dx}{v_{p,k}}.$$

Its effectual velocity at x is therefore

$$\frac{dx}{dt} = \frac{1}{\dfrac{1}{v_{b,k}} + \dfrac{2}{v_{1,k}} + \dfrac{2}{v_{2,k}} + \cdots + \dfrac{2}{v_{p,k}}}.$$

The positive current density at x is given by the equation

$$\overset{+}{j} = \overset{+}{\rho}\frac{dx}{dt},$$

from which

$$\overset{+}{\rho} = \frac{\overset{+}{j}}{\dfrac{dx}{dt}}$$
$$= \overset{+}{j}\left(\frac{1}{v_{b,k}} + \frac{2}{v_{1,k}} + \frac{2}{v_{2,k}} + \cdots + \frac{2}{v_{p,k}}\right).$$

For this vicinity the experimental results indicate that $\bar{j}$ is an extremely small part of j, hence $\bar{\rho}$ is negligible compared with ρ or $\overset{+}{\rho}$. We may then write the last equation

$$\rho = j\left(\frac{1}{v_{b,k}} + \frac{2}{v_{1,k}} + \cdots + \frac{2}{v_{p,k}}\right)$$
$$= \frac{j}{\sqrt{\dfrac{2e}{m}}}\{(V_b - V)^{-1/2} + 2(\kappa^2 V_b - V)^{-1/2} + \cdots + 2(\kappa^{2p} V_b - V)^{-1/2}\}.$$

Substituting this value of ρ in (5)

$$\frac{d^2V}{dx^2} = -\frac{4\pi j}{\sqrt{\frac{2e}{m}}}\{(V_b - V)^{-1/2} + 2(\kappa^2 V_b - V)^{-1/2} + \cdots + 2(\kappa^{2p} V_b - V)^{-1/2}\}.$$

Integrating

$$\left(\frac{dV}{dx}\right)^2 = \frac{16\pi j}{\sqrt{\frac{2e}{m}}}\{(V_b - V)^{1/2} + 2(\kappa^2 V_b - V)^{1/2} + \cdots + 2(\kappa^{2p} V_b - V)^{1/2}\} + \text{constant} \qquad (19)$$

which holds between the turning points p and $p + 1$. This equation cannot be integrated, but remembering that the distance between successive turning points is essentially infinitesimal one may assume the gradient between these points to be linear, its end values given by substituting in this equation.

Letting $V = V_b$ in (19) the integration constant is found equal to

$$\left(\frac{dV}{dx}\right)_b^2,$$

which quantity experiment shows to be negligible when compared with the average value of $(dV/dx)^2$ in the region under consideration.[1] Dropping then $(dV/dx)_b^2$ (19) gives for the first turning point, at x_1, since $V_1 = \kappa^2 V_b$,

$$\left(\frac{dV}{dx}\right)_1^2 = \frac{16\pi j V_b^{1/2}}{\sqrt{\frac{2e}{m}}}(1 - \kappa^2)^{1/2}.$$

For the second turning point, at x_2, $(dV/dx)_1^2$ being now the integration constant,

$$\left(\frac{dV}{dx}\right)_2^2 = \frac{16\pi j V_b^{1/2}}{\sqrt{\frac{2e}{m}}}\{(1 - \kappa^4)^{1/2} + 2\kappa(1 - \kappa^2)^{1/2} + (1 - \kappa^2)^{1/2}\}.$$

For the third, at x_3, $(dV/dx)_2^2$ being the integration constant,

$$\left(\frac{dV}{dx}\right)_3^2 = \frac{16\pi j V_b^{1/2}}{\sqrt{\frac{2e}{m}}}\{(1 - \kappa^6)^{1/2} + 2\kappa(1 - \kappa^4)^{1/2} + 2\kappa(1 - \kappa^2)^{1/2} + (1 - \kappa^4)^{1/2} + 2\kappa(1 - \kappa^2)^{1/2} + (1 - \kappa^2)^{1/2}\},$$

and so on.

[1] For example, in case of the normal cathode fall with an aluminium cathode in hydrogen, the square of the gradient per mean free path of an electron at x_b is about 170, while the square of the average value between x_b and the cathode is about 150,000.

The mean gradients between consecutive turning points, being the average of these end values, may then be written, respectively,

$$\left(\frac{dV}{dx}\right)_{b,\,1} = -\sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}}\cdot j^{1/2}V_b^{1/4}c_{b,\,1},$$

$$\left(\frac{dV}{dx}\right)_{1,\,2} = -\sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}}\cdot j^{1/2}\cdot V_b^{1/4}\cdot c_{1,\,2},$$

and so forth—$c_{b,\,1}$, $c_{1,\,2}$, $\cdots$ being constants whose values depend simply on κ.

Obviously then

$$\int_b^k \frac{dV}{dx}dx = -\sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}}\cdot j^{1/2}V_b^{1/4}\{c_{b,\,1}(x_1 - x_b) + c_{1,\,2}(x_2 - x_1) + \cdots + c_{k-1,\,k}(x_k - x_{k-1})\}. \quad (20)$$

That is

$$V_b = \sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}}\cdot j^{1/2}V_b^{1/4}C(x_k - x_b), \quad (21)$$

in which C is a factor which depends not only on κ but increases with the total number of rebounds, hence should increase with V_b.

We have already assumed that $(x_k - x_b)$ is equal to the mean free path of the positive ions, hence proportional to the mean free path $\bar{\lambda}$ of the electrons. Then

$$(x_k - x_b) = a\bar{\lambda}.$$

Substituting in (21) and writing

$$P = aC,$$

we have for the fall of potential in the apparent cathode film

$$V_b = \left[\frac{16\pi}{\sqrt{\frac{2e}{m}}}\right]^{2/3}\cdot P^{4/3}(j\bar{\lambda}^2)^{2/3}, \quad (22)$$

in which P should be constant with constant values of V_b and κ, but increase with V_b. The first condition allows an interesting test of the theory to be made, namely, for the case under which the cathode fall is found to be constant over a wide range of gas pressures.

The Total Cathode Fall.—Adding (18) and (22) we have the cathode fall

$$V_0 < \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right)\right\}^{3/2} + \left[\frac{16\pi}{\sqrt{\frac{2e}{m}}}\right]^{2/3} \cdot P^{4/3}(j\bar{\lambda}^2)^{2/3}$$

$$> \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right) - \epsilon^{2/3}\right\}^{3/2} + \epsilon + \left[\frac{16\pi}{\sqrt{\frac{2e}{m}}}\right]^{2/3} \cdot P^{4/3}(j\bar{\lambda}^2)^{2/3}. \quad (23)$$

These limits for V_0 suggest the simplest condition under which the cathode fall should remain constant with varying gas pressure. For this case the variable quantities in both expressions are

$$(j\bar{\lambda}^2) \quad \text{and} \quad \left(\frac{x_b - x_0}{\bar{\lambda}}\right),$$

the second being, within the errors of measurement, *the number of mean free paths of the electrons between the cathode and the outer edge of the bright striation of the negative glow, where the gradient is a minimum.*

Providing the above limits of V_0 are not too greatly different,[1] V_0 should be constant if $(j\bar{\lambda}^2)$ and $[(x_b - x_0)/\bar{\lambda}]$ are constant.

The results of an experimental study of these magnitudes follow.

Experimental.

Apparatus.—A discharge tube (diam. 3 cm.) was provided with a circular disk cathode (area 3 sq. cm.) accurately adjustable in a direction perpendicular to its face by a screw actuated through a ground joint. A glass hood confined the current to the front face of the cathode. Two fine parallel probe wires (aluminium) lying in the equipotential surfaces, and sheathed to near their ends with very fine glass tubing, served to indicate (in connection with an electrometer) the point of minimum gradient in the negative glow.

It was soon found that, except at the lowest pressures and current densities, the point of minimum gradient, or more exactly the point at which the gradient suddenly began to increase from a zero value, was very closely marked by the luminosity as described on page 000. This being a much more convenient method of setting, it was largely used in the measurements.[2]

[1] Calculations made from observations on an aluminium cathode in hydrogen show these limits of V_0 to differ at most by 12 per cent. (see Table VI.).

[2] Both methods of setting are subject to appreciable error at those low gas pressures where the minimum cathode fall begins to run above its so-called normal value.

Special care was taken to prevent, by sufficient insulation, any leakage current from the probe wires, as the corresponding fall of potential, if the wire for the leakage current is a cathode, has a very marked effect on the indicated value.[1]

The electric current was furnished by a battery of small storage cells, was measured by either a milli-ammeter or a more sensitive galvanometer, and regulated by a solution of cadmium iodide in amyl alcohol.

The cathode fall was measured by a Kelvin electrometer, being placed between the needle and one pair of quadrants, while a definite potential (10–20 volts) was placed between the quadrant pairs.

Normal Cathode Fall, Current Density and Distance to Negative Glow.—The gas chosen was hydrogen because of the relatively large distance it gives between the cathode and negative glow. The metals chosen were aluminium and steel which have respectively (among the common metals) the smallest and largest values of the cathode fall.

The hydrogen was generated from aluminium in a solution of potassium hydrate, then carefully dried in a chamber containing phosphorous pentoxide, and finally stored in a convenient bulb (freed of occluded gases), from which the supply was drawn as needed.

The cathode was always polished with infusorial earth and rubbed clean with new cloth just before mounting in the discharge tube. In addition its discharge surface was always cleaned by a heavy current (passed through the old gas just before evacuating and introducing the fresh). All measurements were made as soon as possible after this last cleaning was done. For each day's observations the cathode was always polished anew.

Tables I. and II. give a representative series of measurements for the two metals respectively. Each recorded pressure (first column) represents an entirely fresh gas filling.

With gas pressures ranging from one to five millimeters the normal cathode fall (second column) fluctuates irregularly about a constant value.

The normal current density (third column) was obtained either by increasing the current to the point where the cathode fall began to increase, or until the negative glow appeared of uniform intensity over the face of the cathode and did not curl away from it at any part—the latter method of setting being usually the more sensitive.

The distance from cathode to the position of minimum gradient was measured by the number of turns of the screw required to shift the cathode from contact with the fixed probe wire to where the wire was at the point of minimum gradient, located as already described.

[1] In order to eliminate errors arising from leakage all lines to the electrometer including the battery used to charge the quadrants, and the electrometer itself had to be placed on sealing wax insulators—no "ground" to the electrometer being used.

TABLE I.

Aluminium Cathode in Hydrogen. Area of Cathode: 3.0 sq. cm.

Gas Pressure (Mm.).	Normal Fall (Volts).	Normal Current Density (j_n)	Distance to Neg. Glow ($x_k - x_0$).	Mean Free Path ($\bar{\lambda}$)	($j_n\bar{\lambda}^2$)	$\left(\frac{x_k - x_0}{\bar{\lambda}}\right)$
1.12	205	.10 m.a.	9.6 mm.	.64 mm.	.041	15
1.65	196	.19	6.8	.44	.037	15.5
2.27	194	.42	4.6	.315	.042	14.6
2.28	196	.43	4.5	.315	.043	14.3
2.41	206	.43	4.7	.30	.039	15.7
3.13	195	.67	3.6	.235	.037	15.3
3.58	194	.90	——	.20	.036	——
4.18	194	1.40	2.7	.17	.040	15.8
5.36	197	——	2.15	.135	——	15.9
Mean	197		Mean		.039	15.3
Max. dev.	4.5%		Max. dev.		10%	6.5%
Mean dev.	3%		Mean dev.		5%	3%

The mean free path of the electrons (column 5) which is inversely proportional to the gas pressure was obtained by multiplying the m.f.p. of the molecule by $4\sqrt{2}$.
ments of this magnitude.

Columns six and seven give respectively the calculated values of the quantities $(j\bar{\lambda}^2)$ and $[(x_k - x_0)/\bar{\lambda}]$ which practically $= [(x_b - x_0)/\bar{\lambda}]$.

The results show that, with constant cathode fall, gas pressures ranging from one to four times, and the normal current densities ranging from one

TABLE II.

Steel Cathode in Hydrogen. Area of Cathode: 3.0 sq. cm.

Gas Pressure (Mm.).	Normal Fall (Volts).	Normal Current Density (j_n).	Distance to Neg. Glow ($x_k - x_0$)	Mean Free Path ($\bar{\lambda}$)	($j_n\bar{\lambda}^2$)	$\left(\frac{x_k - x_0}{\bar{\lambda}}\right)$
1.30	310	.087 m.a.	11.7	.55	.0265	20.1
1.48	306	.10	10.2	.49	.024	20.8
1.73	305	.167	8.4	.42	.0295	20.0
1.95	313	.207	7.6	.37	.0285	20.5
2.11	306	.25	7.1	.34	.029	20.9
2.43	305	.300	5.7	.30	.027	19.0
2.72	305	.367	5.5	.265	.025	20.8
2.93	302	.467	5.1	.245	.028	20.8
3.31	298	.567	4.6	.215	.026	21.4
3.72	298	.767	4.1	.195	.029	21.0
4.00	302	.85	3.7	.18	.0275	20.6
4.77	289	1.20	2.9	.15	.027	19.3
5.00	304	1.33	3.0	.145	.028	20.8
Mean	303.5			Mean	.0273	20.5
Max. dev.	5%			Max. dev.	12%	7%
Mean dev.	2%			Mean dev.	5%	2½%

to sixteen, both of these magnitudes fluctuate irregularly about constant values—the deviation falling on the average well within the errors of observation.

Thus we find, as suggested by the foregoing theoretical deductions, that the *normal current density is inversely proportional to* $\bar{\lambda}^2$, *that is, proportional to the square of the gas pressure*[1]—nearly one and a half times as large with aluminium as it is with steel. Further, *the normal cathode fall extends from the cathode, at all gas pressures, the same number of mean free paths* of the electron. With a steel cathode this number is one and a third times that with an aluminium cathode, its fall being one and a half times as large.

These experimental results being favorable to the theory a more thorough investigation of its appplicability was desirable.

The Potential Curve in the Gas.—Preliminary measurements of the potential difference between the cathode and various distances $(x_k - x)$ in the gas (out to the negative glow), at various pressures and current densities, indicated that for the same values of $(j\bar{\lambda}^2)$ and $[(x_k - x)/\bar{\lambda}]$ the potential at x was always the same. That is, the potentials in the gas plotted against the number of mean free paths from the cathode, fell, for constant values of $(j\bar{\lambda}^2)$, on the same curve.

A systematic investigation of this was carried out with an aluminium cathode in hydrogen. The results are incorporated in Tables III., IV. and V. Each series of observations were made with fresh gas and a current-cleaned cathode. A series consisted in measuring the difference of potential between the cathode and a single probe wire (al., diam. .24 mm.) when the latter was placed at two, three, four, etc., mean free paths (measured to the center of the probe wire) from the cathode—the current density during that time being maintained at "normal," "twice-normal," or "four-normal" value. A disturbing source of error arises from the change in the gas, with duration of current, causing an increase in the potential difference. We are inclined to attribute this change to the presence of metal vapor from the cathode, as its effect is appreciably smaller with aluminium than with steel, and it is well known that the former gives off distinctly less material by the cathode spray than the other metals.

To avoid this error the observations were hurried through as rapidly as possible by three observers—one regulating the current, the second observing the electrometer deflection (to one side only), and the third setting the probe wire.

[1] H. A. Wilson (Phil. Mag., Vol. VI, 4, p. 608) from a study of the current density with wire cathodes in air, concluded that at a determinate distance from the cathode surface the current density is proportional to the gas pressure.