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THE THOMSON EFFECTS IN TUNGSTEN, TANTALUM AND CARBON AT INCANDESCENT TEMPERATURES DETERMINED BY AN OPTICAL PYROMETER METHOD.

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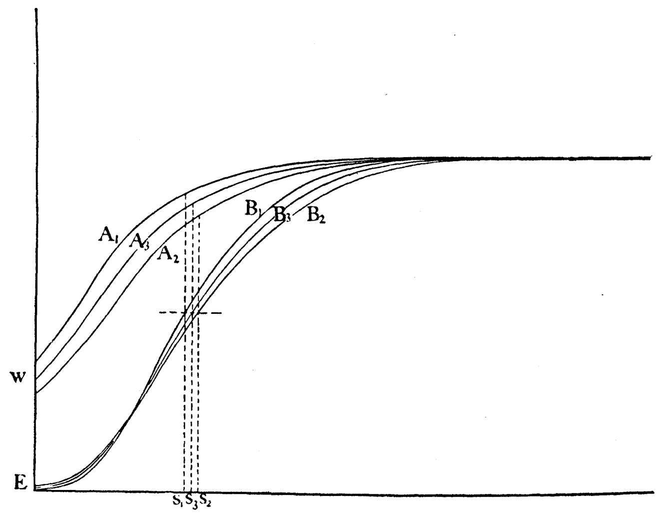
INTRODUCTION.

THE present paper is a direct outcome of certain effects noted in connection with a study by Hyde, Cady and Worthing[1] of the energy losses in electric incandescent lamps. Favorable conditions for observing the Thomson effect exist in such lamps in consequence of the high temperature gradients in the neighborhood of the leading-in junctions and of the large current densities. W. König,[2] using a visual method, has obtained qualitative results on Pt, Cu, Fe and constanstan. However, so far as the writer knows, no quantitative results have been obtained hitherto in connection with incandescent temperatures. An optical pyrometer method for such measurements and its application are here described.

THEORY.

Consider a small filament of uniform surface and cross-section mounted in an evacuated bulb and heated to incandescence by an electric current. For the temperature distributions in the neighborhood of a cooling junction for a given intensity of heating current, three cases are to be distinguished depending on whether the heating current through the filament (1) is a direct current directed away from the junction, (2) is a direct current directed toward the junction, or (3) is an alternating current. In Fig. 1 (a diagrammatic representation for tungsten) the curves A_1, A_2 and A_3 respectively represent the distributions of the rates of production of heat per unit of radiating surface, w_1, w_2 and w_3 (Table I.); and the curves B_1, B_2 and B_3 respectively the radiation intensities E_1, E_2 and

[1] Illum. Eng. (Lond.), *4*, p. 389, 1911. Trans. Illum. Eng. Soc. (U. S.), *6*, p. 238, 1911.
[2] Phys. Zeit., *11*, 1913, 1910.



Distance along filament.

Fig. 1.

Diagrammatic representation for a portion of a tungsten filament near a cooling junction, of the distributions of the rates of development of heat in the elements of filament per unit of radiating surface (A_1, A_2, A_3) and of the radiation intensities (B_1, B_2, B_3) respectively for the conditions that the heating current is (1) directed away from the cooling junction, (2) directed toward the cooling junction, and (3) alternating.

TABLE I.

Symbols and Quantities Used.

w = Rate of development of heat in an element of filament per unit of radiating surface.
E = Radiation intensity.
l = Distance along filament.
r = Radius of filament.
I = Current.
ρ = Resistivity.
T = Temperature.
σ = Coefficient of the Thomson effect.
$\frac{dH}{dT}$ = Rate of heat conduction along the filament at a given cross-section.
k = Thermal conductivity.
p = Constant defined by equation (8).
μ = Constant of equation (12).
E_m and T_m = Maximum values of E and T.

E_3. Methods for obtaining curves B_1, B_2, B_3 and A_3 are fully described by the author[1] elsewhere. A method which may be employed in obtaining curves A_1 and A_2 will appear from later considerations. The crossing of the B curves has no particular significance from the standpoint

[1] PHYS. REV., II., 4, p. 535, 1914.

of the Thomson effect. It is merely a consequence of the lateral displacements of these curves due to the Peltier effect at the junction.

For the rates w_1 and w_2 corresponding to the condition $E_1 = E_2$ and thus necessarily $T_1 = T_2$ (positions S_1 and S_2, Fig. 1), we have, from the standpoint that the heating per unit length of filament must equal the sum of the Joulean and the Thomson heatings,

$$2\pi r w_1 = \frac{I^2\rho}{\pi r^2} - \sigma I \frac{dT}{dl_1}, \tag{1}$$

and

$$2\pi r w_2 = \frac{I^2\rho}{\pi r^2} + \sigma I \frac{dT}{dl_2}, \tag{2}$$

in which the sign of the coefficient σ is taken as positive when the gradient of the Thomson E.M.F. coincides in direction with the temperature gradient. For tungsten as is shown in Fig. 1, σ is negative. (1) and (2) give directly

$$\left[\sigma = \frac{2\pi r(w_1 - w_2)}{I\left(\dfrac{dT}{dl_1} + \dfrac{dT}{dl_2}\right)}\right]_{T_1=T_2=T}. \tag{3}$$

In order to make use of (3), a method of obtaining the distributions of w_1 and w_2 must be devised. Since the rate of heat production in an element of filament must equal the sum of the rate of radiation of energy from it and of the *net* rate of heat conduction from it, there results

$$2\pi r w = 2\pi r E - \frac{d}{dl}\left(\frac{dH}{dt}\right), \tag{4}$$

where dH/dt represents the rate of conduction of heat across the cross-section of filament at l. Since

$$\frac{dH}{dt} = \pi r^2 k \frac{dT}{dl}, \tag{5}$$

there follows

$$\frac{d}{dl}\left(\frac{dH}{dt}\right) = \pi r^2\left[k\frac{d^2T}{dl^2} + \frac{dk}{dT}\left(\frac{dT}{dl}\right)^2\right]. \tag{6}$$

Equations (4) and (6) give

$$w = E - \frac{r}{2}\left[k\frac{d^2T}{dl^2} + \frac{dk}{dT}\left(\frac{dT}{dl}\right)^2\right]. \tag{7}$$

Determinations of k as a function of T and of T and E as functions of l, such as the writer has reported in the papers already referred to, enable one to obtain the desired distributions of w_1 and w_2 as functions first of E_1 and E_2 and then of l. The relations thus obtained suffice for obtaining σ.

A simplification results if a certain condition as to the distributions of E_1 and E_2 exists. This condition, which was found experimentally to hold, is

$$\left[\frac{dT}{dl_2} = p\frac{dT}{dl_1}\right]_{T_1=T_2=T}, \tag{8}$$

in which p is a constant depending on the material of the filament and on the maximum filament temperature. Then since the distribution of E_3 is approximately if not accurately a mean of the distributions of E_1 and E_2, this condition may be rewritten as

$$\left[\sqrt{p}\frac{dT}{dl_1} = \frac{1}{\sqrt{p}}\frac{dT}{dl_2} = \frac{dT}{dl_3}\right]_{T_1=T_2=T_3=T}. \tag{9}$$

Since as indicated in (8) and (9) the points considered on the different temperature distribution curves refer to cross-sections having the same temperature, l_1 and l_2 may be expressed as functions of l_3. This leads to

$$\left[p\frac{d^2T}{dl_1^2} = \frac{1}{p}\frac{d^2T}{dl_2^2} = \frac{d^2T}{dl_3^2}\right]_{T_1=T_2=T_3=T}. \tag{10}$$

Equations (3) and (7) when combined subject to (9) and (10) give finally

$$\sigma = \frac{\pi r^2}{2I}\left(p - \frac{1}{p}\right)\frac{k\dfrac{d^2T}{dl_3^2} + \dfrac{dk}{dT}\left(\dfrac{dT}{dl_3}\right)^2}{\dfrac{dT}{dl_3}}. \tag{11}$$

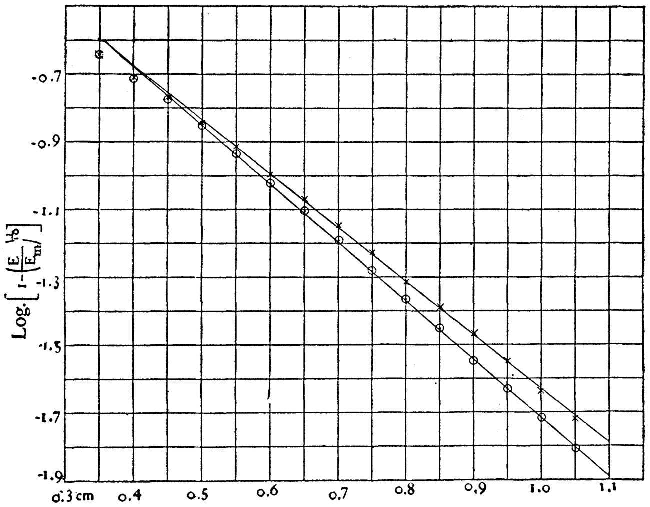
RESULTS.

As indicated in the paper already referred to, the temperature distribution along an incandescent tungsten filament in vacuo is of the type

$$\frac{T}{T_m} = [1 - e^{-\mu(l+l_0)}]^{1.87}. \tag{12}$$

Such a temperature distribution curve and the corresponding radiation intensity distribution curve were platted for a particular tungsten filament (r = 0.01045 cm.) heated by a current of 4.32 amps. to a maximum temperature of 2315° K. (scale of Mendenhall and Forsythe).[1] In Fig. 2 there are platted the two individual distributions of radiation intensity depending on the direction of flow of the D.C. heating currents, from which the previously mentioned curves were obtained. As platted they are approximately straight lines. As has been stated before in connection with the other related curves, the deviations, with the exception of those near the cooling junction, are accidental. The conclusion as to the constancy of p resulted from a large number of such tests. In Table II.

[1] Astrophys. Jour., 37, p. 380, 1913.



Distance along filament.

Fig. 2.

The radiation intensity distribution for a particular tungsten filament (r = 0.01045 cm.) heated to a maximum temperature of 2315° K., by (1) a current of 4.32 amps. directed away from the cooling junction (⊙ ⊙ ⊙), and (2) by a current of the same intensity directed toward the cooling junction (× × ×).

TABLE II.

Data Necessary for the Application of (11) to a Particular Tungsten Filament, and Computed Values of the Thomson E.M.F. Coefficient.

Condition.	Maximum Filament Temperature.	Average Value of p.	r	I	μ
1	2315° K.	0.920	0.01045 cm.	4.32 amp.	3.80 $\frac{\text{I}}{\text{cm.}}$
2	1890	0.955	0.01045	2.88	2.65

Condition 1.		Condition 2.	
Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degree}}$.	Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degree}}$.
2200	−35	1800	−16
2000	−28	1600	−11
1800	−21	1500	−10

there are given the average values of p for two maximum filament temperatures. The value for 1890° K. is considerably more uncertain than that for 2315° K. There are given also other data necessary in the application of these two values by means of (11), together with the results. The lower temperature in each case refers to a region in which the fit of (12) indicates the application of (11) as being still justifiable. The lack of agreement of the two values for 1800° K., on the assumption that σ is independent of the current density, is probably ascribable to the comparatively great difficulty experienced in condition 2 in determining the value of p. That thermionic currents can not account for the difference seems evident when one considers the maximum potential difference of approximately 20 volts between the terminals of the lamp filament in conjunction with the work of Langmuir.[1]

The results obtained for filaments of tantalum and of untreated carbon

TABLE III.

Thomson Effect in Tantalum and Carbon.

Tantalum.		Carbon.	
Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degrees}}$.	Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degree}}$.
2100	+24	2100	−22
1900	+20	2000	−21
1700	+16	1800	−19

are indicated in Table III. The temperature calibrations here used are also those of Mendenhall and Forsythe.[2]

The greatest uncertainty in this work lies in the fact that the relations $E = \varphi(T)$ and $E = f\ (l)$ [relations (5) and (6) of previous paper on thermal conduction] are determined at different portions of the filament under experiment. This means a difference in the two cases in the glassware between the background filament and the pyrometer filament and also possible inequalities in the cross-sections of the filament. Due to the many filaments of tungsten which have been investigated, the uncertainty there has been largely reduced but such is not the case with carbon or tantalum. It is possible to eliminate largely this source of error by using metal cases with plate glass windows, with a device for clamping and unclamping the filament in the neighborhood to be studied.

SUMMARY.

1. A method of studying quantitatively the Thomson effects in filaments mounted in evacuated chambers, with the aid of an optical pyrometer, has been developed.

2. The Thomson E.M.F. coefficients have been determined for tungsten, tantalum and carbon (Tables II. and III.) for temperatures ranging from 1500° K. to 2200° K.

[1] PHYS. REV., II., 2, p. 450, 1913.

[2] Astrophys. Jour., 37, p. 380, 1913.

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