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THE MECHANICAL EQUIVALENT OF LIGHT.

By Herbert E. Ives, W. W. Coblentz and E. F. Kingsbury.

Introduction.

THE mechanical equivalent of light has been the subject of investigation by Tumlirz,[1] Angstrom[2] and others. Knowing as we do that

[1] Annalen der Physik, 38, p. 650, 1889.
[2] Annalen der Physik, 67, p. 648, 1899.

objective light and radiant energy are one and the same thing, it is but natural that attempts should be made to obtain the one in terms of the absolute units in which the other is specifiable. These early measurements, with which the term "mechanical equivalent of light" has been associated, were most unfortunately based upon a hazy and immature idea of what constitutes "light." Under this condemnation must fall as well numerous determinations of "luminous efficiency," for the two quantities are of necessity closely interrelated. In brief, light has been considered merely as radiation that can be seen, quite irrespective of the widely different capacities of the various "visible" radiations to excite the subjective sensation of light. As a consequence the physically determined "luminous efficiencies" and "mechanical equivalents" (this latter different for every light source) have no definite relationship to the "efficiencies" and "specific consumptions" used by the engineer. These latter are rational and consistent quantities (which the physical ones are not), though unfortunately related to the chance dimensions of the first measured candle instead of to the C.G.S. units. The present investigation was undertaken to establish on a consistent scheme, in terms of the fundamental physical units, the real values of the light units now in practical use.

The relationship between light and power upon which this work is based are developed in a paper by one of the present writers under the title: "The Primary Standard of Light,"[1] to which reference may be made by those unfamiliar with the subject. In that paper it is proposed that light be defined as radiant energy flux evaluated according to its capacity to produce the sensation of light. It is further proposed that the standard of luminous flux be one watt of radiation of maximum luminous efficiency. More mature thought on the subject suggests, however, that this statement of the proposed standard is more complicated than need be. Accepting the definition of light there given, it is quite sufficient to say that *the unit of luminous flux shall be the watt.* The determination of the mechanical equivalent of light, therefore, becomes merely the fixing of the lumen in terms of the watt.

2. Definitions of Light and Luminous Quantities.

The discussion of this subject can be much shortened by adopting at the start a set of definitions of the quantities frequently entering in. These are taken partly from those now in technical use, and partly either taken or adapted from a list suggested by Ives.[2]

[1] Ives, Astrophysical Journal, XXXVI., No. 4, Nov., 1912, p. 322.
[2] Lighting Journal, Vol. 1, Oct., 1913, p. 250.

Power consumed by a light source = P; expressed in watts, a portion of which is dissipated by radiation, the remainder by conduction and convection.

Power radiated by a source = $R = \int_0^\infty R_\lambda d\lambda$ = power emitted by a light source in the form of radiation between wave-lengths 0 and ∞, expressed in watts.

Radiation efficiency = R/P = ratio of the power dissipated as radiation to the total amount of power consumed by the source (a pure numeric).

Luminous flux = F = radiant power evaluated according to its capacity to produce the sensation of light.

Light evaluating factor or stimulus coefficient of any radiation is the ratio of the luminous flux, in its appropriate units, to the radiant power producing it, in its appropriate units.

The luminous efficiency of any radiation = L_R, = the relative capacity of the radiation to produce the sensation of light, compared with the capacity of the same quantity of radiation of the maximum possible light producing capacity (a pure numeric).

The total luminous efficiency of a light source = L_r = the relative capacity of the power applied to a light source to produce the sensation of light, compared with the capacity of the same quantity of power in the form of radiation of maximum possible luminous efficiency (a pure numeric).

Units.—Luminous flux is connected to radiant power by a numerical evaluating factor. The unit of power is the watt. The present arbitrary practical unit of luminous flux is the lumen. The light evaluating factor or stimulus coefficient is consequently expressed in lumens per watt. If for this evaluating factor is taken the *luminous efficiency* as above defined, the unit of luminous flux is the same as that of radiant power or applied power, namely the *watt.*

In order to go over to the watt as the unit of luminous flux it is necessary to know the:

Mechanical Equivalent of Light = the value of the lumen in watts of luminous flux.

The simplicity of these relationships is illustrated by the equation,

$$\text{Power consumed} \times \text{radiation efficiency} \times \text{radiant luminous efficiency} = \text{luminous flux},$$

in which every quantity of interest in the study of an illuminant finds its place.

This simplification is only possible if there does exist a definite practically establishable "radiant luminous efficiency." Put another way this means that there must exist a definite luminosity curve of the spectrum. While this latter is actually a function of intensity, size of the field of view, etc., reasons have been given elsewhere[1] for believing that the practical situation is adequately met by the adoption of a high intensity luminosity curve as determined by a certain set of photometric conditions. For this will be used the luminosity curve of the normal equal energy spectrum as determined by Ives as the mean of 18 observers and the same curve as determined by Nutting[2] as the mean of 21 observers. As will be seen, the present investigation offers a means of deciding between these slightly different curves.

3. Methods for Determining the Mechanical Equivalent.

In general the determination of the mechanical equivalent of light consists in the measurement of the same luminous flux in both watts and lumens, from which measurement the ratio of the two can at once be determined.

It usually happens, however, that the radiation is not presented to the energy measuring instrument already evaluated as "light," consequently this value must be deduced from the value of the total radiation and its radiant luminous efficiency. It may happen too that the value of the radiation must be deduced from the total input through a known or probable value of the radiation efficiency. According as one or other of these contingencies is met we find three fairly distinct experimental methods of approaching the problem, as follows:

A. Through the graphical evaluation as light of a known energy distribution.

This may be illustrated by calculations on a black body. Thus Nernst[3] has measured the brightness of a black body (solid angular luminous flux density per unit area); the radiation constant (solid angular radiation flux density per unit area) has been the subject of numerous measurements,[4] and the distribution of energy through the spectrum may be calculated from the Wien-Planck equation. Now, by multiplying the latter by the radiant luminous efficiency or luminosity curve of the spectrum, of maximum value unity, a "reduced" area is obtained, the ratio of which to the total area is the radiant luminous efficiency. We then have

[1] Ives, "Studies in the Photometry of Lights of Different Colors," Phil. Mag., July, Sept., Nov., Dec., 1912.

[2] Trans. Illuminating Eng. Soc., Vol. IX., No. 7, p. 633, 1914.

[3] Physikal. Zeit., Vol. 7, p. 380, 1906.

[4] See Coblentz, Bureau of Standards Bulletin, 11, p. 87, 1914.

radiated power × radiant luminous efficiency =
luminous flux in ergs per second or in watts, and (from the candle power measurements) *luminous flux in lumens*,

from which the ratio of the lumen to the watt may be obtained.

A similar case is that presented by the incandescent electric lamp, in which we know the power input, the efficiency losses (approximately), the luminous output in lumens, and the radiant luminous efficiency from energy distribution curves and the luminosity curve of the eye.

Values calculated by this method have been published by Ives and others.[1] Reducing them to what they would be if the luminosity curves here adopted were used, the lumen is found to be about 1/800th of the watt. The defect of this method is that the luminous portion of the spectrum, upon which the evaluating process must be carried out, is an excessively small part of the whole, in which experimental errors of determination or deficiencies in the theoretical formula for energy distribution figure disproportionately.

B. Through mechanical evaluation of a given radiant energy flux as light, by the use of absorbing media or equivalent means.

This method does mechanically what the previous method does indirectly by calculation. Imagine an absorbing screen whose transmission is exactly according to the normal equal energy spectrum luminosity curve of the eye, with a maximum transmission of unity. Measure in absolute units the radiation transmitted through it. Measure also the luminous flux from the same light source in lumens. The figures obtained give the ratio desired.

This method is quite the simplest and most direct, once the spectrum luminosity curve is established and the ideal absorbing medium is at hand. It is the method suggested by Houstoun.[2] Another variation of the same idea is the suggestion of Strache,[3] to form a spectrum, pass it through an opening cut to the shape of the visual luminosity curve, and then measure the radiation in absolute units. No determinations made by this method have been published.

C. The measurement of a selected monochromatic radiation, of known luminious efficiency, as light and power.

This method, which in principle differs in no way from the others, has some advantages. For instance the value obtained for the luminous equivalent of the monochromatic radiation has an independent value in that it can be used with any luminosity curve, not only with the one selected by the experimenter. If the monochromatic radiation is

[1] Electrical World, June 15, 1911, p. 1565.
[3] Proc. Royal Soc., A, 85, 275, 1911.
[3] Proc. American Gas Institute, 2, 401, 1911.

selected near the maximum of visual sensibility the resultant value is largely independent of errors in the determination of the ends of the luminosity curve, since the maximum is fairly well agreed upon.

The great difficulty in the determination by this method is the measurement of the colored light, for which special methods are necessary. By the other methods the colored-light photometry is performed entirely in the determination of the luminosity curve, since the light source measured can always be of the color of the standard.

All the published experimental determinations have been made by this last method. Drysdale,[1] using the spectrally resolved light of the carbon arc, obtained for yellow-green light 210 lumens per watt. Nutting,[2] by a similar procedure obtained 170 lumens per watt for wavelength .566 μ. Buisson and Fabry,[3] measuring the monochromatic green mercury radiation found the value 690. The first two values are unquestionably much too low, probably due on the one hand to scattered radiation, and on the other to the crudity of the methods of measuring the intensity of the colored light. Fabry's value is of the order of magnitude of the calculated figure, but was confessedly weak on the photometric end. The green light was evaluated by simple direct comparison, several observers being used. Their energy standard was probably in error to some extent (see reference 9), but their value is nevertheless remarkably close to the one here obtained.

4. Apparatus and Methods of the Present Investigation.

The aim in the present work has been: first, to develop methods of attack in which the highest attainable accuracy, both photometric and radiometric, may be obtained, and, second, to establish the value of the lumen in terms of the watt with a degree of accuracy sufficient to make that ratio of immediate use in the technology of light production and utilization.

Both experimental methods above outlined (*B* and *C*) were used. A description of the apparatus and method of use follows:

(*a*) *Method B*.—The most important factor in method *B* is the absorbing medium whose transmission shall be the luminosity curve of the normal energy spectrum. The ideal screen would be one whose maximum of transmission was unity and which absolutely matched the curve in question. But neither of these conditions is absolutely necessary. Any other maximum transmission than unity merely involves the correc-

[1] Proc. Royal Soc., 80, 19, 1907.
[2] Electrical World, June 26, 1908.
[3] Compt. Rend., 153, 254, 1911.

tion of the value found for the transmitted energy to what it would be were the value unity. If the transmission is not exactly in accordance with the ideal curve it is possible by graphical calculation to determine with considerable accuracy the correction factor to be applied. For this it is only necessary to know the *shape* of the energy distribution in the visible region, not its value relative to the rest of the emission spectrum as in the case of method *A*.

The screen used in this investigation was a solution of certain inorganic salts contained in a parallel-walled glass tank one centimeter in thickness. The composition of this solution is:

Cupric chloride	60.0 g.
Potassium chromate	1.7 g.
Cobalt ammonium sulphate	7.5 g.
Nitric acid, sp. gr. 1.05	15.0 c.c.
Water to	one liter of solution.

The transmission of this solution through the spectrum was measured against that of clear water by means of the sunlight spectro-radiometer described elsewhere.[1] The two tanks were constructed and manipulated as in the previous investigation with photometric absorbing solution described by us.[2] The values obtained are shown in Fig. 1, Curve *a*; in Curve *b* (circles) they are multiplied by the factor 1.3 and compared with the Curve *c*, which is the Ives luminosity curve it is desired to copy. It is evident that the copy of the Ives curve is quite close.

What we are the most interested in is the correction to be applied in the use of this screen under the conditions of the experiment. This is obtained by multiplying the energy distribution curve of the light source employed, at each wave-length by the value of the ideal curve and by the value of the solution transmission. The correction factor is given by the ratio of the areas of the two resulting curves. This process is gone through in Curves *d* and *e*. The energy distribution of the "4-watt" carbon lamp used as light source is taken as being substantially that of a black body at 2,080 degrees absolute, which is calculated from the Wien equation. The ratio of the areas *d* (ideal curve) to *e* (actual curve) is 1.30, which is the factor by which the observed power must be multiplied to obtain the working value, if the Ives curve is used.

The remaining curves of Fig. 1, namely, *f* and *g*, are the luminosity curves as recently determined by Nutting, and the same applied to the "4-watt" lamp. These are included for the reason that the experimental values to be reported upon can be used equally well to determine the mechanical equivalent on the basis of this luminosity curve, which

[1] Description to be published shortly.

[2] Ives & Kingsbury, Trans. Illuminating Eng. Soc., IX., No. 8, p. 795, 1914.

is slightly different from that of Ives, and can in fact be used to decide between the two, in a manner that will be brought out presently. Nutting's curve, determined in substantially the same way as the older one, differs from it chiefly in approaching its maximum more steeply on each side, by reason of which its area is less. In the case of the "4-watt" lamp this difference of area amounts to eight per cent., so that the correction factor becomes $.92 \times 1.30 = 1.1975$. This difference is perhaps

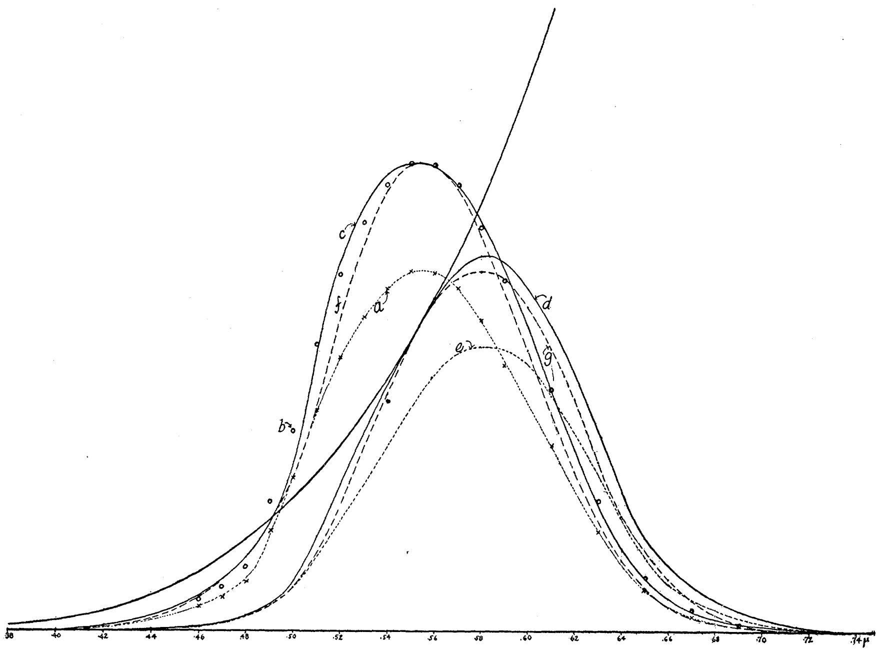

Fig. 1.

Graphical Evaluation of Luminous Flux from "4-watt" Lamp.

due to the characteristics of the two groups of observers, although it has more the appearance of being due to some instrumental difference. Nutting measured the energy distribution of his source at the slit of his observing telescope directly, while Ives had to get his indirectly. While the direct procedure is preferable it may be rendered less accurate by the presence of scattered radiation, apt to be particularly dangerous in the visible spectrum with its small share of the total energy. The Nutting curve agrees, in its shape near the maximum, with the curve determined by Thürmel[1] and others with the Lummer-Pringsheim spectral flicker photometer. As will be shown the agreement or disagreement of methods B and C furnishes a clue as to which curve is the more likely.

[1] Annalen der Physik, XXXIII., p. 1154, 1910.

The apparatus used for method *B* was a portion of the much more complicated arrangement necessary for method *C*, which is shown in Fig. 2. *G* is the surface thermopile, to be described below, turned to face the radiation standard *R* and the light source *P*. The latter was a "point source" 100 candle-power carbon lamp, set to standard "4-watt" color by comparison with a specially furnished standard from the Bureau of Standards. This was carefully measured for candle-power, through a tank of clear water, in terms of two master standards, also from the Bureau of Standards. Since the transmission of the luminosity curve

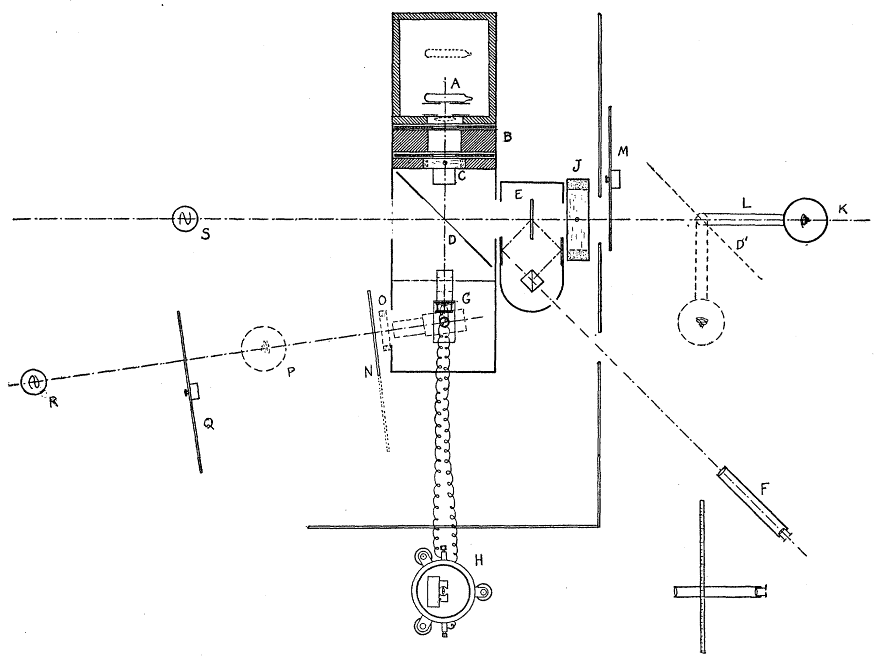

Fig. 2.

Arrangement of Apparatus.

solution was also measured in terms of clear water, the final result is entirely independent of any possible peculiarities of our pair of matched tanks. The luminosity curve solution is shown in position at *O*. At *N* is a shutter, operated from the observing telescope *F*. *Q* is a sector disc, used to reduce the intensity of radiation from the radiation standard, in order to keep all the galvanometer deflections of the same order of magnitude.

The procedure is first to obtain the sensibility of the thermopile by a set of readings on the radiation standard (the lamp *P* and the solution *O* being of course removed): then with lamp and solution in place, to measure the luminous flux. The candle-power of the lamp, divided by the square of the equivalent air distance (allowing for the absorption of the thermopile window if used), gives the lumens per unit area. The watts per unit area are obtained from the thermopile reading corrected by the ratio already obtained from the measurement of the solution transmission. The ratio of the lumen to the watt can then be immediately derived.

(*b*) *Method C*.—The apparatus for this method is substantially that outlined by Ives in his original suggestion for the watt as the unit of luminous flux.[1] The principal improvement is in the means employed to obtain the photometric value of the green light, which is now made a separate determination, by taking advantage of the researches on colored light photometry and photometric absorbing solutions carried out since the original suggestion was made.

In Fig. 2, *A* is a quartz mercury arc (Heræus 110 volt), *B* is a shutter, operated from a distance, consisting of two parallel sheets of heavy brass, separated by a block of wood, and pierced with round holes with beveled edges. The two elements of the shutter move up and down "straddling" a heavy block of wood having a central opening in line with the axis of the apparatus. At *C* is a glass-walled cell containing a solution of cupric chloride, potassium dichromate and neodymium-ammonium-nitrate, with a little nitric acid. This solution, which was made up empirically, transmits nothing from the mercury arc except the green line .5461 μ, as shown by spectrophotographic and spectroradiometric tests.

At *D* is a transparent mirror of clear white glass. Its function is to reflect a small fraction of the radiation to the Lummer-Brodhun photometer head *E*, while the greater part of the radiation falls directly on the thermopile *G*, which latter is connected with the galvanometer *H*. The thermopile is mounted so that it can be rotated about a vertical axis, the mount being adjustable as to its position in the horizontal plane, while the thermopile can be raised or lowered. By means of these adjustments the pile can be set exactly in the axis of the system and at any desired distance from the arc. The method of performing these adjustments is given shortly.

When the thermopile is turned to one adjustable stop it faces the mercury arc, when turned to the other it faces the radiation standard *R*, which is set accurately at a distance of two meters from the opening of the thermopile, and the shutter *N*, operated from *F*. At *S* is an incandescent lamp candle-power standard. At *K* is a "point source" carbon lamp of the type previously described. *M* and *Q* are sector discs to be

[1] Energy Standards of Luminous Efficiency, Trans. Illum. Eng. Soc., April, 1911, p. 258.

used when desired. The photometer field is read by means of the telescope F which is at such a distance from the thermopile that the body of the observer never comes near it. The whole apparatus is most elaborately protected by a large system of metal and cardboard screens not shown in the schematic figure. By these screens all stray light and radiation are completely excluded.

The various incandescent lamp, standard and comparison, are held constant by voltage readings, on a carefully checked laboratory standard voltmeter connected with the lamps by separate voltage leads carried directly to the sockets. The controlling resistances are all at some distance, so that the heat liberated by them shall have the minimum effect on the thermopile. The mercury arc is connected with an ammeter, but as was anticipated when the transparent mirror scheme was adopted for securing simultaneous photo and radiometric observations, the green radiation cannot be held constant within a hundred per cent. by holding a constant current. Immediately after turning on, the green radiation is only a small fraction of what it becomes after several hours' operation, the current being the same.

From the photometric side the most important part of the apparatus is the glass cell J. This is one of a pair, one containing clear water, the other a green solution which transforms the light from lamp K to an exact subjective match with the monochromatic green mercury radiation. The composition and the experimental determination of the transmission of this solution have already been described in this journal. It is, therefore, sufficient here to state that by its use the actual photometry of the green light is performed in this present experiment by comparison of lights of the same color, while the evaluation of the green light in lumens is given in terms of the mean value obtained by the 61 observers who measured the solution by the photometric method recommended in the previous investigations quoted.[1] It is not believed that the use of more than 61 observers, taken at random, would have altered the value obtained by one per cent.

Essential parts of the apparatus are the means for putting all parts in optical alignment and for determining the various constants. The mercury arc is furnished with a diaphragm about two centimeters wide, which is considerably smaller than the rest of the diaphragming system at B and C. In the center of this diaphragm horizontally is a wire cut off so that its point is in the vertical center. In adjusting the position of the thermopile the mercury arc is moved back to the position shown dotted; a lens, also shown dotted, is inserted, which throws an image of

[1] Ives, Astrophysical Journal, XXXVI., No. 4, Nov., 1912, p. 322; "Studies in the Photometry of Light of Different Colors," Phil. Mag., July, Sept., Nov., Dec., 1912.

the wire point on the photometer screen and on the thermopile mount (or preferably on a ground glass-screen placed in the thermopile mount). The thermopile and photometer head are in the same optical line when this image falls on the center of each. The lens is then removed and the mercury arc so placed that it is seen exactly in the line of the sighting crosslines provided in the photometer head. The thermopile mount is supplied with sighting crosshairs as well, and by their aid the thermopile is properly pointed. When these two adjustments are made the pile is correctly placed except for distance. To set it at the same distance as the photometer screen from the arc, recourse is made to a parallax adjustment. The eye is placed at *S*, where the diaphragm over the thermopile and the photometer screen (half drawn out to furnish an edge) are seen superposed. On moving the eye up and down, the two objects separate unless they are in the same plane. This adjustment, provided the mirror *D* is plane parallel, is quite delicate.

The details of these adjustments have been given at some length because the accuracy of the result is directly dependent on their perfection. In some of the preliminary work attempts were made to increase the amount of energy available by concentrating the mercury light with a lens. It was found that it was almost impossible to line up the apparatus twice alike, as shown by the different values obtained for the ratio of illumination to galvanometer deflection. These troubles entirely disappeared with the apparatus as now described.

Before describing the measurement of the various instrument constants it is advisable to describe the procedure in making a measurement. This may be divided into three parts, as follows:

1. The determination of the sensibility of the thermopile.

This is done by turning the thermopile to face the radiation standard, whose radiation is cut down to some convenient value by the disc *Q*.

2. The simultaneous measurement of the green radiation with the photometer and the thermopile.

This is done with the thermopile turned to face the arc, and with the green solution *J* in place. When the shutter *B* is opened one observer notes the galvanometer deflection, the other moves the comparison lamp *K* until, looking through the telescope *F*, a photometric match is made. The position of *K* and the corresponding galvanometer deflection are recorded.

3. The evaluation of the comparison lamp.

This is done by replacing the solution *J* by the clear water, turning on the candle-power standard *S*, placed at some convenient point, and making a photometric setting by the movement of *K*, the light from which is cut down by the sector disc M. In making this measurement there is again no color difference, and as well, the substitution method is used, eliminating the necessity for reversing the photometer head.

The complete formula used to reduce these observations is as follows:

$$\text{Lumens per watt} = \left(\frac{1}{\mu}\right)\left(\frac{T_R}{R_R}\right)\left(\frac{P_S T_R' T_S T_{TL}}{S_B}\right)\left(\frac{D_{C_1}}{D_S}\right)^2\left(\frac{\theta}{D_C^2 \times \Delta \times W}\right),$$

where

- μ = luminous efficiency of the green radiation.
- T_R = transmission of reflector D for green light.
- R_R = reflecting power of reflector D for green light.
- P_S = candle-power of standard S.
- T_R' = transmission of reflector D for "white" light.
- T_S = transmission of green solution J.
- T_{TL} = transmission of glass thermopile window for green light.
- D_{C_1} = air distance at which the comparison lamp K gives the same illumination through the clear water tank as the standard lamp S does through the reflector D when placed at the distance D.
- θ = temperature coefficient of transmission of green solution J.
- S_B = transmission of sector disc M.
- D_C = air distance at which comparison lamp K is set when a photometric match is made through the green solution.
- Δ = galvanometer deflection, when thermopile is exposed to green radiation.
- W = watts per square meter per centimeter deflection.

The last quantity (W) is obtained from the formula:

$$w = \frac{R \times S_A \times T_{TR}}{\delta},$$

where

- R = watts per square meter received from the raditaion standard.
- T_{TR} = transmission of thermopile window to radiation from standard.
- S_A = transmission of sector disc Q.
- δ = deflection of galvanometer when thermopile is exposed to standard.

The measurement of these various constants formed one of the most exacting parts of the investigation. The only measurements of unusual character were those on the reflector D. In order to measure its reflection and transmission, arrangements were made by which it could be moved to the position D', and the lamp K was so mounted that it could be turned about an axis in line with the center of the mirror, as shown by the dotted lines. Needless to say, the greatest care was taken to thoroughly clean the reflector and maintain it clean during both its measurement and its use. The measurements of transmission, reflection, etc., were all made several times, since the accuracy of the result is directly dependent upon the accuracy of these.

The values of the constants as determined and used, are as follows: $\mu = .995$ (value from Ives luminosity curve); $\mu = .985$ (value from Nutting luminosity curve); $T_R/R_R = 1/.1083$; $P_S = 10.0$; $T_R = .895$; $T_S = .0437$; $S_A = .2098$; $S_B = .05285$; $T_{TL} = .91$; $T_{TR} = .777$; $R = .88$ watt per square meter.

Using these values the working formula becomes:

$$\text{Lumens per watt} = \frac{62.2}{\mu} \times \left(\frac{D_{C_1}}{D_S}\right)^2 \times \left(\frac{\theta}{D_C^2 \times \Delta \times w}\right).$$

(*c*) *The Measurement of Radiant Power in Absolute Units.* The Thermopile and Auxiliary Galvanometer.—The thermopile used in making the radiometric measurements was of the surface type, having an area of 12 to 17 sq. mm. This area was somewhat reduced by a diaphragm placed in front of the receivers on the glass window which was used in the actual measurements. This diaphragm was of such size that its aperture was always completely filled by the thermojunction surface. The various distances were measured to it, instead of to the receiving surface four or five millimeters behind, which would have been difficult to locate in the parallax distance adjustment. The thermopile consisted of four units joined in series, with a total resistance of about 31.6 ohms.

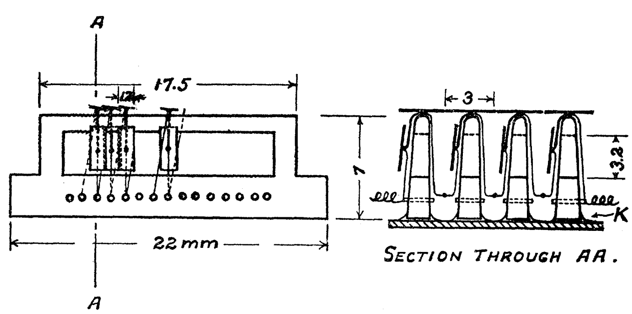

Fig. 3.

Details of Thermopile.

Each unit consisted of 15 thermocouples joined in series as shown in Fig. 3. The receivers were of tin, 1.2 × 3 mm. in area. The thermopile was completely compensated by having receivers upon the unexposed junctions, which were freely suspended in the air, thus admitting a rapid equalization of the temperature, as described elsewhere.[1] By this means drift of the zero reading is reduced to a minimum.

The pile was constructed so that the elements could be joined all (60) in series, or they could be joined one-half (30) in series parallel. When joined all in series the voltage was doubled and the deflections were considerably increased when used with the d'Arsonval galvanometer. When used with a Thomson galvanometer the most efficient combination was the one in which all four units (15 thermocouples in each) were joined in parallel. These incidental details are included here for the completeness of record.[2] The time required to produce a maximum effect upon this thermopile was about 15 seconds, when used with a Thomson galvanometer which had a complete period of about four seconds. This is somewhat longer than usually experienced, and the explanation offered is that the retardation in attaining temperature uniformity in these large receivers is due to the slowness of the heat conduction from the extreme edges.

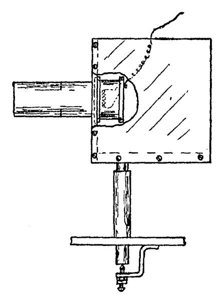

Fig. 4.
Thermopile Mounting

A glass window was used over the thermopile during the actual measurements for the determination of the mechanical equivalent, thus making the pile practically unsusceptible to drafts of air and to changes of background temperatures and other disturbances likely to be caused by the manipulation of the extensive apparatus used in the investigation. The transmission of this glass for the radiation used to calibrate the radiometer (see below), was determined by a separate measurement under the best conditions.

The mounting of the thermopile is shown in the detail sketch, Fig. 4. The protection to radiation and to convection currents is made very complete by the diaphragmed tube T and the enclosing box B, of bright tin. Still further protection is furnished by a large tin enclosing box indicated in the plan of the apparatus Fig. 2, and by the other portions of the screening system previously mentioned.

The auxiliary galvanometer used was of the d'Arsonval type, and was constructed by Leeds & Northrup. Its sensibility was 33 mm. per microvolt, its internal resistance 13.5 ohms, its external critical damping resistance 32.5 ohms and its period 7.5 seconds.

Although the thermopile resistance was very near the critical damping resistance and the thermopile was quite quick acting, it was found

[1] Coblentz, Bull. Bureau of Standards, 11, p. 131, 1914, also 9, p. 7, 1912.

[2] See fuller discussion, Bull. Bureau of Standards, 11, p. 131, 1914.

advisable to allow 45 seconds for the deflection to attain its maximum value before reading. This long period is attributable to the fact that the external resistance was not adjusted to meet the requirements for producing critical damping. In a thermopile the voltage attains about 90 per cent. of its maximum value in two seconds; while in tests on the d'Arsonval galvanometer, as ordinarily used, the maximum voltage is applied at once. The damping is no doubt different in the two cases.

In spite of the compensating construction of the thermopile there was a certain amount of slow drift (sometimes amounting to five per cent. of the deflection), due perhaps to the galvanometer. Any uncertainties due to these characteristics of the system were completely eliminated by the experimental procedure, which was as follows:

The thermopile was first exposed continuously for 15 or 20 minutes to radiation of about the value afterwards to be measured. Measurements when started were made on a time basis—the zero was read, a 45-second exposure was made, and then after another 45 seconds, the zero was again read. The zero used was the mean of the two readings. This procedure eliminated the effects of the slight drift except when this changed direction or rate during the reading, such a change, if large, being sufficient cause to discard the reading. (As it happens, during the final readings no drift change occurred which was considered of sufficient magnitude to call for discarding any readings, although the mean variation of the values would probably have been a little smaller had this been done wherever indicated.)

Another precaution taken was to keep all readings of very nearly the same magnitude. This was accomplished by the use of a sector disc over the ratiation standard. All possible errors due to slowness of deflection or variation from strict proportionality between stimulus and deflection (which there is no reason to expect) are avoided in this way.

The precision attained in making the radiometric measurements was very satisfactory, and is illustrated by the representative set of readings below, being the values obtained in the third run by method *C*.

Stimulus = .88 watt per square meter × window transmission (.777) × disc transmission (.2098) = .1432 watt per square meter.

Deflections in centimeters: 3.83, 3.84, 3.82, 3.83, 3.81.

Interval for reading on green radiation,

3.80, 3.76, 3.84,

Interval for reading on green radiation,

3.88, 3.85, 3.83, 3.82, 3.87.

The mean value is 3.83, from which watts per square meter per centimeter deflection = .03745.

The Radiation Standard.—The galvanometer scale was calibrated to give the intensity of the radiation stimulus in absolute value, by exposing the receiver of the thermopile to a standard of radiation, in the form of a seasoned incandescent lamp. This lamp had been standardized for the intensity of the radiant energy in absolute value, at a distance of two meters from the lamp, by direct comparison with the standard of radiation maintained at the Bureau of Standards.[1] This latter, maintained by a set of incandescent lamps, has been established by comparison against a black body, and also by direct measurement, in absolute value, of the energy radiated. The standard of radiation is thought to be accurate to better than 0.5 per cent. The lamp used in the present work was compared with the Bureau of Standards radiation standard before and after the completion of this research, and was found in agreement within one part in 700. The voltage and current calibration was also found in agreement, showing that the characteristics of the lamp had not changed. A further check was afforded by readings on a second radiation standard for which the transmission of the window over the thermopile was not determined. Assuming this to be the same as for the lamp used when both are at the same current, the watts per square meter per centimeter deflection were determined as .0370, to be compared with the mean value obtained from the chief standard (by many readings) of .372. The accuracy attained in the radiation measurements is believed to be quite as high as that in the photometric.

5. The Measurements.

The greater part of the time devoted to the final measurements was spent upon method *C*. This was done partly because of the independent value of the luminous equivalent of the green mercury radiation, partly because the variation in the value of the radiation from the arc with the consequent burden of simultaneous light and power measurements demanded much more attention and care. Three separate determinations were made by this method, between the first and second of which the apparatus was thrown completely out of adjustment and realigned from the start. Two determinations were made by method *B*, one immediately after the first and one after the third by the other method, using the sensibility values determined from them for the radiometer. No more were considered necessary because the possibilities for latitude in the result by this method lie not in the experimental measurements, which are extremely simple, but in the choice of luminosity curve, the measurements of the luminosity curve solution, etc.

[1] Coblentz, Bull. Bureau of Standards, 11, p. 87, 1914.

The individual readings are recorded in the tables. Under method *C* three sets of calculations are tabulated, namely, the luminous equivalent of one watt of mercury green radiation; the value of a watt of luminous flux in lumens according to the luminous efficiency ascribed to the green radiation from the Ives curve, and the same value when the Nutting curve is used.

These values are:

Lumens per watt of green mercury radiation.........................613.6
Value of one watt of luminous flux in lumens, Ives curve..............616.7
Value of one watt of luminous flux in lumens, Nutting curve...........622.2

The three sets of determinations agree to within one per cent.

Under method *B* two sets of calculations are given, one, the watt in terms of the lumen, using the Ives curve, the other, the same quantity as derived from the Nutting curve. The values are:

Value of one watt of luminous flux in lumens, Ives curve..............563.6
Value of one watt of luminous flux in lumens, Nutting curve...........613.4

The two sets of determinations were in practically perfect agreement.

For reasons given below these figures appear to be decisively in favor of the Nutting curve values. Giving equal weight to the values by the two methods, the value of the mechanical equivalent of light is:

$$1 \text{ lumen} = \frac{1}{617.8} = .00162 \text{ watt of luminous flux.}$$

6. Discussion of Results.

The Luminous Equivalent of the Green Mercury Radiation.—The value to be derived from these observations for the mechanical equivalent of light is dependent on the spectrum luminosity curve which is adopted. The value for the luminous equivalent of the green mercury radiation, on the other hand, is an independent experimental result, depending solely on the method of photometry and the value of the radiation standard. This value—613.6—is thus available for use with any luminosity curve determined by the same photometric method as that used to evaluate the green radiation as light. In view of the fact that all recently determined luminosity curves place the maximum luminosity of the equal energy spectrum close to .55 μ and give to wave-length .5461 μ an efficiency of at least 98 per cent. it appears safe to say that the mechanical equivalent of light is definitely fixed to within two per cent. by the determination of this constant.

The Agreement Between the Two Methods and Its Significance.—The agreement or disagreement of the two methods is quite independent of the radiation standard employed, and might in fact be studied with any arbitrary working standard. It is dependent upon the self consistency of the photometric method used, upon the accuracy of the luminosity curve and upon the similarity of the visual characteristics of the groups of observers who determined the luminosity curve and the transmission of the monochromatic green solution. If the same group of observers had determined the luminosity curve and the transmission of the green solution then the agreement of the two methods would constitute a test of the accuracy of the luminosity curve and of the ability of the photometric method to add luminosities arithmetically. This latter has been previously established.[2] From our experience in the measurement of the monochromatic green solution we judge it extremely improbable that two groups of 18 observers would differ in their average characteristics as much as the difference exhibited by the two luminosity curves in question. There remains then only the question of the accuracy of the determination of these luminosity curves. That curve must be decided the more accurate which gives the best agreement between the two methods. This means the Nutting curve, by which the two methods agree to one and one half per cent., while with the Ives curve there is an outstanding discrepancy of about eight per cent. This difference between the curves, as already pointed out, is chiefly a difference in their area and may probably be traced back to the indirect means employed to determine the energy distribution in the earlier research.

But while the Nutting curve appears to be more correct, by this criterion, it must not be overlooked that this evidence is not alone sufficient to decide its entire correctness. All that is shown is that the ratio of the luminous efficiencies of the green mercury radiation and the "4-watt" lamp as given by this curve is approximately correct. A whole family of curves could be constructed which would meet this test. For instance, a similar curve with its maximum slightly shifted toward the blue would assign a higher value to the luminous efficiency of the green mercury radiation, which would lower the lumen equivalent of the watt of luminous flux; but this same shift would lower the luminous equivalent of the "4-watt" lamp, with a net result that the two methods would give results in closer agreement. Again, the luminosity curve is not as well determined as the transmission of the monochromatic green solution, as only a third the number of observers were used.

The final value of the mechanical equivalent must wait until all uncertainties in the luminosity curve are removed, but, as remarked above, the value can hardly be in doubt by as much as two per cent. unless some error is present in the green-line determination.

[2] Ives, "Studies in the Photometry of Lights of Different Colors," Phil. Mag., July, Sept., Nov., Dec., 1912.

The Weight to be Given to the Two Methods.—Having decided on the use of the values derived from the Nutting curve, the question comes up of the relative weight to be given to the two methods. The precision of both sets of measurements is so good that it is believed the outstanding difference is to be ascribed to the uncertainty of the luminosity curve, perhaps to the difference between it and the curve which would be obtained from the 61 observers who measured the green solution. The change called for might affect each value or both. Thus had the measurement of the monochromatic green solution been stopped at 30 observers, the mean would have been 1½ per cent. lower (perfect agreement). Had only the first 21 been taken (Nutting's number of observers) the mean would have been nearly as much lower. It has therefore seemed permissible to give the two values equal weight, remembering that they both lie within the range that would be calculated from the luminous equivalent of the green radiation by any recent luminosity curve.

Method B.

1st Run:

Watts per cm., mean of 11 settings (mean deflection 3.83 cm.) .... .03745

Candle power of "4-watt" carbon lamp 44.89

Corrected distance, source to thermopile .... .479 meters

Corrected deflection $= \Delta'$. Ives, 1.30Δ; Nutting, 1.1975Δ.

$$\text{Lumens per meter}^2 = \frac{44.89 \times .91}{(.479)^2} = 177.5.$$

Watts per meter^2 $= \Delta' \times .03745$.

$$\text{Working formula. Lumens per watt} = \frac{177.5}{\Delta' \times .03745} = \frac{4740}{\Delta'}.$$

No.	Δ	Δ_I'	Δ_N'	Value of Watt in Lumens.	
				Ives.	Nutting.
1*B*	6.52	8.47	7.80	560	608
2*B*	6.42	8.35	7.67	568	618
3*B*	6.62	8.62	7.91	550	600
4*B*	6.51	8.48	7.78	559	609
5*B*	6.45	8.40	7.71	564	614
6*B*	6.38	8.30	7.62	571	622
7*B*	6.47	8.42	7.74	563	612
8*B*	6.45	8.40	7.71	564	614
9*B*	6.45	8.40	7.71	564	614
10*B*	6.39	8.31	7.64	570	620
				563.3	613.3 mean

Various Checks on the Order of the Magnitude of the Results.—A check on the order of magnitude of the result may be obtained by using various data on the efficiency and efficiency losses in incandescent lamps. The greater part of the power input in such lamps is transformed into radiation, and such losses as occur can be fairly closely determined. A loss occurs due to conduction of heat away through the leading-in wires and filament supports. This loss has been measured by Hyde, Cady and Worthing[1] and amounts, in the case of a carbon lamp of the oval anchored filament type, operated at 4.85 w. p. m. s. c. to between four and five per cent. in efficiency. Another loss is caused by the absorption of radiation by the glass bulb. This absorption is much greater for the long-wave heat radiation than for light. The absorbed radiation is in part carried away by convection and conduction. Drysdale[2] found by experiment that this loss amounted to two or three per cent. of the applied power. Another part of the absorbed radiation is re-directed

2d Run:

Watts per cm. (13 settings, mean value 3.83 cm.) .... .03745
Candle power of "4 watt" lamp 44.89
Corrected distance, lamp to thermopile .... .564

$$\text{Lumens per meter}^2 = \frac{44.89}{(.564)^2} \times .91 = 128.0.$$

$$\text{Watts per meter}^2 = \Delta' \times .03745.$$

$$\text{Working formula. Lumens per watt} = \frac{128}{\Delta' \times .03745} = \frac{342}{\Delta'}.$$

No.	Δ	Δ_I'	Δ_N'	Value of Watt in Lumens.	
				Ives.	Nutting.
11*B*	4.63	6.02	5.54	568	617
12*B*	4.59	5.97	5.48	573	624
13*B*	4.67	6.08	5.58	562	612
14*B*	4.71	6.13	5.63	558	608
15*B*	4.65	6.05	5.56	556	614
16*B*	4.67	6.08	5.58	562	612
17*B*	4.70	6.12	5.62	559	608
				564.0	613.6 mean

as radiation of much longer wave-length. The distribution of intensity of this radiation around the lamp will be somewhat different from that of the light. It will be more nearly spherical, with a consequent still further loss of power in certain directions, notably the horizontal. This long-wave radiation will also suffer some loss by absorption through the air. There is, therefore, a difference to be expected between the total efficiency of an incandescent lamp of this type and its radiant efficiency of probably not less than seven or eight per cent.

The radiation standard lamp used is of the type of filament just described. It matches the candle-power standards at 103.5 volts. At this voltage it gives an illumination of 2.785 lumens per square meter at

[1] Trans. Illum. Eng. Soc., 6, p. 238, 1911.
[2] Proc. Royal Soc., A, 85, 275, 1911.

Method C.

1st Run:

Temperature at beginning, 21°; at end 22.5°, mean = 21.75°................ θ =.989

W, mean of eleven readings in three sets, during and at end of run (mean value of deflection 3.83 cm.).. =.03745

$$\left(\frac{D_{c_1}}{D_s}\right)^2 = \left(\frac{.405}{.798}\right)^2 = .258$$

Working formula, $\frac{62.2 \times .258 \times .989}{\mu \times .03745 \times \Delta \times D^2}$, giving, lumens per watt

of green mercury radiation................................ = $m = \frac{424}{\Delta \times D_c^2}$,

ratio of the lumen to the watt of luminous flux, Ives curve..... = $M_I = \frac{426}{\Delta \times D_c^2}$,

ratio of the lumen to the watt of luminous flux, Nutting curve = $M_N = \frac{430}{\Delta \times D_c^2}$.

	Δ	D_c	$\Delta \times D_c^2$	m	M_I	M_N
1*C*	1.47	.702	.726	584	587	592
2*C*	1.63	.647	.683	621	625	630
3*C*	1.85	.618	.707	600	603	608
4*C*	2.91	.487	.692	613	616	622
5*C*	3.48	.4455	.691	614	617	622
6*C*	3.80	.4275	.695	610	613	619
7*C*	3.93	.421	.697	609	612	617
8*C*	4.04	.418	.707	600	603	608
9*C*	4.04	.4415	.685	619	622	628
10*C*	4.05	.413	.691	614	517	623
11*C*	4.01	.414	.687	617	620	626
12*C*	4.02	.413	.686	619	622	627
13*C*	4.10	.4115	.695	610	613	619
14*C*	3.98	.4185	.697	609	612	617
15*C*	3.92	.4155	.677	626	629	635
16*C*	4.05	.417	.705	602	605	610
17*C*	3.91	.4195	.688	616	619	625
18*C*	3.95	.416	.684	620	623	629
19*C*	3.96	.4165	.687	617	620	626
20*C*	3.98	.4185	.698	607	610	616
				611.4	614.4	619.9 mean

two meters' distance. It also gives .975 watt per square meter at this distance. Hence its radiated lumens per watt = 2.858. The lumens per watt input = 2.597. The efficiency loss is therefore nine per cent. of the order of magnitude indicated by the considerations above. This measurement gives a check merely on the radiation standard. A check on the value for the mechanical equivalent is obtained by a supplementary measurement of luminous efficiency.

The large point-source carbon lamp at "4-watt" color was measured for radiant luminous efficiency by determining the ration of the radiant power to the radiant power transmitted by the luminosity curve solution (the latter being, according to definition, luminous flux).[1]

Correcting for the actual transmission curve of the solution as compared with the Nutting curve, the radiant luminous efficiency was found to be .0045. Now .0045 times 417.7 is 2.78 = radiated lumens per watt. The lumens per watt consumption = 2.59. From this the efficiency

2d Run:

Temperature throughout 23°...θ =.982

W, mean of 8 settings in two groups (mean deflection 3.88 cm.)................ =.0366

$$\left(\frac{D_{c_1}}{D_s}\right)^2 = \left(\frac{.406}{.798}\right)^2 = .259.$$

Working formulas:

$$m = \frac{432}{\Delta \times D_c^2},$$

$$M_I = \frac{434}{\Delta \times D_c^2},$$

$$M_N = \frac{438.5}{\Delta \times D_c^2}.$$

	Δ	D_c	$\Delta \times D_c^2$	m	M_I	M_N
21*C*	3.43	.456	.714	607	610	614
22*C*	5.41	.360	.702	616	619	625
23*C*	5.88	.342	.688	629	632	637
24*C*	6.03	.338	.690	627	630	635
25*C*	6.12	.340	.708	611	614	619
26*C*	5.99	.346	.717	603	606	611
27*C*	6.06	.3395	.699	619	622	627
28*C*	5.84	.345	.696	622	625	630
29*C*	5.91	.3455	.706	612	615	621
30*C*	5.98	.3465	.719	602	605	609
				614.8	617.8	622.8 mean

loss is seven per cent.—a satisfactory check with the other values. This check is of course practically method *B*, except that the power transmitted by the luminosity curve solution is obtained indirectly by two separate experiments.

The Reproducible Character of the Measurements Given.—A feature of the work here reported, which is believed worthy of emphasis, is that unlike most previous measurements of a similar nature, every element entering into the result may be copied and checked by other observers. The determination is, in short, of a strictly reproducible physical character. This is made possible by recording the factors most difficult to measure in material standards of reproducible or maintainable form.

[1] A set of determinations of luminous efficiency have recently been made in this manner by Karrer, Physical Review, p. 189, Vol. V., N. S., No. 3, 1915.

Thus the difficult measurement of green light is recorded in the reproducible green solution. The measurement or radiation in absolute value is confided to long-lived incandescent lamps. If in the future either the standard of radiation is changed, or the photometric method here employed is superseded, the ratio of the new to the old value can be applied directly to the value obtained in this investigation without the necessity for repeating the whole piece of work.

The chief uncertainties in the result are on the physiological side,

3d Run:

Mean temperature 21.5°.. θ = .991

W, mean of 13 settings (mean deflection 3.83 cm.)........................ = .03745

$$\left(\frac{D_{c_1}}{D_s}\right)^2 = \left(\frac{.404}{.798}\right)^2 = .256.$$

Working formulas:

$$m = \frac{421}{\Delta \times D_c^2},$$

$$M_I = \frac{423}{\Delta \times D_c^2},$$

$$M_N = \frac{427.5}{\Delta \times D_c^2}.$$

	Δ	D_c	$\Delta \times D_c^2$	m	M_I	M_N
31C	3.11	.4675	.680	620	623	629
32C	4.14	.405	.680	620	623	629
33C	4.68	.380	.677	622	625	632
34C	4.75	.381	.690	610	613	619
35C	4.76	.3785	.682	618	621	627
36C	4.62	.384	.681	619	622	628
37C	4.54	.387	.681	619	622	628
38C	4.61	.386	.687	614	617	622
39C	4.64	.386	.692	609	612	618
40C	4.56	.386	.680	620	623	629
				617.1	620.1	626.1
Mean of all observations (c)............				613.6	616.7	622.2

which is here relegated to entirely independent investigations. A general agreement on photometric methods, a definite answer to the question: "What is light?" by the establishment of a representative average eye spectrum luminosity curve—there lies the work of the future. When that is completed the chief uncertainty of the present work can be removed. This uncertainty is however believed to be quite small.

7. Summary.

The subject matter of this paper may be summarized as follows:

1. Rational definitions have been given for light quantities. In

accordance with these the mechanical equivalent of light is defined as the value of the lumen in watts.

2. An experimental determination by two different methods gives for the mechanical equivalent of light a mean value of 0.00162 watt per lumen.

The discussion of the significance and importance of this quantity may be brief by reason of the full discussion in the various publications to which reference has been made. Suffice it to say that here luminous flux, on the basis of the accepted definition, can be measured in C.G.S. units, *e. g.*, in watts, and that consequently the watt is a rational standard of luminous flux ("primary standard of light").

The measurement of luminous flux in watts and the establishment of the watt as the standard are dependent on the evaluation of the present standard and units in terms of the watt. The mechanical equivalent of light is therefore the most fundamental quantity in the establishment of light measurement on a physical basis.

Physical Laboratory,
The United Gas Improvement Company,
Philadelphia,
December, 1914.