THEORY AND USE OF THE MOLECULAR GAUGE.

BY SAUL DUSHMAN.

SOME time ago Dr. I. Langmuir described the construction of a "molecular" gauge for the measurement of very small gas pressures.[1] At the suggestion of Dr. Langmuir the writer undertook a more detailed study of the theory and use of the instrument, and the following paper contains the results of a number of measurements that were carried out with the aid of this gauge.

THEORETICAL.

If a plane is moving in a given direction with velocity u relatively to another plane situated parallel to it at a distance d, there is exerted on the latter a dragging action whose magnitude may be calculated from considerations based on the kinetic theory of gases.

At comparatively higher pressures where the mean free path of the gas molecules is considerably smaller than the distance between the plates, the rate of transference of momentum across unit area is given by the equation

$$B = \frac{\eta u}{d}, \qquad (1)$$

where η denotes the coefficient of viscosity.

According to the kinetic theory of gases this coefficient ought to be independent of the pressure. The confirmation of this deduction over a very large range of pressures has been looked upon as one of the most striking arguments for the validity of the assumptions on which the kinetic theory of gases is based.

It was found, however, by Kundt and Warburg,[2] that at very low pressures, where the mean free path of the molecules becomes of the same order of magnitude as the distance between a moving and stationary surface placed in the gas, there is distinct evidence of a slipping of gas molecules over the planes. The amount of this slip was found to be inversely proportional to the pressure.

[1] PHYSICAL REVIEW, *1*, 337 (1913). See also abstract, PHYS. REV., *2* (1913).

[2] Pogg. Ann., *155*, 340 (1875). Poynting and Thomson, Properties of Matter, p. 220.

Denoting the coefficient of slip by δ, it may be defined by the relation

$$v_g - v_s = \frac{\eta}{\zeta}\frac{\partial v}{\partial x} = \delta \cdot \frac{\partial v}{\partial x}, \tag{2}$$

where v_g = velocity of gas molecules at the surface,
v_s = velocity of surface,
ζ = coefficient of external viscosity.

It follows from hydrodynamical considerations that the amount of momentum transferred per unit area is

$$B = \frac{\eta u}{d + 2\delta}. \tag{3}$$

Thus, owing to slip there is an apparent increase in the thickness of the gas layer between the two surfaces. This increase amounts to $\delta = \eta/\zeta$ for each surface.

Experiments on the conduction of heat at low pressures led to similar observations in this case. According to the kinetic theory the heat conductivity should be independent of the pressure. Accurate determinations showed that at very low pressures the conductivity apparently decreases. This led to the conception that at the surface there occurs a very steep temperature gradient (Temperatursprung) so that the amount of heat, Q, conducted between two surfaces maintained at temperatures T_1 and T_2 is given by

$$Q = \frac{T_1 - T_2}{d + 2\gamma}, \tag{4}$$

where d is the distance between the two plates and γ represents the apparent increase in thickness of the layer of gas at each surface.

The definition of γ may be expressed by the following relation, which by its analogy with equation (2) helps to exhibit the complete parallelism of the phenomena observed in both the case of heat conduction and that of viscosity effect. Denoting the drop in temperature at the surface by ΔT, and the temperature gradient there by $\partial T/\partial x$, the definition of γ follows from the relation

$$\Delta T = \gamma \cdot \frac{\partial T}{\partial x}. \tag{5}$$

An interpretation of this temperature drop on the basis of the kinetic theory of gases was first advanced by Maxwell and subsequently developed still further by Smoluchowski.[1]

It is assumed that of the molecules striking a heated surface only a fraction f is absorbed and then emitted with an average kinetic energy

[1] Ann. Phys., *35*, 983, where references to previous literature are given.

corresponding to that of the surface. The remainder $1 - f$ is reflected according to the laws of elastic collision. If T_1 denote the temperature of the molecules striking the surface, T_2, the temperature of the latter, and T_2^1 the temperature of the molecules leaving the surface, then:

$$T_2^1 - T_2 = (1 - f)(T_1 - T_2). \tag{6}$$

The constant f is known as the coefficient of equalization (Smoluchowski) or accommodation (Knudsen),[1] for it is evident that f is unity when the average temperature of the molecules leaving the heated surface corresponds to the temperature of the latter.

In consequence of this lack of complete equalization of temperatures, there is produced an apparent temperature drop at the surface, which is related to the coefficient of equalization by the following equation:

$$\gamma = \frac{2 - f}{f} \cdot \frac{15}{4\pi} L, \tag{7}$$

where L is the mean free path of the molecules at the given pressure.

The same method of interpretation was extended to the case of transference of momentum from one surface to another at very low gas pressures. Of the molecules striking a moving surface a portion β is "absorbed" and then "emitted" with velocities that range according to Maxwell's distribution law. The direction of emission is perfectly independent of the direction of incidence. On the other hand, the fraction $1 - \beta$ is "reflected" according to the laws of elastic collision.[2]

A number of investigators have concerned themselves with the mode of determination of these coefficients β and f, which may be designated as the coefficients of accommodation for viscosity and heat conduction respectively.

Knudsen,[3] who has carried out a large number of investigations on the behavior of gases at very low pressures, concludes that while the value of this coefficient is less than unity for heat conduction (and differs with the nature of the gas) it is equal to unity in all those cases where transference of momentum is concerned. He assumes, in other words, that all the molecules are emitted from a moving surface in directions which are absolutely independent of the original directions of incidence and that these molecules then obey Maxwell's law of distribution of velocities.

Timiriazeff[4] makes the assumption that the coefficient of accommodation has the same value, both for viscosity measurements and for the

[1] Ann. Phys., *34*, 593 (1910).

[2] A. Timiriazeff, Ann. Phys., *40*, 978 (1913).

[3] Ann. Physik, *28*, 75 (1908); *31*, 205 (1909); *33*, 1435 (1910); *34*, 593, 823 (1911); *35*, 389 (1911); *36*, 871 (1911).

[4] *Ann. Physik*, *37*, 233 (1912).

determination of heat conduction in gases at low pressures and deduces from this assumption the relation

$$\delta = \frac{8}{15}\gamma \tag{8}$$

where δ has the significance assigned to it in equation (3) above.

From equations (7) and (8) it follows that

$$\delta = \frac{2}{\pi} \cdot \frac{2-f}{f} \cdot L. \tag{9}$$

More generally, we can write

$$\delta = a \cdot L$$

where a is a constant whose exact value depends upon the particular assumptions made regarding the value of the accommodation coefficient.

At extremely low pressures, where L is large compared to d, equation (3) reduces to

$$B = \frac{\eta u}{2aL} \tag{10}$$

or, since

$$\frac{\eta}{L} = 0.31 p \sqrt{\frac{8M}{\pi RT}},$$

$$B = \frac{2 \times 0.31}{a} pu \sqrt{\frac{M}{2\pi RT}}. \tag{11}$$

Substituting for a the value deduced by Timiriazeff, see equation (9), it follows that

$$B = \frac{f}{2-f} \times 0.31 \pi u p \sqrt{\frac{M}{2\pi RT}}. \tag{12}$$

A relation of the same form as this may also be deduced by means of considerations similar to those used by Knudsen. This method of derivation has the advantage that it does not involve any extrapolation of equation (3), but starts from fundamentally different premises.

At very low pressures, the mass of gas striking unit area of a surface per unit time is equal to

$$\frac{1}{4}\rho\Omega = p\sqrt{\frac{M}{2\pi RT}}$$

where

ρ = density of gas,

Ω = average (arithmetical) velocity.

Assuming, as Knudsen does, that the coefficient of accommodation is

unity, it follows that the rate of transference of momentum per unit area from a surface moving with velocity u is

$$B = up\sqrt{\frac{M}{2\pi RT}}. \tag{13}$$

Equations (12) and (13) agree in the conclusion that at very low pressures B is proportional to $p\sqrt{M/RT}$.

According to Gaede,[1] Knudsen's assumption that the accommodation coefficient is equal to unity in the case of viscosity measurements at very low pressures is justified only at pressures below about 1.33 bars (.001 mm. of mercury). In a very recent paper, Baule has discussed the work of Smoluckowski, Knudsen, Timirazeff and others in detail[2] and by introducing some very plausible assumptions as to the actual mechanism by which a gas molecule exchanges energy with a molecule of the surface against which it strikes, he arrives at the relation

$$B = up\left(\frac{(1 - \alpha'v)}{(1 + \alpha'v)}\right)\sqrt{\frac{M}{2\pi RT}}, \tag{14}$$

where $\alpha'v$ is a function of the masses and diameters of the molecules of the gas and solid, and the distances between the molecules in the plane surface. It is thus evident that no two writers are agreed upon the manner in which the coefficient f is to be calculated.

We are, however, justified in concluding that there exists a relation between B and $p\sqrt{M/RT}$ of the general form

$$B = kup\sqrt{\frac{M}{RT}}, \tag{15}$$

where k is a *constant* whose value depends upon the nature of the gas and that of the surface with which it is in contact.

This is the fundamental relation upon which is based the construction of the "molecular gauge" described in the following section.

DESCRIPTION OF GAUGE.

The construction of the gauge is shown in Fig. 1. It consists of a glass bulb B in which are contained a rotating disc A and, suspended above it, another disc C. The disc A is made of thin aluminum and is attached to a steel or tungsten shaft mounted on jewel bearings and carrying a magnetic needle NS. Where the gauge is to be used for measuring the pressure of corrosive gases like chlorine, the shaft and disc may be made of platinum. The disc B is of very thin mica, about

[1] Ann. Physik, *41*, 289.

[2] B. Baule, Ann. d. Physik, *44*, 145 (1914).

.0025 cm. thick and 3 cm. in diameter. A small mirror, *M*, about 0.5 cm. square is attached to the mica disc by a framework of very thin aluminum. This framework carries a hook with square notch which fits into another hook similarly shaped, so that there is no tendency for one hook to turn on the other. The upper hook is attached to a quartz fiber, about 2×10^{-3} cm. diameter, and 15 cm. long.

"The lower disc can be rotated by means of a rotating magnetic field produced outside the bulb. This field is most conveniently obtained by

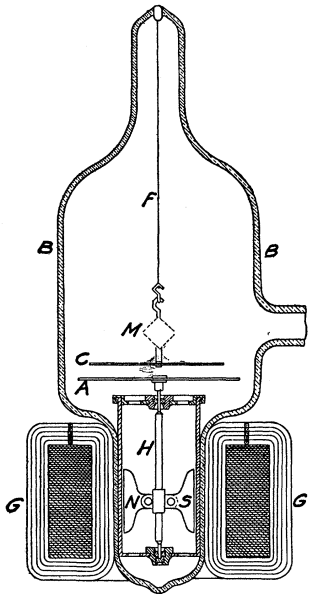

Fig. 1.

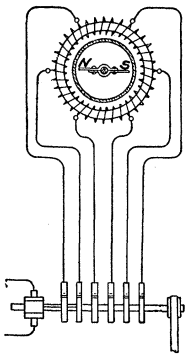

Fig. 2.

Rotating Commutator and Connections to Gramme Ring *G–G* of Fig. 1.

a Gramme ring (*GG*) supplied with current at six points from a commutating device run by a motor (see Fig. 2). In this way the speed of the motor determines absolutely the speed of the disc, since the two revolve in synchronism. The speed of the disc may thus be varied at will from a few revolutions per minute up to 10,000 or more."

In constructing the gauge, the lower part of the bulb is made to fit an aluminum spring holder which supports the spun aluminum cylinder. The latter contains the upper and lower jewel bearings on which the shaft (*H*) rotates. The bulb is then cut across the widest portion (at *BB*) and the two discs are introduced, care being taken to see that the framework which carries the mirror is not bent during the subsequent re-sealing of the two parts. The quartz fiber is threaded through the

hook and a little glass bead attached on the lower end, while the upper end is fastened to a platinum wire by means of sealing-in glass. The last operation consists in "fishing" for the mica disc by means of the hook on the quartz fiber, and after the distance between the two discs has been adjusted, so that the upper disc hangs centrally over the lower disc in a perfectly horizontal plane at a distance of less than 1 cm., the glass at the top of the bulb is closed up around the platinum wire.

One of the great advantages of the gauges as constructed in the above manner is *the complete absence of any parts that cannot be heated up to a temperature of about 300° C. No cement, shellac or other source of vapor should be used in attaching the mirror or the quartz fiber.*

CALIBRATION OF GAGE.

Let r = radius of rotating disc,

ω = angular velocity of rotating disc,

α = angle of torque of upper disc,

D = "Direktions-kraft" on upper disc

$$= \frac{\pi^2}{t^2} K,$$

where K = moment of inertia of upper disc,

t = period of oscillation.

From equation (15) it follows that the momentum transferred per unit time to upper disc is

$$B_0 = \int_0^r \omega r^2 \cdot 2\pi r dr \cdot kp \sqrt{\frac{M}{RT}}$$

$$= \frac{k\pi r^4 \omega p}{2} \cdot \sqrt{\frac{M}{RT}} \qquad (16)^1$$

$$= \alpha D = \frac{\alpha \pi^2}{t^2} \cdot K.$$

Consequently

$$\alpha = \left(\frac{kt^2 r^4}{2\pi_k}\right) pw \sqrt{\frac{M}{RT}}. \qquad (17)$$

Hence the torque on the upper disc is proportional to the product of the speed of rotation of the aluminum disc and the function $p\sqrt{M/RT}$.

Upon this equation depends the use of the instrument as a sensitive vacuum gauge.

[1] This equation is only rigorously true if the diameter of the rotating disc is very large compared with that of the upper disc, so that errors due to "edge effect" are avoided.

By properly designing the dimensions of the discs it is evident that equation (17) could be used for *very accurate determinations of the value of k*. In this manner the conclusions of Knudsen, Smoluckowski and Baule on the correction for slip could readily be tested. As the present investigation was carried out mainly with the view of determining the utility of the instrument as a gauge, no such accuracy was attempted so that definite conclusions could not be drawn regarding the value of k. In each case the gauge was calibrated at pressures of about .001 to .01 mm. of mercury against a McLeod gauge. The following data give, however, some idea of the degree of sensitiveness to be expected (and actually obtained) from a gauge constructed on the above principles.

For this particular gauge, the weight of the mica disc was 0.1 gm., $r = 2$ cm., $t = 12$ seconds.

Consequently

$$K = \tfrac{1}{2} W^2 = \tfrac{1}{2} \times 0.1 \times 4 = 0.2.$$

Assuming a speed of 1,000 r.p.m.,

$$\omega = \frac{2\pi}{60} \times 1{,}000.$$

In the case of air at a pressure of *1 bar*[1] and 300° Abs., it is found by making the proper substitutions in equation (17) and assuming $k = 1/\sqrt{2\pi}$ that

$$\alpha_{\text{calc.}} = 150^\circ \text{ per bar.}$$

By illuminating the mirror and using a similar arrangement to that used for galvanometers, it is possible to detect a deflection of 1 mm. at a distance of 50 cm. or

$$\frac{1}{500} \times \frac{180}{\pi} \times \frac{1}{150} \text{ bar} = 0.8 \times 10^{-3} \text{ bar.}$$

Increasing the speed to 10,000 r.p.m. increases the sensitiveness tenfold and under these conditions it ought, therefore, to be possible to measure a pressure of about 10^{-4} bar.

CORRECTION FACTORS.

In using the instrument there are, however, several points regarding which special care ought to be taken.

1. *Correction Due to Eddy Currents in Metal Parts of Mica Disc.*—

[1] In accordance with most recent practice, we have adopted in this paper as unit of pressure 1 dyne per cm.^{2}. This is known as a *bar*. The relation between this unit and the conventional unit ($1\mu = 10^{-3}$ mm.) is very simple. For all purposes the relation $1\mu = 4/3$ bar is accurate enough. The exact relation is that 1 micron of mercury at 45° latitude and sea-level is equal to 1.01327/.76 = 1.33325 bar.

Owing to the rotation of the magnetic field produced by the Gramme ring, eddy currents are set up in the metal framework used to hold the mirror on the mica disc. Denoting the current through the commutator and Gramme ring by i, the torque actually produced on the upper disc may be expressed as additively composed of two terms, one due to the gas molecules from the rotating disc, and the other due to eddy currents in the metal parts of the upper disc. Consequently, equation (17) assumes the form

$$\alpha = \left(\frac{kt^2r^4}{2\pi K} p\omega \sqrt{\frac{\overline{M}}{RT}} \right) + k_1 i^2 \omega, \qquad (18)$$

where k_1 is a constant for the gauge.

The magnitude of the correction term may be diminished by using metal parts whose electrical resistance is very high and by placing the Gramme ring at a greater distance below the upper disc.

On the other hand, there is really no need for any metal parts whatever in connection with the upper disc. The mirror could be supported in a mica or glass holder, and where extreme accuracy is desired such a construction could no doubt be worked out in detail. For ordinary purposes where it is desired to measure pressures that are not less than 0.001 bar, a framework of thin aluminum wires for holding the mirror introduces no measurable errors.

2. *Synchronism.*—From the construction of the apparatus, it is evident that the aluminum disc rotates five times as fast as the magnetic field. In order to maintain the disc and commutator in synchronism, a rotating sector with five slots in it may be attached to the commutator so as to enable the operator to view a mark on the aluminum disc which should obviously appear to remain stationary if the two are in synchronism. An equally good check is to take readings of the deflection at different speeds. If the speed of the commutator is increased very slowly, there is no difficulty in maintaining the disc and commutator in synchronism.

4. *Relative Position of Discs.*—At a pressure of 1 bar and ordinary temperatures, the mean free path for air is about 10 cm. Consequently, in order that equation (17) should be valid at this pressure, the discs ought to be placed at a distance of less than 1 cm. apart. Care should also be taken to see that the upper disc is located centrally over the lower one. Regarding which disc should be the larger, the following considerations are of interest. In the operation of the gauge there is always a tendency for the upper disc to start swinging or at least get away from its symmetrical position with respect to the lower disc. If the latter is large compared to the mica disc, there obviously results a much greater torque on one side of the disc than on the other and the tendency to

swing is increased until finally the disc hits the walls of the bulb. As the damping at low pressures is very feeble, it is very difficult to stop this oscillation when once started, except by imparting to the bulb itself an opposing motion by hand. After a little experience it is easy in this manner to stop any tendency for the disc to vibrate.

Where it is intended to use the instrument as an absolute gauge or for the determination of k, it is obviously necessary to have the rotating disc much larger. On the other hand, for most purposes, that is where the instrument can be calibrated against say a McLeod gauge at pressures above 1 bar, and used to extrapolate the indications of the latter for very low pressures, it is more advantageous to have the upper disc larger since, in this manner, the tendency to swing is diminished considerably. It must be remembered, however, that as the area of the upper disc is increased beyond that of the lower disc the sensitiveness is decreased.

EXPERIMENTAL.

1. *Preliminary Experiments.*—The gauge used in these experiments contained a much heavier mica disc (weight about 0.5 gm.) and a phosphor-bronze suspension similar to those used in galvanometers. The deflection was determined directly by noting the position of a mark on the mica disc with respect to a circular scale outside the bulb. The molecular gauge was connected in series with a liquid air trap to a Gaede mercury pump and ordinary McLeod gauge.

The following data show that the deflections observed are proportional to the rate of rotation of the aluminum disc. Under p is given the pressure in bars; under r, the rate of rotation in r.p.m., and under A the deflection in degrees. The fourth column gives $D = (A/r) \times 1{,}000$, while the last cloumn gives $D_0 = (A/r) \times 1{,}000/p$, that is, the *deflection per bar at 1,000 r.p.m.*

TABLE I.

p	r	A	D	D_0
0.97	850	18	21	21.3
	1,200	28	23	23
	2,000	44	22	22.5
5.74	1,050	80	76	13.5
	1,750	120	70	12

The reason for the larger value of D at low pressures is probably due to the presence of water-vapor and other condensible gases in the gauge, as the bulb had not been previously baked out.

After allowing dry air to enter the system until the pressure was over

5 mm. of mercury, readings were taken of both the McLeod and molecular gauges, as the pressure was decreased by pumping.

The results of the observations are recorded in Table II.

TABLE II.

p	D	D_0
1,707	990	0.6
960	870	0.9
540	840	1.6
304	820	2.7
171	745	4.0
96	655	6.8
55	490	9
31	360	11.6
24.5	280	11.4
14.0	180	12.8
8	106	13.5
4.7	67	14.3
3.5	46	14.3

It will be observed that up to about 20 bar the deflection was proportional to the pressure. At this pressure the mean free path in air is about 0.5 cm., and this was about the distance between the two discs.

2. *Vapor Pressures of Mercury and Ice.*—For the observations recorded in this section, the sensitiveness of the molecular gauge used was such that $D_0 = 9°$ corresponding to 180 mm. on scale.

The gauge was baked out for one hour at 330° C. and observations then taken on both the McLeod gauge and the molecular gauge under different conditions.

TABLE III.

Press. in McLeod.	D	Press. in Mol. Gage (Calc.).	Remarks.
0.49 bar	23°.7	2.6 bars	Liquid air on trap. Pump not exhausting.
0.8	105.	11.2	Removed liquid air.
0.27	36.4 mm.	0.27	Liquid air on trap and pump exhausting.
1.33	84°	9.3	Removed liquid air.
0.27	36.4	0.27	Liquid air on trap.
0.033	4.5	0.033	Liquid air on trap and pump exhausting.
Removed liquid air.		Stopped exhausting.	
	170 mm.	1.26	At end of 1 minute.
	100°	11.3	At end of 8 minutes.
Put on liquid air again			
	4.5 mm.	0.033	At end of 5 minutes.

It will be noted that the molecular gauge followed changes in pressure which were altogether lost as far as the McLeod was concerned. The

pressure of about 10 bar observed on removing the liquid air is evidently due to mercury vapor and non-condensible gases. Allowing about 1 bar for the pressure of the latter (indicated on McLeod) it follows that the pressure due to mercury vapor alone was about 9 bar at room temperature (298° Abs.).

According to Smith and Menzies[1] the vapor tension of mercury between 20° and 30° C. is as follows. (The pressure in bar was obtained by multiplying the pressure in mm. by $4/3 \times 1{,}000 \times \sqrt{200/28.8}$.)

Temperature.	Press. in Mm.	Press. in Bar.
20°	.0013	4.57
24	.00183	6.58
28	.00254	8.92
30	.00299	10.5

The determination of the vapor pressure of mercury as given above is in fair accord with the data for 28°–30° C.

A determination was also made of the vapor tension of ice at − 78° C. A bath of acetone with solid carbon dioxide was put around the liquid air trap and the pressure in the gauge measured while the pump was exhausting. The average of three determinations was 0.9 bar. Allowing for the difference in the temperature of gauge and liquid air trap, the pressure in the latter must have been $0.9\sqrt{195/298} = 0.78$ bar.

Extrapolating from the data given by Scheel and Heuse[2] for the vapor tension of ice at temperatures down to − 68° C., and allowing for the difference in molecular weight of air and water vapor, the pressure at − 78° C. is calculated to be about 0.2 bar. The pressure due to non-condensible gases was not over .05 bar in the above measurement.

3. *Calibration of the Gauge with Hydrogen.*—The theoretical conclusion that the indications of the gauge at constant pressure ought to vary with the square root of the molecular weight was tested by introducing hydrogen into the gauge instead of air.

The gauge used gave a deflection, with air, of 135 mm. per bar at 1,000 r.p.m. The sensitiveness with hydrogen should therefore have been $135 \times \sqrt{2/28.8} = 35$ mm. per bar at 1,000 r.p.m. The actual experiments gave values ranging from 37 to 42 mm.; the discrepancy being probably due to the presence of small quantities of air in the hydrogen used.

4. *Pressure in Tungsten Lamp.*—An ordinary 60-watt type Mazda lamp bulb was connected to a molecular gauge and after exhausting

[1] Jour. Am. Chem. Soc., *32*, 1447 (1910).

[2] Ann. Phys., *29*, 723 (1909).

them for one and a half hours at 250° C. they were sealed off and pressure observations taken at intervals during the life of the lamp.

The gauge used had a sensitiveness of 1,100 mm. per bar at 1,000 r.p.m. After sealing off and before lighting the filament, the pressure indicated was about 0.8 bar, but as soon as the filament was lighted the pressure decreased and inside of less than an hour it went down to below 10^{-3} bar. The lowest pressure was certainly well below 5×10^{-4} bar, and this determination may be regarded as an upper limit of the probable pressure in a tungsten lamp. It must be noted that the lamp used in these measurements was not given nearly as good a heat treatment as in the usual lamp exhaust.

The fact that the vacuum in a tungsten lamp improves when the filament is lighted has been known for some time, and the causes of this "clean-up" effect have been discussed in a number of papers published during the past three years by I. Langmuir.[1]

As the volume of the gauge was only slightly greater than that of the lamp bulb, we can conclude that the pressure obtained in a well-exhausted tungsten lamp, when the filament is lighted, is certainly well below 10^{-3} bar.

5. *Experiments with the Gaede Molecular Pump.*—A Gaede molecular pump[2] was run in series with an oil pump which in turn was connected to the "rough vacuum" line. A McLeod gauge was inserted between the oil pump and molecular pump in order to read the pressure on the rough side of the latter.

A liquid air trap was arranged between the gauge and the pump so that the diffusion of vapor of stopcock grease or of water could be prevented.

The following table shows the results obtained under different conditions:

No.	Press. on Rough Side of Molecular Pump.	Press. on Fine Side, Read by Molecular Gauge.	Conditions of Experiment.
1	13.3 bar	0.20 bar	After exhausting for 1 hour. No heating of gage; no liquid air.
2	13.3	0.09	Put on liquid air.
3	13.3	0.033	Heated gage to 300° C. for 1 hour, but did not heat glass tubing between gage and pump.
4	1,333	0.033	Let in dry air on rough side. Press. on fine side remained constant. Ratio = 40,000:1.
5	20,000	0.4	Let in more air on rough side. Ratio = 50,00:1.
6	20	$\ll$0.0007	Ratio 30,000:1.

[1] J. Am. Chem. Soc., *35*, 107 (1913), et sub.

[2] W. Gaede, Ann. Physik, *41*, 337 (1913).

It is evident from experiment (3) that heating the gauge alone was not sufficient to reduce the pressure in it owing to the constant diffusion of water-vapor from the tubing between the gauge and liquid air trap. But after this tubing had been also heated to 330° C. (experiment 6) the pressure in the gauge went down below 7×10^{-4} bar.

The sensitiveness of the gauge employed was such that 1 bar gave a deflection of 525 mm. at 1,000 r.p.m. At very low pressures a correction term had to be introduced for the eddy current effect in the framework of the mirror. The equation connecting pressure (p) and deflection (D) may be written in the form

$$D = \frac{525}{1000} p.r. + ki^2r, \tag{19}$$

where r = revolutions per minute, and k is a constant. See equation (18).

By noting D for different values of r and i, while p is maintained constant, it is possible to determine the value of k, and hence introduce the proper correction into the calculation for p. A special series of experiments showed that in the case of the above gauge, the equation for calculating p was of the form:

$$p = \frac{D}{0.525r} - 29 \times 10^{-5} i^2.$$

The value of i varied from 3 to 5 amperes. It is evident that the presence of this eddy current effect limited the sensitiveness of the gauge, for even at zero pressure, the deflection at 4 amperes and 10,000 r.p.m. would be 24 mm. At 7×10^{-4} bar, the deflection at 10,000 r.p.m. and 4 amperes would be 27.6 mm.

Under the best vacuum conditions, that is, using liquid air, and heating the gauge and all connecting parts to over 330° C. for about one and a half hours or longer, the deflections actually obtained were only slightly greater (1 or 2 mm. more) than the correction due to the eddy current effect. Allowing for experimental errors and for the difficulty in reading to an accuracy of 2 mm. when the disc was rotated at very high speeds, it is probably correct to conclude that the vacuum obtained was less than 7×10^{-4} bar. Assuming the ratio of 50,000 : 1 as holding down to the very highest vacuum conditions, the vacuum attained in the gauge with a pressure of 20 bar on the rough side should have been 4×10^{-4}. The experimental observations are in satisfactory agreement with this calculation.

It is worth noting in this connection that in his paper describing the construction of the molecular pump, Gaede states that (at 8,200 r.p.m.)

with a rough pump pressure of 1 mm. he obtained a pressure of .02 μ on the fine pump side corresponding to a ratio of 50,000 : 1. Both pressures were read by means of McLeod gauges.

OTHER VACUUM GAUGES.[1]

In this connection it might not be amiss to mention briefly some of the other vacuum gauges that have been suggested for the measurement of pressures below 1 bar.

1. The radiometer has been used by a large number of investigators. Dewar has stated the case for this instrument as follows:[2] "The radiometer may be used as an efficient instrument of research for the detection of small gas pressures. For quantitative measurements the torsion balance or bifilar suspension must be employed."

Some years ago Mr. W. E. Ruder, of this laboratory, developed a method of using the radiometer for the measurement of the gas pressure in incandescent lamps. "It was found that when exhausted to the degree required in an incandescent lamp the radiometer could not be made to revolve, even in the brightest sunlight. In order to get a measure of the vacuum, the radiometer vanes were revolved rapidly by shaking the lamp and the time required to come to a complete stop was therefore a measure of the resistance offered to the vanes by the gas, together with the frictional resistance of the bearing. The latter quantity was found to be so small in most cases that a direct comparison of the rates of decay of speed of the vanes gave a satisfactory measure of the degree of evacuation. In this manner a complete set of curves was obtained which showed the change in vacuum in an incandescent bulb during its whole life and under a variety of conditions of exhaust.

"The chief objections to this method of measuring vacua were the difficulty in calibrating the radiometer and the difference in frictional resistance offered by differnet radiometers. For *comparative* results, however, the method was entirely satisfactory."[3]

2. Scheele and Heuse[4] devised a manometer which has been used successfully for the accurate determination of the vapor pressures of mercury and ice at very low temperatures. This gage consists of two chambers separated by a copper membrane. One of the chambers is

[1] A good description of some of the gages mentioned in this summary is given in K. Jellinek's recently published "Lehrbuch der Physikalischen Chemie," I, 1.

Shortly after this paper was sent to the printer, a description of a modified Knudsen manometer was published by J. W. Woodrow, Phys. Rev. 4, 491 (1914). The sensitiveness of this gage is stated to be about 4×10^{-5} bar.

[2] Proc. Roy. Soc., *A*, *79*, 529 (1907).

[3] This account was kindly prepared by Mr. Ruder at the request of the writer.

[4] Ber. d. deutsch. phys. Ges., 1909, 1–13.

maintained at constant pressure while the other is connected to the system under investigation. The membrane presses on a glass plate and the variation in the thickness of the film is measured by noting the number of interference bands.

3. The McLeod gauge can be constructed so that it is sensitive to 0.01 bar. Its field of application is however necessarily limited.

4. Pirani[1] has suggested a resistance manometer which depends upon the fact that at low pressures the heat conductivity of gases is a function of the pressure. In consequence of this change in the heat conductivity, the apparent resistance of the wire changes with pressure of the gas surrounding it. The method has been improved by C. F. Hale[2] and the manometer has been found to give reliable results down to 0.00001 mm. of mercury, that is to about 0.01 bar.

5. Very recently W. Rohn has described a vacuum meter based on almost the same principles.[3] In this case the effect on the thermo-electromotive force of varying gas pressure is used as a method of determining very low gas pressures. The instrument is most sensitive between about 100 and 1 bar (0.075 mm. and 0.00075 mm. of mercury) the electromotive force varies approximately linearly with the logarithm of the pressure. At lower pressures, the sensitiveness diminishes quite rapidly.

6. Haber and Kerschbaum have used vibrating quartz fibers to measure the pressure of mercury and iodine.[4] This method was originally suggested by I. Langmuir[5] for measuring the residual gas pressure in sealed-off tungsten lamps and has been in use in this laboratory for about three or four years. As the pressure decreases the duration of the oscillations increases. Haber and Kerschbaum have deduced a relation between the pressure p and the interval t in which the vibration decreases to half its original amplitude, as follows:

$$\Sigma(p\sqrt{M}) + a = \frac{b}{t},$$

where a and b are constants and Σ denotes that the sum of the products of partial pressure and square root of the molecular weight is to be taken for each gas present. The lowest pressures actually measured by the above authors were about 0.015 bar, but the method has been used in this laboratory by Dr. Fonda to measure pressures considerably smaller than this.

[1] Ber. d. deutsch. physikal. Ges., 1906, 686.
[2] Trans. Am. Electrochem. Soc., *20*, 243 (1911).
[3] Z. f. Elektrochem., *20*, 539 (1914).
[4] Z. f. Elektrochem., *20*, 296, 1914.
[5] J. Am. Chem. Soc., *35*, 107, 1913.

7. W. Sutherland[1] and, subsequently, J. L. Hogg[2] derived simple relations between the pressure and the logarithmic decrement of a vibrating mica disc. The gauge based on this principle requires calibration at two known pressures, and is not very sensitive below about 0.01 bar.

8. M. Knudsen as the result of an elaborate study of the laws of "Molekularströmung" devised an absolute form of manometer[3] which depends upon the fact that at very high vacua there exists a very simple relation between the pressure and the torque imparted to a movable surface by the molecules flowing to it from a hotter surface. This relation has the form

$$K = \frac{p}{2}\left(\sqrt{\frac{T_1}{T_2}} - 1\right),$$

where K denotes the force of repulsion between two surfaces maintained at temperatures T_1 and T_2 respectively in a gas at pressure p. From the dimensions of the movable disc and the period of oscillation of the suspension, the value of K may be calculated and the gauge may therefore be used without any previous calibration. Knudsen uses this guage to indicate pressures as low as 2×10^{-3} bar.

9. Still more recently[4] Knudsen has devised a simplified form of vacuum guage, based on the same principles as the above, which he states to be sensitive to 2×10^{-4} bar.

Concluding Remarks.

The vacuum gauge described above might obviously be used to determine the magnitude of the "accommodation coefficient" for different gases, and thus test out the deductions advanced by different investigators.

Another line of investigation for which the gauge would be useful is the determination of the vapor tension of oils, waxes, etc., such as are used in connection with vacuum work. Owing to pressure of other work the writer has been prevented till now from carrying on such an investigation; but the results would be of great practical utility, as these materials are being constantly used by experimenters in connection with so-called "high vacuum" experiments.

In conclusion the author desires to express his appreciation of the

[1] Phil. Mag., *43*, 83 (1897).

[2] Proc. Am. Acad., *42*, 6 (1906); Phil. Mag., *19* (1906); Proc. Am. Acad., *45*, No. 1, Aug., 1909.

[3] Am. Physik *32*, 809 (1910).

[4] Am. Physik *44*, 525 (1914).

kindly interest shown by Dr. I. Langmuir during the progress of the investigation and for helpful suggestions.

SUMMARY.

A theoretical consideration of the behavior of gases at very low pressures shows that a rotating disc exerts a torque on a disc suspended symmetrically above it, that is proportional to the quantity $\Sigma(p\sqrt{M/RT})$. Here p denotes the partial pressure and M the molecular weight of each constituent present in the gas and R and T have their usual signification.

The paper contains the description of a vacuum gauge based upon this principle, and also the results of a number of measurements carried out with its aid.

It was found that in order to obtain the best possible results with a Gaede molecular pump, it is necessary not only to heat the vessel to be exhausted and connecting tubing to a temperature at which most of the moisture adsorbed in the walls is driven out, but also to insert a liquid air trap to prevent the diffusion backwards of condensible vapors.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.